Exercise 14(A)

1. Find, which of the following points lie on the line x - 2y + 5 = 0:

(i) (1, 3) (ii) (0, 5)

(iii) (-5, 0) (iv) (5, 5)

(v) (2, -1.5) (vi) (-2, -1.5)

Solution:

Given line equation is $x - 2y + 5 = 0$.

(i) On substituting x = 1 and y = 3 in the given line equation, we have
L.H.S. = 1 - 2(3) + 5 = 1 - 6 + 5 = 6 - 6 = 0 = R.H.S.
Hence, the point (1, 3) lies on the given line.

(ii) On substituting x = 0 and y = 5 in the given line equation, we have
L.H.S. = 0 – 2(5) + 5 = -10 + 5 = -5 ≠ R.H.S.
Hence, the point (0, 5) does not lie on the given line.

(iii) On substituting x = -5 and y = 0 in the given line equation, we have
L.H.S. = -5 – 2(0) + 5 = -5 - 0 + 5 = 5 - 5 = 0 = R.H.S.
Hence, the point (-5, 0) lie on the given line.

(iv) On substituting x = 5 and y = 5 in the given line equation, we have
L.H.S. = 5 – 2(5) + 5 = 5 - 10 + 5 = 10 - 10 = 0 = R.H.S.
Hence, the point (5, 5) lies on the given line.

(v) On substituting x = 2 and y = -1.5 in the given line equation, we have
L.H.S. = 2 - 2(-1.5) + 5 = 2 + 3 + 5 = 10 ≠ R.H.S.
Hence, the point (2, -1.5) does not lie on the given line.

(vi) On substituting x = -2 and y = -1.5 in the given line equation, we have
L.H.S. = -2 - 2(-1.5) + 5 = -2 + 3 + 5 = 6 ≠ R.H.S.
Hence, the point (-2, -1.5) does not lie on the given line.

2. State, true or false:

(i) the line x/2 + y/3 = 0 passes through the point (2, 3).

(ii) the line x/2 + y/3 = 0 passes through the point (4, -6).

(iii) the point (8, 7) lies on the line y - 7 = 0.

(iv) the point (-3, 0) lies on the line x + 3 = 0.

(v) if the point (2, a) lies on the line 2x - y = 3, then a = 5.

Solution:

(i) The given line is $x/2 + y/3 = 0$
Substituting x = 2 and y = 3 in the given equation,
L.H.S = 2/2 + 3/3 = 1 + 1 = 2 ≠ R.H.S
Hence, the given statement is false.

(ii) The given line is $x/2 + y/3 = 0$
Substituting $x = 4$ and $y = -6$ in the given equation,
L.H.S = $4/2 + (-6)/3 = 2 – 2 =$ R.H.S
Hence, the given statement is true.

(iii) The given line is $y - 7 = 0$
Substituting $y = 7$ in the given equation,
L.H.S = $y - 7 = 7 - 7 = 0$ = R.H.S.
Hence, the given statement is true.

(iv) The given line is $x + 3 = 0$
Substituting $x = -3$ in the given equation,
L.H.S. = $x + 3 = -3 + 3 = 0$ = R.H.S
Hence, the given statement is true.

(v) The point (2, a) lies on the line $2x - y = 3$.
So, $2(2) - a = 3$
$4 - a = 3$
$a = 4 - 3 = 1$
Hence, the given statement is false.

3. The line given by the equation $2x – y/3 = 7$ passes through the point (k, 6); calculate the value of k.
Solution:

Given line equation is $2x – y/3 = 7$ passes through the point (k, 6).
So, on substituting $x = k$ and $y = 6$ in the given equation, we have
$2k – 6/3 = 7$
$6k – 6 = 21$
$6k = 27$
$k = 27/6 = 9/2$
$k = 4.5$

4. For what value of k will the point (3, -k) lie on the line $9x + 4y = 3$?
Solution:

The given line equation is $9x + 4y = 3$.
On putting $x = 3$ and $y = -k$, we have
$9(3) + 4(-k) = 3$
$27 - 4k = 3$
$4k = 27 - 3 = 24$
$k = 6$

5. The line $3x/5 – 2y/3 + 1 = 0$ contains the point (m, 2m - 1); calculate the value of m.
Solution:

The equation of the given line is 3x/5 – 2y/3 + 1 = 0
On putting x = m, y = 2m - 1, we have

$$\frac{3m}{5} - \frac{2(2m-1)}{3} + 1 = 0$$

$$\frac{3m}{5} - \frac{4m-2}{3} = -1$$

$$\frac{9m - 20m + 10}{15} = -1$$

9m – 20m + 10 = -15
-11m = -25
m = 25/11
m = $2\frac{3}{11}$

6. Does the line 3x - 5y = 6 bisect the join of (5, -2) and (-1, 2)?
Solution:

It's known that the given line will bisect the join of A (5, -2) and B (-1, 2), if the co-ordinates of the mid-point of AB satisfy the line equation.
The co-ordinates of the mid-point of AB are
(5-1/2, -2+2/2) = (2, 0)
On substituting x = 2 and y = 0 in the given line equation, we have
L.H.S. = 3x - 5y = 3(2) - 5(0) = 6 - 0 = 6 = R.H.S.
Therefore, the line 3x - 5y = 6 bisect the join of (5, -2) and (-1, 2).

Exercise 14(B) Page No: 190

1. Find the slope of the line whose inclination is:
(i) 0^o (ii) 30^o
(iii) 72^o 30' (iv) 46^o
Solution:

We know that, the slope of a line is given by the tan of its inclination.
(i) Slope = tan 0^o = 0
(ii) Slope = tan 30^o = $1/\sqrt{3}$
(iii) Slope = tan 72^o 30' = 3.1716
(iv) Slope = tan 46^o = 1.0355

2. Find the inclination of the line whose slope is:
(i) 0 (ii) $\sqrt{3}$
(iii) 0.7646 (iv) 1.0875
Solution:

(i) Slope = tan θ = 0
⇒ θ = 0^o
(ii) Slope = tan θ = $\sqrt{3}$
⇒ θ = 60^o
(iii) Slope = tan θ = 0.7646
⇒ θ = 37^o 24'
(iv) Slope = tan θ = 1.0875
⇒ θ = 47^o 24'

3. Find the slope of the line passing through the following pairs of points:
(i) (-2, -3) and (1, 2)
(ii) (-4, 0) and origin
(iii) (a, -b) and (b, -a)
Solution:

