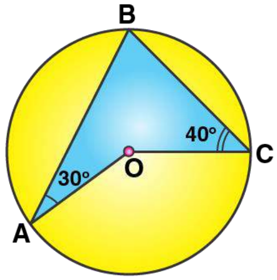
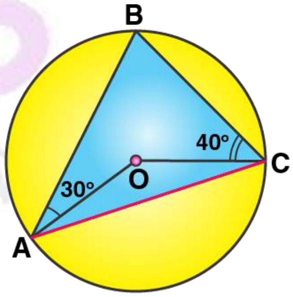
Exercise 17(A) Page No: 257

1. In the given figure, O is the center of the circle. ∠OAB and ∠OCB are 30º and 40º respectively. Find ∠AOC Show your steps of working.

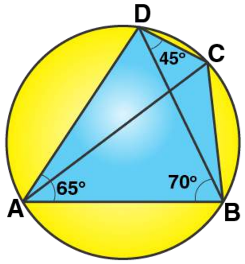


Solution:



Firstly, let's join AC.
And, let ∠OAC = ∠OCA = x [Angles opposite to equal sides are equal]
So, ∠AOC = 180º – 2x
Also,
∠BAC = 30º + x
∠BCA = 40º + x
Now, in ΔABC
∠ABC = 180º - ∠BAC - ∠BCA [Angles sum property of a triangle]
= 180º – (30º + x) – (40º + x)
= 110º – 2x
And, ∠AOC = 2∠ABC
[Angle at the center is double the angle at the circumference subtend by the same chord]
180º – 2x = 2(110º – 2x)
2x = 40º
x = 20º
Thus, ∠AOC = 180º – 2x20º = 140º

2. In the given figure, ∠BAD = 65°, ∠ABD = 70°, ∠BDC = 45°



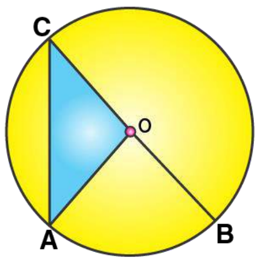
(i) Prove that AC is a diameter of the circle.
(ii) Find ∠ACB.
Solution:

(i) In ΔABD,
∠DAB + ∠ABD + ∠ADB = 180º

$65^o + 70^o + \angle ADB = 180^o$
$135^o + \angle ADB = 180^o$
$\angle ADB = 180^o - 135^o = 45^o$
Now,
$\angle ADC = \angle ADB + \angle BDC = 45^o + 45^o = 90^o$
As ∠ADC is the angle of semi-circle for AC as the diameter of the circle.

(ii) $\angle ACB = \angle ADB$ [Angles in the same segment of a circle]
Hence, $\angle ACB = 45^o$

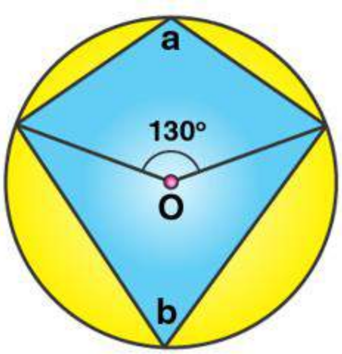
3. Given O is the centre of the circle and ∠AOB = 70º.



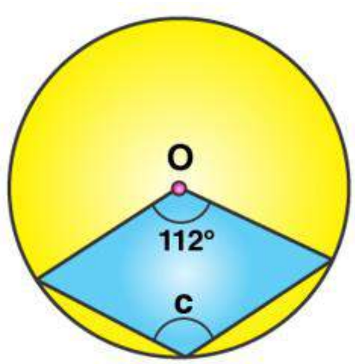
Calculate the value of:
(i) ∠OCA,
(ii) ∠OAC.
Solution:

Here, $\angle AOB = 2\angle ACB$
[Angle at the center is double the angle at the circumference subtend by the same chord]
$\angle ACB = 70^o/ 2 = 35^o$
Now, OC = OA [Radii of same circle]
Thus,
$\angle OCA = \angle OAC = 35^o$

4. In each of the following figures, O is the centre of the circle. Find the values of a, b and c.
Solution:



(i)



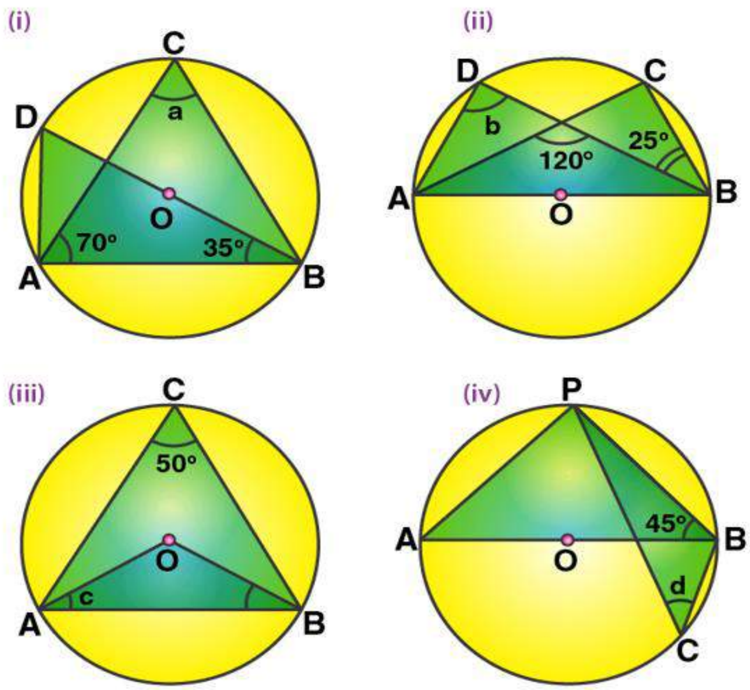
(ii)

(i) Here, $b = ½ \times 130^o$
[Angle at the center is double the angle at the circumference subtend by the same chord]
Thus, $b = 65^o$

Now,
a + b = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
a = 180° – 65° = 115°

(ii) Here, c = ½ x Reflex (112°)
[Angle at the center is double the angle at the circumference subtend by the same chord]
Thus, c = ½ x (360° – 112°) = 124°

5. In each of the following figures, O is the center of the circle. Find the values of a, b, c and d.



Solution:

(i) Here, ∠BAD = 90° [Angle in a semi-circle]
So, ∠BDA = 90° – 35° = 55°
And,
a = ∠ACB = ∠BDA = 55°
[Angles subtended by the same chord on the circle are equal]

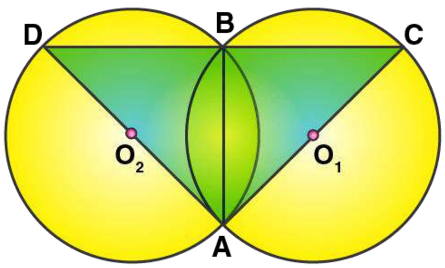
(ii) Here, ∠DAC = ∠CBD = 25°
[Angles subtended by the same chord on the circle are equal]
And, we have
120° = b + 25°
[Exterior angle property of a triangle]
b = 95°

(iii) ∠AOB = 2∠AOB = 2 x 50° = 100°
[Angle at the center is double the angle at the circumference subtend by the same chord]
Also, OA = OB

∠OBA = ∠OAB = c
c = (180° - 100°)/ 2 = 40°

(iv) We have, ∠APB = 90° [Angle in a semicircle]
∠BAP = 90° – 45° = 45°
Now, d = ∠BCP = ∠BAP = 45°
[Angles subtended by the same chord on the circle are equal]

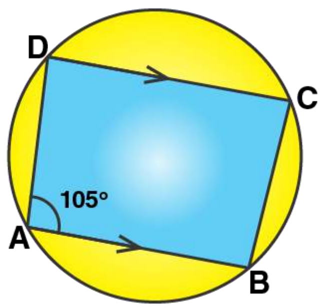
6. In the figure, AB is common chord of the two circles. If AC and AD are diameters; prove that D, B and C are in a straight line. O_1 and O_2 are the centers of two circles.



Solution:

It's seen that,
∠DBA = ∠CBA = 90° [Angle in a semi-circle is a right angle]
So, adding both
∠DBA + ∠CBA = 180°
Thus, DBC is a straight line i.e. D, B and C form a straight line.

7. In the figure, given below, find:
(i) ∠BCD,
(ii) ∠ADC,
(iii) ∠ABC.
Show steps of your working.



Solution:

From the given fig, it's seen that
In cyclic quadrilateral ABCD, DC || AB
And given, ∠DAB = 105°

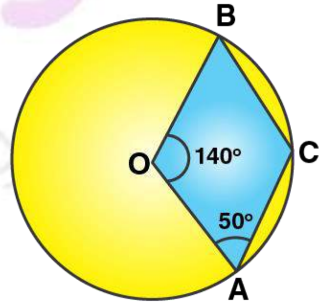
(i) So,
∠BCD = 180° – 105° = 75°
[Sum of opposite angles in a cyclic quadrilateral is 180°]

(ii) