

Exercise 13(B)

1. Find the mid-point of the line segment joining the points:

(i) (-6, 7) and (3, 5)

(ii) (5, -3) and (-1, 7)

Solution:

(i) Let A (-6, 7) and B (3, 5)

So, the mid-point of AB = $(-6+3/2, 7+5/2) = (-3/2, 6)$

(ii) Let A (5, -3) and B (-1, 7)

So, the mid-point of AB = $(5-1/2, -3+7/2) = (2, 2)$

2. Points A and B have co-ordinates (3, 5) and (x, y) respectively. The mid-point of AB is (2, 3). Find the values of x and y.

Solution:

Given, mid-point of AB = (2, 3)

Thus,

$$(3+x/2, 5+y/2) = (2, 3)$$

$$3 + x/2 = 2 \quad \text{and} \quad 5 + y/2 = 3$$

$$3 + x = 4 \quad \text{and} \quad 5 + y = 6$$

$$x = 1 \quad \text{and} \quad y = 1$$

3. A (5, 3), B (-1, 1) and C (7, -3) are the vertices of triangle ABC. If L is the mid-point of AB and M is the mid-point of AC, show that $LM = \frac{1}{2} BC$.

Solution:

It's given that, L is the mid-point of AB and M is the mid-point of AC.

Co-ordinates of L are,

$$\left(\frac{5-1}{2}, \frac{3+1}{2}\right) = (2, 2)$$

Co-ordinates of M are,

$$\left(\frac{5+7}{2}, \frac{3-3}{2}\right) = (6, 0)$$

Using distance formula, we have:

$$BC = \sqrt{(7+1)^2 + (-3-1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$LM = \sqrt{(6-2)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Thus, $LM = \frac{1}{2} BC$

4. Given M is the mid-point of AB, find the co-ordinates of:

(i) A; if M = (1, 7) and B = (-5, 10)

(ii) B; if A = (3, -1) and M = (-1, 3).

Solution:

(i) Let's assume the co-ordinates of A to be (x, y).

$$\text{So, } (1, 7) = (x-5/2, y+10/2)$$

$$1 = x-5/2 \quad \text{and} \quad 7 = y+10/2$$

$$2 = x - 5 \quad \text{and} \quad 14 = y + 10$$

$$x = 7 \quad \text{and} \quad y = 4$$

Thus, the co-ordinates of A are (7, 4).

(ii) Let's assume the co-ordinates of B be (x, y).

$$\text{So, } (-1, 3) = (3+x/2, -1+y/2)$$

$$-1 = 3+x/2 \quad \text{and} \quad 3 = -1+y/2$$

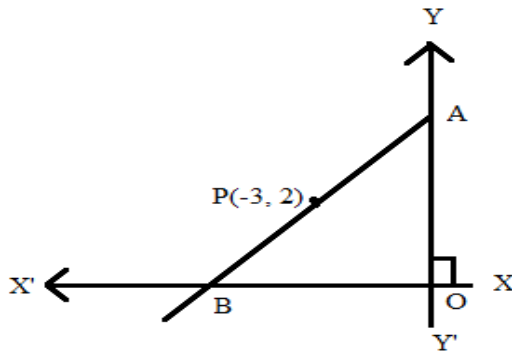
$$-2 = 3 + x \quad \text{and} \quad 6 = -1 + y$$

$$x = -5 \quad \text{and} \quad y = 7$$

Thus, the co-ordinates of B are (-5, 7).

5. P (-3, 2) is the mid-point of line segment AB as shown in the given figure. Find the co-ordinates of points A and B.

Solution:



It's seen that,

Point A lies on y-axis, hence its co-ordinates is taken to be (0, y).

Point B lies on x-axis, hence its co-ordinates is taken to be (x, 0).

Given, P (-3, 2) is the mid-point of line segment AB.

So, by the mid-point section we have

$$(-3, 2) = (0+x/2, y+0/2)$$

$$(-3, 2) = (x/2, y/2)$$

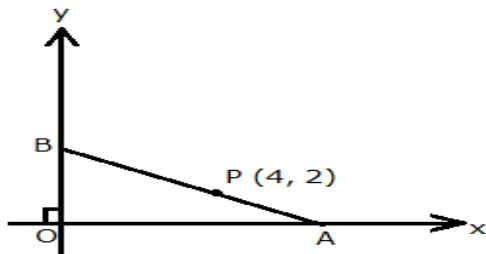
$$-3 = x/2 \quad \text{and} \quad 2 = y/2$$

$$x = -6 \quad \text{and} \quad y = 4$$

Therefore, the co-ordinates of points A and B are (0, 4) and (-6, 0) respectively.

