Exercise 13(A) Page No: 177

1. Calculate the co-ordinates of the point P which divides the line segment joining:
(i) A (1, 3) and B (5, 9) in the ratio 1: 2.
(ii) A (-4, 6) and B (3, -5) in the ratio 3: 2.
Solution:

(i) Let's assume the co-ordinates of the point P be (x, y)
Then by section formula, we have
$P(x, y) = (m_1x_2 + m_2x_1)/ (m_1 + m_2), (m_1y_2 + m_2y_1)/ (m_1 + m_2)$

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1\times5+2\times1}{1+2} = \frac{7}{3}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\times9+2\times3}{1+2} = \frac{15}{3} = 5$$

Hence, the co-ordinates of point P are (7/3, 5).

(ii) Let's assume the co-ordinates of the point P be (x, y)
Then by section formula, we have
$P(x, y) = (m_1x_2 + m_2x_1)/ (m_1 + m_2), (m_1y_2 + m_2y_1)/ (m_1 + m_2)$

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{3\times3+2\times(-4)}{3+2} = \frac{1}{5}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{3\times(-5)+2\times6}{3+2} = \frac{-3}{5}$$

Hence, the co-ordinates of point P are (1/5, -3/5).

2. In what ratio is the line joining (2, -3) and (5, 6) divided by the x-axis.
Solution:

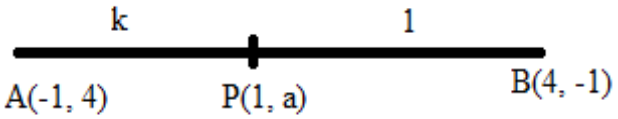
Let's assume the joining points as A(2, -3) and B(5, 6) be divided by point P(x ,0) in the ratio k: 1.
Then we have,
$y = ky_2 + y_1/ (k + 1)$
$0 = 6k + (-3)/ (k + 1)$
$0 = 6k – 3$
$k = ½$
Hence, the required ratio is 1: 2.

3. In what ratio is the line joining (2, -4) and (-3, 6) divided by the y-axis.
Solution:

Let's assume the line joining points A(2, -4) and B(-3, 6) be divided by point P (0, y) in the ratio k: 1.
Then we have,
$x = kx_2 + x_1/ (k + 1)$
$0 = k(-3) + (1x2)/ (k + 1)$
$0 = -3k + 2$
$k = 2/3$

Hence, the required ratio is 2: 3.

4. In what ratio does the point (1, a) divided the join of (-1, 4) and (4, -1)? Also, find the value of a.
Solution:



Let's assume the point P (1, a) divide the line segment AB in the ratio k: 1.
Then by section formula, we have
1 = (4k - 1)/ (k + 1),
k + 1 = 4k – 1
2 = 3k
k = 2/3 ….. (1)
And,
a = (-k + 4)/ (k + 1)
a = (-(2/3) + 4)/ ((2/3) + 1) [using (1)]
a = 10/5 = 2
Thus, the required ratio is 2: 3 and a = 2.

5. In what ratio does the point (a, 6) divide the join of (-4, 3) and (2, 8)? Also, find the value of a.
Solution:

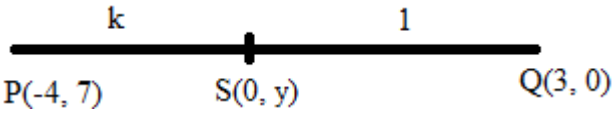
Let's assume the point P (a, 6) divides the line segment joining A (-4, 3) and B (2, 8) in the ratio k: 1.
Then by section formula, we have
6 = (8k + 3)/ (k + 1),
6k + 6 = 8k + 3
3 = 3k
k = 3/2 …. (1)
a = (2k – 4)/ (k + 1)
a = (2(3/2) – 4)/ ((3/2) + 1)
a = -2/5
Thus, the required ratio is 3: 2 and a = -2/5

6. In what ratio is the join of (4, 3) and (2, -6) divided by the x-axis. Also, find the co-ordinates of the point of intersection.
Solution:

Let's assume the point P (x, 0) on x-axis divides the line segment joining A (4, 3) and B (2, -6) in the ratio k: 1.
Then by section formula, we have
0 = (-6k + 3)/ (k + 1)
0 = -6k + 3
k = ½
Hence, the required ratio is 1: 2

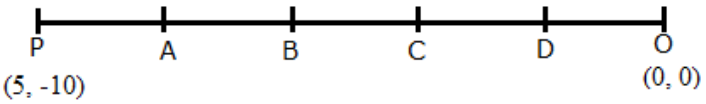
And,
x = (2k + 4)/ (k + 1)
= {2(1/2) + 4}/ {k + 1}
= 10/3
Therefore, the required co-ordinates of the point of intersection are (10/3, 0).

7. Find the ratio in which the join of (-4, 7) and (3, 0) is divided by the y-axis. Also, find the coordinates of the point of intersection.
Solution:



Let's assume S (0, y) be the point on y-axis which divides the line segment PQ in the ratio k: 1.
Then by section formula, we have
0 = (3k – 4)/ (k + 1)
3k = 4
k = 4/3 … (1)
y = (0 + 7)/ (k + 1)
y = 7/ (4/3 + 1) [From (1)]
y = 3
Thus, the required ratio is 4: 3 and the required point is S(0, 3).

8. Points A, B, C and D divide the line segment joining the point (5, -10) and the origin in five equal parts. Find the co-ordinates of A, B, C and D.
Solution:



Point A divides PO in the ratio 1: 4.
So, the co-ordinates of point A are

$$\left(\frac{1\times0+4\times5}{1+4},\frac{1\times0+4\times(-10)}{1+4}\right)=\left(\frac{20}{5},\frac{-40}{5}\right)=(4,-8)$$

Next, point B divides PO in the ratio 2: 3.
So, co-ordinates of point B are

$$\left(\frac{2\times0+3\times5}{2+3},\frac{2\times0+3\times(-10)}{2+3}\right)=\left(\frac{15}{5},\frac{-30}{5}\right)=(3,-6)$$

And, point C divides PO in the ratio 3: 2.
Co-ordinates of point C are

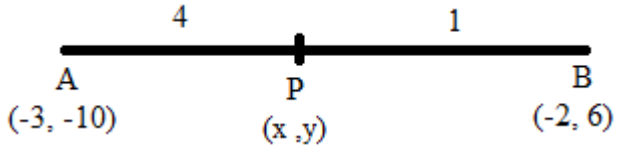
$$\left(\frac{3\times0+2\times5}{3+2},\frac{3\times0+2\times(-10)}{3+2}\right)=\left(\frac{10}{5},\frac{-20}{5}\right)=(2,-4)$$

Lastly, point D divides PO in the ratio 4: 1.
Co-ordinates of point D are

$$\left(\frac{4\times0+1\times5}{4+1},\frac{4\times0+1\times(-10)}{4+1}\right)=\left(\frac{5}{5},\frac{-10}{5}\right)=(1,-2)$$

9. The line joining the points A (-3, -10) and B (-2, 6) is divided by the point P such that PB/AB = 1/5 Find the co-ordinates of P.
Solution:

Let the coordinates of point P be taken as (x, y).



Given,
PB: AB = 1: 5
So, PB: PA = 1: 4
Hence, the coordinates of P are

$$(x, y) = \left(\frac{4\times(-2)+1\times(-3)}{5}, \frac{4\times6+1\times(-10)}{5}\right) = \left(-\frac{11}{5}, \frac{14}{5}\right)$$

10. P is a point on the line joining A (4, 3) and B (-2, 6) such that 5AP = 2BP. Find the co-ordinates of P.
Solution:

5AP = 2BP
So, AP/BP = 2/5
Hence, the co-ordinates of the point P are
((2x(-2) + 5x4)/(2 + 5), (2x6 + 5x3)/ (2 + 5))
(16/7, 27/7)

11. Calculate the ratio in which the line joining the points (-3, -1) and (5, 7) is divided by the line x = 2. Also, find the co-ordinates of the point of intersection.
Solution:

We know that,
The co-ordinates of every point on the line x = 2 will be of the type (2, y).
So from section formula, we have
$x = m_1 \times 5 + m_2 \times (-3)/ (m_1 + m_2)$
$2 = 5m_1 – 3m_2/ m_1 + m_2$
$2m_1 + 2m_2 = 5m_1 – 3m_2$
$5m_2 = 3m_1$
Hence, the required ratio is 5: 3.
$y = (m_1x7 + m_2x(-1))/ (m_1 + m_2)$
y = 5x7 + 3(-1)/ 5 + 3
y = 35 – 3/ 8
y = 32/8 = 4
Therefore, the required co-ordinates of the point of intersection are (2, 4).

12. Calculate the ratio in which the line joining A (6, 5) and B (4, -3) is divided by the line y = 2.
Solution:

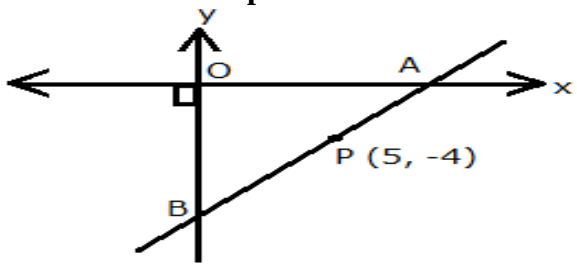
We know that,
The co-ordinates of every point on the line y = 2 will be of the type (x, 2).
So, by section formula, we have

$$y = \frac{m_1 \times (-3) + m_2 \times 5}{m_1 + m_2}$$

$$2 = \frac{-3m_1 + 5m_2}{m_1 + m_2}$$

$2m_1 + 2m_2 = -3m_1 + 5m_2$
$5m_1 = 3m_2$
$m_1/ m_2 = 3/5$
Hence, the required ratio is 3: 5.

13. The point P (5, -4) divides the line segment AB, as shown in the figure, in the ratio 2: 5. Find the co-ordinates of points A and B. Given AP is smaller than BP



Solution:

From the diagram we can see that,
Point A lies on x-axis. So, its co-ordinates can be taken as A (x, 0).
Point B lies on y-axis. So, its co-ordinates can be taken as B be (0, y).
And, P divides AB in the ratio 2: 5. (Given)
Now, we have

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$5 = \frac{2 \times 0 + 5 \times x}{2 + 5}$$

5 = 5x/7
x = 7
Hence, the co-ordinates of point A are (7, 0).

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$-4 = \frac{2 \times y + 5 \times 0}{2 + 5}$$

-4 = 2y/7
-2 = y/7
y = -14
Hence, the co-ordinates of point B are (0, -14)