We know that,
Slope = $(y_2 - y_1)/ (x_2 - x_1)$
(i) Slope = (2 + 3)/ (1 + 2) = 5/3
(ii) Slope = (0 - 0)/ (0 + 4) = 0
(iii) Slope = (-a + b)/ (b - a) = 1

4. Find the slope of the line parallel to AB if:
(i) A = (-2, 4) and B = (0, 6)
(ii) A = (0, -3) and B = (-2, 5)
Solution:

(i) Slope of AB = (6 - 4)/ (0 + 2) = 2/2 = 1
Hence, the slope of the line parallel to AB should be = Slope of AB = 1

(ii) Slope of AB = (5 + 3)/ (-2 - 0) = 8/-2 = -4
Hence, the slope of the line parallel to AB should be = Slope of AB = -4

5. Find the slope of the line perpendicular to AB if:
(i) A = (0, -5) and B = (-2, 4)
(ii) A = (3, -2) and B = (-1, 2)
Solution:

(i) Slope of AB = (4 + 5)/ (-2 - 0) = -9/2
Slope of the line perpendicular to AB = -1/Slope of AB = -1/(-9/2) = 2/9
(ii) Slope of AB = (2 + 2)/ (-1 - 3) = 4/-4 = -1
Slope of the line perpendicular to AB = -1/slope of AB = -1/-1 = 1

6. The line passing through (0, 2) and (-3, -1) is parallel to the line passing through (-1, 5) and (4, a). Find a.
Solution:

Slope of the line passing through (0, 2) and (-3, -1) = (-1 - 2)/ (-3 - 0) = -3/-3 = 1
Slope of the line passing through (-1, 5) and (4, a) = (a - 5)/ (4 + 1) = (a -5)/ 5
As, the lines are parallel.
The slopes must be equal.
1 = (a - 5)/ 5
a – 5 = 5
a = 10

7. The line passing through (-4, -2) and (2, -3) is perpendicular to the line passing through (a, 5) and (2, -1). Find a.
Solution:
Slope of the line passing through (-4, -2) and (2, -3) = (-3 + 2)/ (2 + 4) = -1/6
Slope of the line passing through (a, 5) and (2, -1) = (-1 – 5)/ (2 - a) = -6/ (2 –a)
As, the lines are perpendicular we have
-1/6 = -1/(-6/ 2 – a)
-1/6 = (2 - a)/6
2 – a = -1
a = 3

8. Without using the distance formula, show that the points A (4, -2), B (-4, 4) and C (10, 6) are the vertices of a right-angled triangle.
Solution:

Given points are A (4, -2), B (-4, 4) and C (10, 6).
Calculating the slopes, we have
Slope of AB = (4 + 2)/ (-4 - 4) = 6/ -8 = -3/4
Slope of BC = (6 – 4)/ (10 + 4) = 2/ 14 = 1/7
Slope of AC = (6 + 2)/ (10 - 4) = 8/ 6 = 4/3
It’s clearly seen that,

Slope of AB = -1/ Slope of AC
Thus, AB ⊥ AC.
Therefore, the given points are the vertices of a right-angled triangle.

9. Without using the distance formula, show that the points A (4, 5), B (1, 2), C (4, 3) and D (7, 6) are the vertices of a parallelogram.
Solution:

Given points are A (4, 5), B (1, 2), C (4, 3) and D (7, 6).
Slope of AB = (2 - 5)/ (1 - 4) = -3/ -3 = 1
Slope of CD = (6 - 3)/ (7 - 4) = 3/3 = 1
As the slope of AB = slope of CD
So, we can conclude AB || CD
Now,
Slope of BC = (3 - 2)/ (4 - 1) = 1/3
Slope of DA = (5 – 6)/ (4 - 7) = -1/-3 = 1/3
As, slope of BC = slope of DA
Hence, we can say BC || DA
Therefore, ABCD is a parallelogram.

10. (-2, 4), (4, 8), (10, 7) and (11, -5) are the vertices of a quadrilateral. Show that the quadrilateral, obtained on joining the mid-points of its sides, is a parallelogram.
Solution:

Let the given points be A (-2, 4), B (4, 8), C (10, 7) and D (11, -5).
And, let P, Q, R and S be the mid-points of AB, BC, CD and DA respectively.
So,
Co-ordinates of P are

$$\left(\frac{-2+4}{2}, \frac{4+8}{2}\right) = (1, 6)$$

Co-ordinates of Q are

$$\left(\frac{4+10}{2}, \frac{8+7}{2}\right) = \left(7, \frac{15}{2}\right)$$

Co-ordinates of R are

$$\left(\frac{10+11}{2}, \frac{7-5}{2}\right) = \left(\frac{21}{2}, 1\right)$$

Co-ordinates of S are

$$\left(\frac{11-2}{2}, \frac{-5+4}{2}\right) = \left(\frac{9}{2}, \frac{-1}{2}\right)$$

$$Slope\ of\ PQ = \frac{\frac{15}{2} - 6}{7-1} = \frac{\frac{15-12}{2}}{6} = \frac{3}{12} = \frac{1}{4}$$

$$Slope\ of\ RS = \frac{\frac{-1}{2} - 1}{\frac{9}{2} - \frac{21}{2}} = \frac{\frac{-1-2}{2}}{\frac{9-21}{2}} = \frac{-3}{-12} = \frac{1}{4}$$

As, slope of PQ = Slope of RS, we can say PQ || RS.
Now,

$$Slope\ of\ QR = \frac{1 - \frac{15}{2}}{\frac{21}{2} - 7} = \frac{\frac{2-15}{2}}{\frac{21-14}{2}} = \frac{-13}{7}$$

$$Slope\ of\ SP = \frac{6 + \frac{1}{2}}{1 - \frac{9}{2}} = \frac{\frac{12+1}{2}}{\frac{2-9}{2}} = \frac{13}{-7} = \frac{-13}{7}$$

As, slope of QR = Slope of SP, we can say QR || SP.
Therefore, PQRS is a parallelogram.

11. Show that the points P (a, b + c), Q (b, c + a) and R (c, a + b) are collinear.
Solution:

We know that,
The points P, Q, R will be collinear if the slopes of PQ and QR are the same.
Calculating foe the slopes, we get
Slope of PQ = (c + a – b – c)/ (b – a) = (a - b)/ (b - a) = -1
Slope of QR = (a + b – c - a)/ (c - b) = (b – c)/ (c - b) = -1
Therefore, the points P, Q, and R are collinear.

