Exercise 13(C)

1. Given a triangle ABC in which A = (4, -4), B = (0, 5) and C = (5, 10). A point P lies on BC such that BP: PC = 3: 2. Find the length of line segment AP.
Solution:

Given, BP: PC = 3: 2
Then by section formula, the co-ordinates of point P are given as:
$$\left(\frac{3\times5+2\times0}{3+2}, \frac{3\times10+2\times5}{3+2}\right)$$
= (15/5, 40/5)
= (3, 8)
Now, by using distance formula, we get
$$AP = \sqrt{(3-4)^2+(8+4)^2} = \sqrt{1+144} = \sqrt{145} = 12.04$$

2. A (20, 0) and B (10, -20) are two fixed points. Find the co-ordinates of a point P in AB such that: 3PB = AB. Also, find the co-ordinates of some other point Q in AB such that AB = 6AQ.
Solution:

Given, 3PB = AB
So,
AB/PB = 3/1
(AB - PB)/ PB = (3 - 1)/ 1
AP/PB = 2/1
By section formula, we get the coordinates of P to be

$$P(x, y) = P\left(\frac{2\times10+1\times20}{2+1}, \frac{2\times(-20)+1\times0}{2+1}\right)$$
= P (40/3, -40/3)
Also given that, AB = 6AQ
AQ/AB = 1/6
AQ/(AB - AQ) = 1/(6 - 1)
AQ/ QB = 1/5
Now, again by using section formula we get
The coordinates of Q as
$$Q(x, y) = Q\left(\frac{1\times10+5\times20}{1+5}, \frac{1\times(-20)+5\times0}{1+5}\right)$$
= Q(110/6, -20/6)
= Q(55/3, -10/3)

3. A (-8, 0), B (0, 16) and C (0, 0) are the vertices of a triangle ABC. Point P lies on AB and Q lies on AC such that AP: PB = 3: 5 and AQ: QC = 3: 5. Show that: PQ = 3/8 BC.
Solution:

Given that, point P lies on AB such that AP: PB = 3: 5.

So, the co-ordinates of point P are given as

$$\left(\frac{3\times 0+5\times(-8)}{3+5},\frac{3\times 16+5\times 0}{3+5}\right)$$

= (-40/8, 48/8)
= (-5, 6)
Also given that, point Q lies on AB such that AQ: QC = 3: 5.
So, the co-ordinates of point Q are given as

$$\left(\frac{3\times 0+5\times(-8)}{3+5},\frac{3\times 0+5\times 0}{3+5}\right)$$

= (-40/8, 0/8)
= (-5, 0)
Now, by distance formula we get

$$PQ = \sqrt{(-5+5)^2+(0-6)^2} = \sqrt{0+36} = 6$$

$$BC = \sqrt{(0-0)^2+(0-16)^2} = \sqrt{0+(16)^2} = 16$$

Thus,
3/8 x BC = 3/8 x 16 = 6 = PQ
- Hence proved.

4. Find the co-ordinates of points of trisection of the line segment joining the point (6, -9) and the origin.
Solution:

Let's assume P and Q to be the points of trisection of the line segment joining A (6, -9) and B (0, 0).
So, P divides AB in the ratio 1: 2.
Hence, the co-ordinates of point P are given as

$$\left(\frac{1\times 0+2\times 6}{1+2},\frac{1\times 0+2\times(-9)}{1+2}\right)$$

= (12/3, -18/3)
= (4, -6)
And, Q divides AB in the ratio 2: 1.
Hence, the co-ordinates of point Q are

$$\left(\frac{2\times 0+1\times 6}{2+1},\frac{2\times 0+1\times(-9)}{2+1}\right)$$

= (6/3, -9/3) = (2, 3)
Therefore, the required coordinates of trisection of PQ are (4, -6) and (2, -3).

5. A line segment joining A (-1, 5/3) and B (a, 5) is divided in the ratio 1: 3 at P, point where the line segment AB intersects the y-axis.
(i) Calculate the value of 'a'.
(ii) Calculate the co-ordinates of 'P'.
Solution:

As, the line segment AB intersects the y-axis at point P, let the co-ordinates of point P be taken as (0, y).

And, P divides AB in the ratio 1: 3.
So,

$$(0, y) = \left(\frac{1 \times a + 3 \times (-1)}{1+3}, \frac{1 \times 5 + 3 \times \frac{5}{3}}{1+3}\right)$$

(0, y) = (a-3/4, 10/4)
0 = a-3/4 and y = 10/4
a -3 = 0 and y = 5/2
a = 3
Therefore, the value of a is 3 and the co-ordinates of point P are (0, 5/2).