6. In the given figure, P (4, 2) is mid-point of line segment AB. Find the co-ordinates of A and B.

Solution:



Let point A lies on x-axis, hence its co-ordinates can be $(x, 0)$.

And, Point B lies on y-axis, hence its co-ordinates can be $(0, y)$.

Given, $P(4, 2)$ is the mid-point of line segment AB.

So,

$$(4, 2) = (x+0/2, 0+y/2)$$

$$4 = x/2 \quad \text{and} \quad 2 = y/2$$

$$8 = x \quad \text{and} \quad 4 = y$$

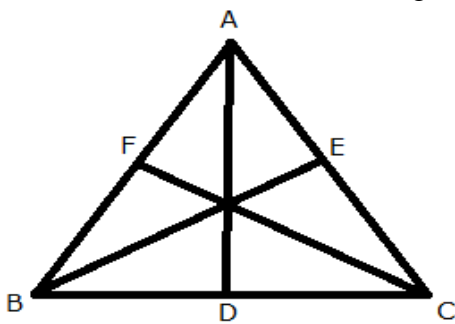
Thus, the co-ordinates of points A and B are $(8, 0)$ and $(0, 4)$ respectively.

7. $(-5, 2)$, $(3, -6)$ and $(7, 4)$ are the vertices of a triangle. Find the length of its median through the vertex $(3, -6)$.

Solution:

Let A $(-5, 2)$, B $(3, -6)$ and C $(7, 4)$ be the vertices of the given triangle.

And let AD be the median through A, BE be the median through B and CF be the median through C.



We know that, median of a triangle bisects the opposite side.

So, the co-ordinates of point F are

$$\left(\frac{-5+3}{2}, \frac{2-6}{2}\right) = \left(\frac{-2}{2}, \frac{-4}{2}\right) = (-1, -2)$$

The co-ordinates of point D are

$$\left(\frac{3+7}{2}, \frac{-6+4}{2}\right) = \left(\frac{10}{2}, \frac{-2}{2}\right) = (5, -1)$$

And, co-ordinates of point E are

$$\left(\frac{-5+7}{2}, \frac{2+4}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$

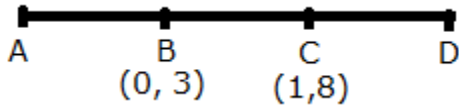
The median of the triangle through the vertex B $(3, -6)$ is BE.

Therefore, by distance formula we get

$$BE = \sqrt{(1-3)^2 + (3+6)^2} = \sqrt{4+81} = \sqrt{85} = 9.22$$

8. Given a line ABCD in which AB = BC = CD, B = (0, 3) and C = (1, 8). Find the co-ordinates of A and D.

Solution:



Given, AB = BC = CD

So, B is the mid-point of AC. Let the co-ordinates of point A be (x, y).

$$(0, 3) = (x+1/2, y+8/2)$$

$$0 = (x+1)/2 \quad \text{and} \quad 3 = (y+8)/2$$

$$0 = x + 1 \quad \text{and} \quad 6 = y + 8$$

$$x = -1 \quad \text{and} \quad y = -2$$

Hence, the co-ordinates of point A are (-1, -2).

Also given, C is the mid-point of BD. And, let the co-ordinates of point D be (p, q).

$$(1, 8) = (0+p/2, 3+q/2)$$

$$1 = 0+p/2 \quad \text{and} \quad 8 = 3+q/2$$

$$2 = 0 + p \quad \text{and} \quad 16 = 3 + q$$

$$p = 2 \quad \text{and} \quad q = 13$$

Hence, the co-ordinates of point D are (2, 13).

9. One end of the diameter of a circle is (-2, 5). Find the co-ordinates of the other end of it, if the centre of the circle is (2, -1).

Solution:

We know that,

The centre is the mid-point of any diameter of a circle.

Let assume the required co-ordinates of the other end of mid-point to be (x, y).

$$\therefore (2, -1) = \left(\frac{-2+x}{2}, \frac{5+y}{2} \right)$$

$$\Rightarrow 2 = \frac{-2+x}{2} \quad \text{and} \quad -1 = \frac{5+y}{2}$$

$$\Rightarrow 4 = -2+x \quad \text{and} \quad -2 = 5+y$$

$$\Rightarrow 6 = x \quad \text{and} \quad -7 = y$$

$$2 = (-2+x)/2 \quad \text{and} \quad -1 = (5+y)/2$$

$$4 = -2+x \quad \text{and} \quad -2 = 5+y$$

$$x = 6 \quad \text{and} \quad y = -7$$

Therefore, the required co-ordinates of the other end of the diameter are (6, -7).