Exercise 14(C)

1. Find the equation of a line whose:
y - intercept = 2 and slope = 3.
Solution:

Given,
y - intercept = c = 2 and slope = m = 3.
The line equation is given by: $y = mx + c$
On substituting the values of c and m, we get
$y = 3x + 2$
The above is the required line equation.

2. Find the equation of a line whose:
y - intercept = -1 and inclination = 45°.
Solution:

Given,
y - intercept = c = -1 and inclination = 45°.
So, slope = m = tan 45° = 1
Hence, on substituting the values of c and m in the line equation $y = mx + c$, we get
$y = x - 1$
The above is the required line equation.

3. Find the equation of the line whose slope is -4/3 and which passes through (-3, 4).
Solution:

Given, slope = -4/3
The line passes through the point $(-3, 4) = (x_1, y_1)$
Now, on substituting the values in $y - y_1 = m(x - x_1)$, we have
$y - 4 = -4/3 (x + 3)$
$3y - 12 = -4x - 12$
$4x + 3y = 0$
Hence, the above is the required line equation.

4. Find the equation of a line which passes through (5, 4) and makes an angle of 60° with the positive direction of the x-axis.
Solution:

The slope of the line, $m = \tan 60° = \sqrt{3}$
And, the line passes through the point $(5, 4) = (x_1, y_1)$
Hence, on substituting the values in $y - y_1 = m(x - x_1)$, we have
$y - 4 = \sqrt{3} (x - 5)$
$y - 4 = \sqrt{3}x - 5\sqrt{3}$
$y = \sqrt{3}x + 4 - 5\sqrt{3}$, which is the required line equation.

5. Find the equation of the line passing through:
(i) (0, 1) and (1, 2) (ii) (-1, -4) and (3, 0)
Solution:

(i) Let (0, 1) = (x_1, y_1) and (1, 2) = (x_2, y_2)
So,
Slope of the line = (2 - 1)/ (1 - 0) = 1
Now,
The required line equation is given by,
$y - y_1 = m(x - x_1)$
y - 1 = 1(x - 0)
y - 1 = x
y = x + 1

(ii) Let (-1, -4) = (x_1, y_1) and (3, 0) = (x_2, y_2)
So,
Slope of the line = (0 + 4)/ (3 + 1) = 4/4 = 1
The required line equation is given by,
$y - y_1 = m(x - x_1)$
y + 4 = 1(x + 1)
y + 4 = x + 1
y = x – 3

6. The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively. Find:
(i) the gradient of PQ;
(ii) the equation of PQ;
(iii) the co-ordinates of the point where PQ intersects the x-axis.
Solution:

Given,
The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively.
(i) Gradient of PQ = (5 - 6)/ (-3 - 2) = -1/-5 = 1/5

(ii) The line equation of PQ is given by
$y - y_1 = m(x - x_1)$
y - 6 = 1/5 (x - 2)
5y - 30 = x - 2
5y = x + 28

(iii) Let the line PQ intersect the x-axis at point A (x, 0).
So, on putting y = 0 in the line equation of PQ, we get
0 = x + 28
x = -28
Hence, the co-ordinates of the point where PQ intersects the x-axis are A (-28, 0).

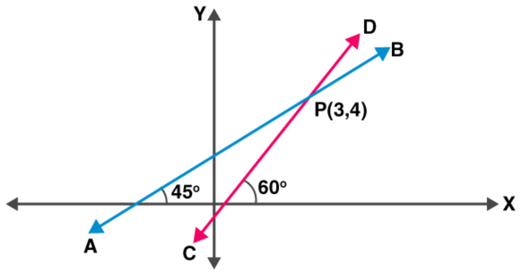
7. The co-ordinates of two points A and B are (-3, 4) and (2, -1). Find:

(i) the equation of AB;
(ii) the co-ordinates of the point where the line AB intersects the y-axis.
Solution:

(i) Given, co-ordinates of two points A and B are (-3, 4) and (2, -1).
Slope = (-1 - 4)/ (2 + 3) = -5/5 = -1
The equation of the line AB is given by:
$y - y_1 = m(x - x_1)$
y + 1 = -1(x - 2)
y + 1 = -x + 2
x + y = 1

(ii) Let's consider the line AB intersect the y-axis at point (0, y).
On putting x = 0 in the line equation, we get
0 + y = 1
y = 1
Thus, the co-ordinates of the point where the line AB intersects the y-axis are (0, 1).

8. The figure given below shows two straight lines AB and CD intersecting each other at point P (3, 4). Find the equation of AB and CD.



Solution:

The slope of line AB = tan 45° = 1
And, the line AB passes through P (3, 4).
Hence, the equation of the line AB is given by
$y - y_1 = m(x - x_1)$
y - 4 = 1(x - 3)
y - 4 = x - 3
y = x + 1

The slope of line CD = tan 60° = √3
And, the line CD passes through P (3, 4).
Hence, the equation of the line CD is given by
$y - y_1 = m(x - x_1)$
y - 4 = √3 (x - 3)
y - 4 = √3x - 3√3
y = √3x + 4 - 3√3

9. In ΔABC, A = (3, 5), B = (7, 8) and C = (1, -10). Find the equation of the median through A.
Solution:

Given,
Vertices of ΔABC, A = (3, 5), B = (7, 8) and C = (1, -10).

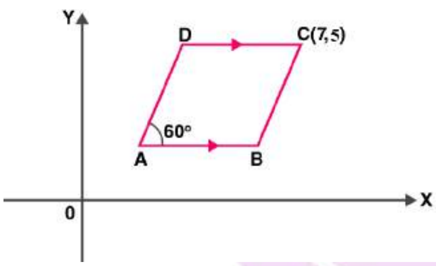
The mid - point D of BC $= \left(\frac{7+1}{2}, \frac{8+(-10)}{2}\right)$

$= \left(\frac{8}{2}, \frac{-2}{2}\right)$

$= (4, -1)$

The slope of AD = (-1 - 5)/ (4 - 3) = -6/1 = -6
Hence, the equation of the median AD through A is given by
y – 5 = -6(x - 3)
y – 5 = -6x + 18
6x + y = 23

10. The following figure shows a parallelogram ABCD whose side AB is parallel to the x-axis, ∠A = 60° and vertex C = (7, 5). Find the equations of BC and CD.



Solution:

Given, ∠A = 60° and vertex C = (7, 5)
As, ABCD is a parallelogram, we have
∠A + ∠B = 180° [corresponding angles]
∠B = 180° - 60° = 120°
Slope of BC = tan 120° = tan (90° + 30°) = cot 30° = √3
So, the equation of line BC is given by
$y - y_1 = m(x - x_1)$
y - 5 = √3 (x - 7)
y - 5 = √3x - 7√3
y = √3x + 5 - 7√3
As, CD || AB and AB || x-axis the slope of CD = Slope of AB = 0 [As slope of x-axis is zero]
So, the equation of the line CD is given by
$y - y_1 = m(x - x_1)$
y - 5 = 0(x - 7)
y = 5

11. Find the equation of the straight line passing through origin and the point of intersection of the lines x + 2y = 7 and x - y = 4.
Solution:

The given line equations are:
$x + 2y = 7$ ....(1)
$x - y = 4$ ....(2)
On solving the above line equations, we can find the point of intersection of the two lines.
So, subtracting (2) from (1), we get
$3y = 3$
$y = 1$
Now,
$x = 4 + y = 4 + 1 = 5$ [From (2)]
It's given that,
The required line passes through (0, 0) and (5, 1).
The slope of the line = (1 - 0)/ (5 - 0) = 1/5
Hence, the required equation of the line is given by
$y – y_1 = m(x – x_1)$
$y – 0 = 1/5(x - 0)$
$5y = x$
$x – 5y = 0$

Exercise 14(D)

1. Find the slope and y-intercept of the line:
(i) y = 4
(ii) ax - by = 0
(iii) 3x - 4y = 5
Solution:

(i) $y = 4$
On comparing the given equation with $y = mx + c$, we get
Slope = m = 0
y - intercept = c = 4

(ii) $ax - by = 0 \Rightarrow by = ax \Rightarrow y = (a/b)x$
On comparing the above equation with $y = mx + c$, we get
Slope = m = a/b
y - intercept = c = 0

(iii) $3x - 4y = 5 \Rightarrow 4y = 3x – 5 \Rightarrow y = \frac{3}{4} x - 5/4$
On comparing the above equation with $y = mx + c$, we get
Slope = m = 3/4
y-intercept = c = -5/4

2. The equation of a line x - y = 4. Find its slope and y-intercept. Also, find its inclination.
Solution:

Given equation of a line: $x - y = 4$
$\Rightarrow y = x - 4$
Comparing the above equation with $y = mx + c$, we get
Slope = m = 1
y - intercept = c = -4
Let the inclination be θ.
Slope = 1 = $\tan \theta = \tan 45^o$
$\theta = 45^o$

3. (i) Is the line 3x + 4y + 7 = 0 perpendicular to the line 28x - 21y + 50 = 0?
(ii) Is the line x - 3y = 4 perpendicular to the line 3x - y = 7?
(iii) Is the line 3x + 2y = 5 parallel to the line x + 2y = 1?
(iv) Determine x so that the slope of the line through (1, 4) and (x, 2) is 2.
Solution:

(i) Given,
$3x + 4y + 7 = 0$
$\Rightarrow 4y = -3x – 7$
$\Rightarrow y = (-3/4)x – 7/4$
Slope of this line = -3/4

And, for
28x - 21y + 50 = 0
⇒ 21y = 28x + 50
⇒ y = (28/21)x + 50/21
⇒ y = (4/3)x + 50/21
Slope of this line = 4/3
As, the product of slopes of the two lines = 4/3 x -3/4 = -1
Therefore, the lines are perpendicular to each other.

(ii) Given,
x - 3y = 4
⇒ 3y = x – 4
⇒ y = (1/3)x – 4/3
So, the slope of this line = 1/3
And, for
3x - y = 7
y = 3x - 7
So, the slope of this line = 3
Now, the slopes of the two lines = 1/3 x 3 = 1 and not equal to -1.
Hence, the lines are not perpendicular to each other.

(iii) Given,
3x + 2y = 5
2y = -3x + 5
y = (-3/2) x + 5/2
So, the slope of this line = -3/2
And, for
x + 2y = 1
2y = -x + 1
y = -1/2x + 1/2
So, the slope of this line = -1/2
The slopes of the two lines are not equal.
Hence, the lines are not parallel to each other.

(iv) Given, the slope of the line through (1, 4) and (x, 2) is 2.
So,

$$\frac{2-4}{x-1} = 2$$

$$\frac{-2}{x-1} = 2$$

$$\frac{-1}{x-1} = 1$$

-1 = x – 1
x = 0

4. Find the slope of the line which is parallel to:
(i) x + 2y + 3 = 0 (ii) x/2 – y/3 – 1 = 0
Solution:

(i) x + 2y + 3 = 0
2y = -x - 3
y = -1/2x – 3/2
Slope of this line = -1/2
Thus, slope of the line which is parallel to the given line = slope of the given line = -1/2

(ii) x/2 – y/3 – 1 = 0
y/3 = x/2 – 1
y = (3/2)x - 3
So, the slope of this line = 3/2
Thus, slope of the line which is parallel to the given line = Slope of the given line = 3/2

5. Find the slope of the line which is perpendicular to:
(i) x – y/2 + 3 = 0 (ii) x/3 – 2y = 4
Solution:

(i) x – y/2 + 3 = 0
y/2 = x + 3
y = 2x + 6
So, the slope of this line = 2
We know that,
Slope of the line which is perpendicular to the given line = -1/(Slope of the given line) = -1/2

(ii) x/3 – 2y = 4
2y = x/3 – 4
y = x/6 - 2
So, the slope of this line = 1/6
We know that,
Slope of the line which is perpendicular to the given line = -1/(Slope of the given line) = -1/(1/6) = -6

6. (i) Lines 2x - by + 3 = 0 and ax + 3y = 2 are parallel to each other. Find the relation connecting a and b.
(ii) Lines mx + 3y + 7 = 0 and 5x - ny - 3 = 0 are perpendicular to each other. Find the relation connecting m and n.
Solution:

(i) We know that, if two lines are parallel then, the slopes of the two lines must be equal.
For, 2x - by + 3 = 0
by = 2x + 3
y = (2/b)x + 3/b
So, the slope of this line = 2/b

And, ax + 3y = 2
3y = -ax + 2
y = (-a/3)x + 2/3
So, the slope of this line = -a/3
Now, equating the slopes we get
2/b = -a/3
ab = -6

(ii) We know that, if two lines are perpendicular to each other then, the product of their slopes = -1.
For, mx + 3y + 7 = 0
3y = -mx - 7
y = -m/3x – 7/3
Slope of this line = -m/3
And, 5x - ny - 3 = 0
ny = 5x - 3
y = (5/n)x – 3/n
Slope of this line = 5/n
Products of slopes is
(-m/3) x (5/n) = -1
5m = 3n

7. Find the value of p if the lines, whose equations are 2x - y + 5 = 0 and px + 3y = 4 are perpendicular to each other.
Solution:

Given line equations,
2x - y + 5 = 0 …. (1)
y = 2x + 5 …… (2)
Now,
Slope of line (2) = 2
Then, (2) can rewritten as
3y = -px + 4
y = (-p/3) + 4/3
So, the slope of this line = -p/3
For 2 lines to be perpendicular to each other, the product of their slopes must be -1.
So,
(2) x (-p/3) = -1
2p/3 = 1
p = 3/2

8. The equation of a line AB is 2x - 2y + 3 = 0.
(i) Find the slope of the line AB.
(ii) Calculate the angle that the line AB makes with the positive direction of the x-axis.
Solution:

(i) Given, equation of the line

2x - 2y + 3 = 0
2y = 2x + 3
y = x + (3/2)
So, the slope of the line AB = 1

(ii) Required to find the angle of the line AB = θ
We have,
Slope = tan θ = 1
And, tan 45^o = 1
Hence, θ = 45^o

9. The lines represented by 4x + 3y = 9 and px - 6y + 3 = 0 are parallel. Find the value of p.
Solution:

Given line equations are,
4x + 3y = 9
3y = -4x + 9
y = (-4/3)x + 3
Slope of this line = -4/3
And,
px - 6y + 3 = 0
6y = px + 3
y = (p/6)x + ½
Slope of this line = p/6
For two lines to be parallel, their slopes must be equal.
-4/3 = p/6
-4 = p/2
p = -8

10. If the lines y = 3x + 7 and 2y + px = 3 are perpendicular to each other, find the value of p.
Solution:

Given line equations,
y = 3x + 7
Slope of this line = 3
And,
2y + px = 3
2y = -px + 3
y = (-p/2) + 3
So, the slope of this line = -p/2
For these two lines to be perpendicular to each other, the product of their slopes must be -1.
(3) x (-p/2) = -1
3p/2 = 1
p = 2/3

11. The line through A(-2,3) and B(4,b) is perpendicular to the line 2x - 4y =5. Find the value of b.

Solution:

Given,
Points A (-2, 3) and B (4, b)
And, line equation: $2x - 4y = 5$
$4y = 2x - 5$
$y = (1/2)x - 5/4$
So, the slope of this line = ½
From the question, it's said that
The line through A and B is perpendicular to above given line.
We know that, when two lines are perpendicular their product of slopes is -1
Hence, the slope of the line through A and B must be -2.
Now,
The slope of the line through A and B is given by,
Slope of AB = $(b - 3)/(4 + 2)$
$= (b - 3)/6$
Thus,
$(b - 3)/6 = -2$
$b - 3 = -12$
$b = -9$

12. Find the equation of the line through (-5, 7) and parallel to:
(i) x-axis (ii) y-axis
Solution:

(i) We know that, the slope of a line parallel to x-axis is 0.
Here, $(x_1, y_1) = (-5, 7)$ and $m = 0$
So, the required line equation is
$y - y_1 = m(x - x_1)$
$y - 7 = 0(x + 5)$
$y = 7$

(ii) We know that, the slope of a line parallel to y-axis is not defined. ($\tan 90^\circ$)
So, the given line is parallel to y-axis.
Here, $(x_1, y_1) = (-5, 7)$
So, the required equation of the line is
$x - x_1 = 0$
$x + 5 = 0$

Exercise 14(E)

1. Point P divides the line segment joining the points A (8, 0) and B (16, -8) in the ratio 3: 5. Find its co-ordinates of point P.
Also, find the equation of the line through P and parallel to 3x + 5y = 7.
Solution:

Given points, A (8, 0) and B (16, -8)
By section formula, the co-ordinates of the point P which divides AB in the ratio 3: 5 is given by
$$\left(\frac{3\times 16+5\times 8}{3+5},\frac{3\times(-8)+5\times 0}{3+5}\right)$$
= (11, -3) = (x_1, y_1)
Given line equation is,
3x + 5y = 7
5y = -3x + 7
y = (-3/5) x + 7/5
So, the slope of this line = -3/5
The line parallel to the line 3x + 5y = 7 will have the same slope,
Hence, the slope of the required line = Slope of the given line = -3/5
Thus,
The equation of the required line is
y - y_1 = m(x - x_1)
y + 3 = (-3/5)(x - 11)
5y + 15 = -3x + 33
3x + 5y = 18

2. The line segment joining the points A(3, -4) and B (-2, 1) is divided in the ratio 1: 3 at point P in it. Find the co-ordinates of P. Also, find the equation of the line through P and perpendicular to the line 5x - 3y + 4 = 0.
Solution:

Given points, A (3, -4) and B (-2, 1)
By section formula, the co-ordinates of the point P which divides AB in the ratio 1: 3 is given by
$$\left(\frac{1\times(-2)+3\times 3}{1+3},\frac{1\times 1+3\times(-4)}{1+3}\right)$$
= (7/4, -11/4) = (x_1, y_1)
Given line equation is,
5x - 3y + 4 = 0
3y = 5x + 4
y = (5/3) x + 4/3
So, the slope of this line = 5/3
The line perpendicular to the given line will have slope
Slope of the required line = -1/(5/3) = -3/5
Hence,
The equation of the required line is given by

y - y1 = m(x - x1)

$$y + \frac{11}{4} = \frac{-3}{5}\left(x - \frac{7}{4}\right)$$

$$\frac{4y + 11}{4} = \frac{-3}{5}\left(\frac{4x - 7}{4}\right)$$

20y + 55 = -12x + 21
12x + 20y + 34 = 0
6x + 10y + 17 = 0

3. A line 5x + 3y + 15 = 0 meets y-axis at point P. Find the co-ordinates of point P. Find the equation of a line through P and perpendicular to x - 3y + 4 = 0.
Solution:

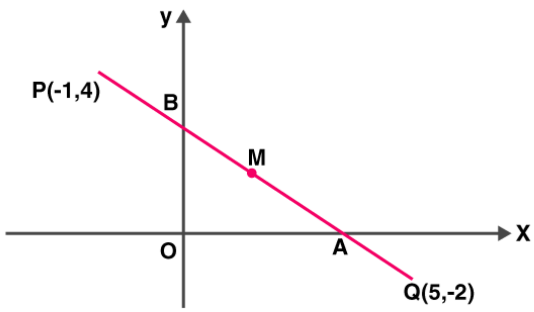
As the point P lies on y-axis,
Putting x = 0 in the equation 5x + 3y + 15 = 0, we get
5(0) + 3y + 15 = 0
y = -5
Hence, the co-ordinates of the point P are (0, -5).
Given line equation,
x - 3y + 4 = 0
3y = x + 4
y = (1/3)x + 4/3
Slope of this line = 1/3
From the question, the required line equation is perpendicular to the given equation: x - 3y + 4 = 0.
So, the product of their slopes is -1.
Slope of the required line = -1/(1/3) = -3
And,
$(x_1, y_1) = (0, -5)$
Therefore,
The required line equation is
y - y1 = m(x - x1)
y + 5 = -3(x - 0)
3x + y + 5 = 0

4. Find the value of k for which the lines kx - 5y + 4 = 0 and 5x - 2y + 5 = 0 are perpendicular to each other.
Solution:

Given line equations,
kx - 5y + 4 = 0
5y = kx + 4
⇒ y = (k/5) + 4/5
So, the slope of this line = m1 = k/5
And, for 5x - 2y + 5 = 0
⇒ 2y = 5x + 5

y = (5/2) x + 5/2
Slope of this line = m2 = 5/2
As, the lines are perpendicular to each other
$m_1 \times m_2 = -1$
(k/5) x (5/2) = -1
k = -2

5. A straight line passes through the points P (-1, 4) and Q (5, -2). It intersects the co-ordinate axes at points A and B. M is the mid-point of the segment AB. Find:
(i) the equation of the line.
(ii) the co-ordinates of A and B.
(iii) the co-ordinates of M.



Solution:

(i) Given points, P (-1, 4) and Q (5, -2)
Slope of PQ = (-2 - 4)/ (5 + 1) = -6/6 = -1
Equation of the line PQ is given by,
$y - y_1 = m(x - x_1)$
y - 4 = -1(x + 1)
y - 4 = -x - 1
x + y = 3

(ii) For point A (on x-axis), y = 0.
So, putting y = 0 in the equation of PQ, we have
x = 3
Hence, the co-ordinates of point A are (3, 0).
For point B (on y-axis), x = 0.
So, putting x = 0 in the equation of PQ, we have
y = 3
Hence, the co-ordinates of point B are (0, 3).

(iii) M is the mid-point of AB.
Thus, the co-ordinates of point M are
(3+0/2, 0+3/2) = (3/2, 3/2)

6. (1, 5) and (-3, -1) are the co-ordinates of vertices A and C respectively of rhombus ABCD. Find

the equations of the diagonals AC and BD.
Solution:

Given, A = (1, 5) and C = (-3, -1) of rhombus ABCD.
We know that in a rhombus, diagonals bisect each other at right angle.
Let's take O to be the point of intersection of the diagonals AC and BD.
Then, the co-ordinates of O are

$$\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2)$$

Slope of AC = (-1 - 5)/ (-3 - 1) = -6/-4 = 3/2
Then, the equation of the line AC is
y - y1 = m (x - x1)
y - 5 = (3/2) (x - 1)
2y - 10 = 3x - 3
3x - 2y + 7 = 0
Now, the line BD is perpendicular to AC
Slope of BD = -1/(slope of AC) = -2/3
And, (x_1, y_1) = (-1, 2)
Hence, equation of the line BD is
$y - y_1$ = m (x - x1)
y - 2 = (-2/3) (x + 1)
3y - 6 = -2x - 2
2x + 3y = 4

7. Show that A (3, 2), B (6, -2) and C (2, -5) can be the vertices of a square.
(i) Find the co-ordinates of its fourth vertex D, if ABCD is a square.
(ii) Without using the co-ordinates of vertex D, find the equation of side AD of the square and also the equation of diagonal BD.
Solution:

Given, A (3, 2), B (6, -2) and C (2, -5)
Now, by distance formula
AB = $\sqrt{[(6 - 3)^2 + (-2 - 2)^2]} = \sqrt{(9 + 16)} = 5$
BC = $\sqrt{[(6 - 3)^2 + (-2 - 2)^2]} = \sqrt{(9 + 16)} = 5$
Thus, AC = BC
Then,
Slope of AB = (-2 - 2)/ (6 - 3) = -4/3
Slope of BC = (-5 + 2)/ (2 - 6) = -3/-4 = ¾
Slope of AB x Slope of BC = -4/3 x ¾ = -1
Hence, AB ⊥ BC
Therefore, A, B, C can be the vertices of a square.

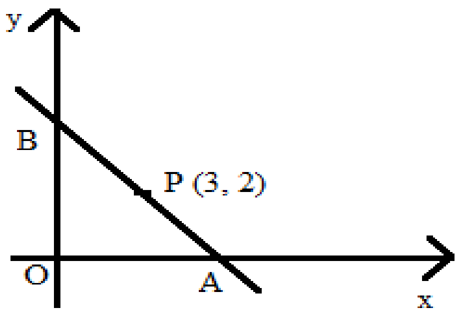
(i) Slope of AB = (-2 - 2)/ (6 - 3) = -4/3 = slope of CD
So, the equation of CD is
y + 5 = -4/3(x - 2)

3y + 15 = -4x + 8
3y = -4x – 7
4x + 3y + 7 = 0 … (1)
Now, slope of BC = (-5 + 2)/ (2 - 6) = -3/-4 = ¾ = Slope of AD
So, the equation of the line AD is
y – 2 = (3/4) (x - 3)
4y – 8 = 3x – 9
3x – 4y = 1 …. (2)
Now, D is the point of intersection of CD and AD.
Solving (1) and (2),
4 x (1) + 3 x (2) ⇒
16x + 12y + 9x – 12y = -28 + 3
25x = -25
x = -1
Putting value of y in (1), we get
4(-1) + 3y + 7 = 0
3y = -3
y = -1
Therefore, the co-ordinates of point D are (-1, -1).

8. A line through origin meets the line x = 3y + 2 at right angles at point X. Find the co-ordinates of X.
Solution:

The given line equation is
x = 3y + 2 ... (1)
3y = x – 2
y = 1/3 x – 2/3
So, slope of this line is 1/3.
And, the required line intersects the given line at right angle.
Thus, slope of the required line = -1/(1/3) = -3
And. the required line passes through (0, 0) = (x_1, y_1)
So, the equation of the required line is:
y - y_1 = m(x - x_1)
y - 0 = -3(x - 0)
3x + y = 0 ... (2)
Next,
Point X is the intersection of the lines (1) and (2).
Using (1) in (2), we get,
3(3y + 2) + y = 0
9y + 6 + y = 0
10y = -6
y = -6/10 = -3/5
And, finally
x = 3(-3/5) + 2 = -9/5 + 2 = 1/5
Thus, the co-ordinates of the point X are (1/5, -3/5).

9. A straight line passes through the point (3, 2) and the portion of this line, intercepted between the positive axes, is bisected at this point. Find the equation of the line.
Solution:



Let the line intersect the x-axis at point A (x, 0) and y-axis at point B (0, y).
Since, P is the mid-point of AB, we have:
$\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (3, 2)$
(x/2, y/2) = (3, 2)
x = 6, y = 4
Thus, A = (6, 0) and B = (0, 4)
Slope of line AB = (4 - 0)/ (0 - 6) = 4/-6 = -2/3
And, let (x_1, y_1) = (6, 0)
So, the required equation of the line AB is given by
$y - y_1 = m(x - x_1)$
y - 0 = (-2/3) (x - 6)
3y = -2x + 12
2x + 3y = 12

10. Find the equation of the line passing through the point of intersection of 7x + 6y = 71 and 5x - 8y = -23; and perpendicular to the line 4x - 2y = 1.
Solution:

Given line equations are,
7x + 6y = 71 ⇒ 28x + 24 = 284 ... (1)
5x - 8y = -23 ⇒ 15x - 24y = -69 ... (2)
On adding (1) and (2), we have
43x = 215
x = 5
From (2), we get
8y = 5x + 23 = 25 + 23 = 48
⇒ y = 6
Hence, the required line passes through the point (5, 6).
Given, 4x - 2y = 1
2y = 4x - 1
y = 2x – (1/2)

So, the slope of this line = 2
And, the slope of the required line = -1/2 [As the lines are perpendicular to each other]
Thus, the required equation of the line is
$y - y_1 = m(x - x_1)$
y - 6 = (-1/2) (x - 5)
2y - 12 = -x + 5
x + 2y = 17

11. Find the equation of the line which is perpendicular to the line x/a - y/b = 1 at the point where this line meets y-axis.
Solution:

The given line equation is,
x/a – y/b = 1
y/b = x/a – 1
y = (b/a)x – b
The slope of this line = b/a
So, the slope of the required line = -1/(b/a) = - a/b
Let the line intersect at point P (0, y) on the y-axis.
So, putting x = 0 in the equation x/a – y/b = 1, we get
0 – y/b = 1
y = -b
Hence, P = (0, -b) = (x_1, y_1)
Therefore,
The equation of the required line is
$y - y_1 = m (x - x_1)$
y + b = (-a/b) (x - 0)
$by + b^2 = -ax$
$ax + by + b^2 = 0$

12. O (0, 0), A (3, 5) and B (-5, -3) are the vertices of triangle OAB. Find:
(i) the equation of median of triangle OAB through vertex O.
(ii) the equation of altitude of triangle OAB through vertex B.
Solution:

(i) Let's consider the median through O meets AB at D. So, D will be the mid-point of AB.
Co-ordinates of point D are:
$$\left(\frac{3-5}{2}, \frac{5-3}{2}\right) = (-1, 1)$$
The slope of OD = (1 – 0)/ (-1 - 0) = -1
And, (x_1, y_1) = (0, 0)
Hence, the equation of the median OD is
$y - y_1 = m(x - x_1)$
y - 0 = -1(x - 0)
x + y = 0

(ii) The altitude through vertex B is perpendicular to OA.
We have, slope of OA = (5 - 0)/ (3 - 0) = 5/3
Then, the slope of the required altitude = -1/(5/3) = -3/5
Hence, the equation of the required altitude through B is
$y - y_1 = m(x - x1)$
$y + 3 = (-3/5)(x + 5)$
$5y + 15 = -3x - 15$
$3x + 5y + 30 = 0$

13. Determine whether the line through points (-2, 3) and (4, 1) is perpendicular to the line 3x = y + 1.
Does the line 3x = y + 1 bisect the line segment joining the two given points?
Solution:

Let A = (-2, 3) and B = (4, 1)
Slope of AB = m_1 = (1 - 3)/(4 + 2) = -2/6 = -1/3
So, the equation of line AB is
$y - y_1 = m_1(x - x_1)$
$y - 3 = (-1/3)(x + 2)$
$3y - 9 = -x - 2$
$x + 3y = 7$ ... (1)
Slope of the given line 3x = y + 1 is 3 = m_2.
It's seen that, $m_1 \times m_2 = -1$
Thus, the line through points A and B is perpendicular to the given line.

Given line is 3x = y +1 ... (2)
The co-ordinates of the mid-point of AB are
$$\left(\frac{-2+4}{2}, \frac{3+1}{2}\right) = (1, 2) = P$$
Now, Let check if the point P satisfies the line equation (2)
3(1) = 2 + 1
3 = 3
Hence, the line 3x = y + 1 bisects the line segment joining the points A and B.

14. Given a straight line x cos 30º + y sin 30º = 2. Determine the equation of the other line which is parallel to it and passes through (4, 3).
Solution:

Given line equation, x cos 30º + y sin 30º = 2
$$x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 2$$
$$\sqrt{3}x + y = 4$$
$$y = -\sqrt{3}x + 4$$

So, the slope of this line = $-\sqrt{3}$
Slope of a line which is parallel to this given line = $-\sqrt{3}$
Let (4, 3) = (x_1, y_1)
Therefore, the equation of the required line is given by
$y - y_1 = m_1 (x - x_1)$
$y - 3 = -\sqrt{3} (x - 4)$
$\sqrt{3}x + y = 4\sqrt{3} + 3$

15. Find the value of k such that the line (k - 2)x + (k + 3)y - 5 = 0 is:
(i) perpendicular to the line 2x - y + 7 = 0
(ii) parallel to it.
Solution:

Given line equation,
$(k - 2)x + (k + 3)y - 5 = 0$ .... (1)
$(k + 3)y = -(k - 2)x + 5$
$y = \left(\frac{2-k}{k+3}\right)x + \frac{5}{k+3}$
Slope of this line = $m_1 = (2 - k)/(k + 3)$
(i) Given, $2x - y + 7 = 0$
$y = 2x + 7 = 0$
Slope of this line = $m_2 = 2$
Given that, line (1) is perpendicular to $2x - y + 7 = 0$
$m_1 \times m_2 = -1$
$(2 - k)/(k + 3) \times 2 = -1$
$4 – 2k = -k – 3$
$k = 7$

(ii) Line (1) is parallel to $2x - y + 7 = 0$
So, $m_1 = m_2$
$(2 – k)/(k + 3) = 2$
$2 – k = 2k + 6$
$3k = -4$
$k = -4/3$

16. The vertices of a triangle ABC are A (0, 5), B (-1, -2) and C (11, 7). Write down the equation of BC. Find:
(i) the equation of line through A and perpendicular to BC.
(ii) the co-ordinates of the point, where the perpendicular through A, as obtained in (i), meets BC.

Solution:

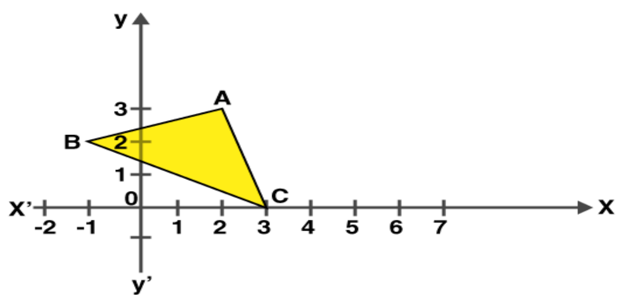
Slope of BC = $(7 + 2)/(11 + 1) = 9/12 = 3/4$
Then the equation of the line BC is
$y + 2 = (x + 1)$ when $x_1 = -1$ and $y_1 = -2$
$4y + 8 = 3x + 3$

3x - 4y = 5 .... (1)

(i) Slope of line perpendicular to BC will be = -1/(3/4) = -4/3
So, the required equation of the line through A (0, 5) and perpendicular to BC is given by
$y - y_1 = m_1 (x - x_1)$
y - 5 = (x - 0)
3y - 15 = -4x
4x + 3y = 15 .... (2)

(ii) Hence, the required point will be the point of intersection of lines (1) and (2).
Solving (1) & (2),
(1) ⇒ 9x - 12y = 15
(2) ⇒ 16x + 12y = 60
Now, adding the above two equations, we get
25x = 75
x = 3
And, 4y = 3x - 5 = 9 - 5 = 4
y = 1
Thus, the co-ordinates of the required point is (3, 1).

17. From the given figure, find:



(i) the co-ordinates of A, B and C.
(ii) the equation of the line through A and parallel to BC.
Solution:

(i) A = (2, 3), B = (-1, 2), C = (3, 0)
(ii) Slope of BC = (0 - 2)/ (3 + 1) = -2/4 = -1/2
Slope of the line which is parallel to BC = Slope of BC = -1/2
(x_1, y_1) = (2, 3)
Hence, the required equation of the line through A and parallel to BC is given by
$y - y_1 = m_1 (x - x_1)$
y - 3 = (-1/2) (x - 2)
2y - 6 = -x + 2
x + 2y = 8

18. P (3, 4), Q (7, -2) and R (-2, -1) are the vertices of triangle PQR. Write down the equation of

the median of the triangle through R.
Solution:

We know that, the median, RX through R will bisect the line PQ.
The co-ordinates of point X are

$$\left(\frac{3+7}{2}, \frac{4-2}{2}\right) = (5, 1)$$

Slope of RX = (1 + 1)/ (5 + 2) = 2/7 = m
$(x_1, y_1) = (-2, -1)$
Then, the required equation of the median RX is given by
$y - y_1 = m_1(x - x_1)$
y + 1 = (2/7) (x + 2)
7y + 7 = 2x + 4
7y = 2x – 3

19. A (8, -6), B (-4, 2) and C (0, -10) are vertices of a triangle ABC. If P is the mid-point of AB and Q is the mid-point of AC, use co-ordinate geometry to show that PQ is parallel to BC. Give a special name of quadrilateral PBCQ.
Solution:

P is the mid-point of AB. Hence, the co-ordinates of point P are

$$\left(\frac{8-4}{2}, \frac{-6+2}{2}\right) = (2, -2)$$

Q is the mid-point of AC. So, the co-ordinate of point Q are

$$\left(\frac{8+0}{2}, \frac{-6-10}{2}\right) = (4, -8)$$

Slope of PQ = (-8 + 2)/ (4 - 2) = -6/2 = -3
Slope of BC = (-10 - 2)/ (0 + 4) = -12/4 = -3
As, the slope of PQ = Slope of BC,
Therefore, PQ || BC
Also,
Slope of PB = (-2 - 2)/ (2 + 4) = -2/3
Slope of QC = (-8 + 10)/ (4 - 10) = 1/2
So, PB is not parallel to QC as their slopes are not equal
Thus, PBCQ is a trapezium.

20. A line AB meets the x-axis at point A and y-axis at point B. The point P (-4, -2) divides the line segment AB internally such that AP: PB = 1: 2. Find:
(i) the co-ordinates of A and B.
(ii) the equation of line through P and perpendicular to AB.
Solution:

(i) Let's assume the co-ordinates of point A, lying on x-axis be (x, 0) and the co-ordinates of point B (lying y-axis) be (0, y).

Given,
P = (-4, -2) and AP: PB = 1:2
By section formula, we get

$$(-4, -2) = \left(\frac{1 \times 0 + 2 \times x}{1+2}, \frac{1 \times y + 2 \times 0}{1+2}\right)$$

$$(-4, -2) = \left(\frac{2x}{3}, \frac{y}{3}\right)$$

-4 = 2x/3 and -2 = y/3
x = -6 and y = -6
Hence, the co-ordinates of A and B are (-6, 0) and (0, -6).

(ii) Slope of AB = (-6 - 0)/ (0 + 6) = -6/6 = -1
Slope of the required line perpendicular to AB = -1/-1 = 1
Here, $(x_1, y_1) = (-4, -2)$
Therefore, the required equation of the line passing through P and perpendicular to AB is given by
$y - y_1 = m(x - x_1)$
y + 2 = 1(x + 4)
y + 2 = x + 4
y = x + 2