6. In what ratio is the line joining A (0, 3) and B (4, -1) divided by the x-axis? Write the co-ordinates of the point where AB intersects the x-axis.
Solution:

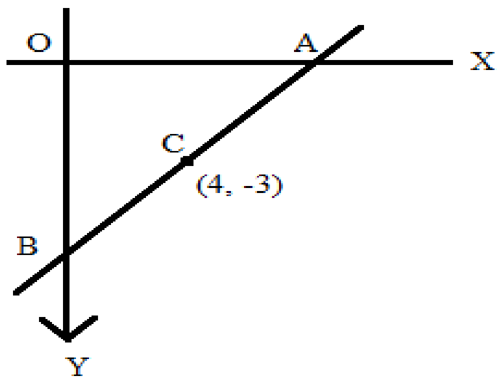
Let assume that the line segment AB intersects the x-axis by point P (x, 0) in the ratio k: 1.

$$\therefore (x, 0) = \left(\frac{k \times 4 + 1 \times 0}{k+1}, \frac{k \times (-1) + 1 \times 3}{k+1}\right)$$

$$(x, 0) = \left(\frac{4k}{k+1}, \frac{-k+3}{k+1}\right)$$

0 = (-k + 3)/ (k + 1)
k = 3
Therefore, the required ratio in which P divides AB is 3: 1.
Also,
x = 4k/(k + 1)
x = (4x3)/ (3 + 1)
x = 12/3 = 3
Hence, the co-ordinates of point P are (3, 0).

7. The mid-point of the segment AB, as shown in diagram, is C (4, -3). Write down the co-ordinates of A and B.



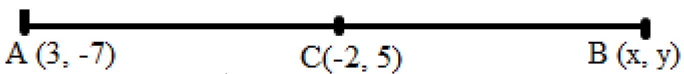
Solution:

As, point A lies on x-axis, we can assume the co-ordinates of point A to be (x, 0).
As, point B lies on y-axis, we can assume the co-ordinates of point B to be (0, y).
And given, the mid-point of AB is C (4, -3).

$$\therefore (4, -3) = \left(\frac{x+0}{2}, \frac{0+y}{2}\right)$$

(4, -3) = (x/2, y/2)
4 = x/2 and -3 = y/2
x = 8 and y = -6
Therefore, the co-ordinates of point A are (8, 0) and the co-ordinates of point B are (0, -6).

8. AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7), find
(i) the length of radius AC
(ii) the coordinates of B.
Solution:



(i) Radius AC = $\sqrt{[(3 + 2)^2 + (-7 - 5)^2]}$
$= \sqrt{[(5^2 + (-12)^2]}$
$= \sqrt{(25 + 144)}$
$= \sqrt{169} = 13$ units

(ii) Let the coordinates of B be (x, y).
Now, by mid-point formula, we get
-2 = (3 + x)/2 and 5 = (-7 + y)/2
-4 = 3 + x and 10 = -7 + y
x = -7 and y = 17
Hence, the coordinates of B are (-7, 17).

9. Find the co-ordinates of the centroid of a triangle ABC whose vertices are:
A (-1, 3), B (1, -1) and C (5, 1)
Solution:

By the centroid of a triangle formula, we get
The co- ordinates of the centroid of triangle ABC as

$$\left(\frac{-1+1+5}{3}, \frac{3-1+1}{3}\right)$$

= (5/3, 1)

10. The mid-point of the line-segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a). Find the values of a and b.
Solution:

Given that the mid-point of the line-segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a).
So, we have

2 = (4a - 4)/2
4 = 4a – 4
8 = 4a
a = 2
Also,
-2a = (2b – 3 + 3b)/ 2
-2 x 2 = (5b – 3)/ 2
-8 = 5b – 3
-5 = 5b
b = -1

11. The mid-point of the line segment joining (2a, 4) and (-2, 2b) is (1, 2a + 1). Find the value of a and b.
Solution:

Given,
The mid-point of (2a, 4) and (-2, 2b) is (1, 2a + 1)
So, by using mid-point formula, we know
(1, 2a + 1) = (2a – 2/2, 4 + 2b/2)
1 = 2a – 2/2 and 2a + 1 = 4 + 2b/2
2 = 2a – 2 and 4a + 2 = 4 + 2b
4 = 2a and 4a = 2 + 2b
a = 2 and 4(2) = 2 + 2b [using the value of a]
8 = 2 + 2b
6 = 2b
b = 3

Hence, the value of a = 2 and b = 3.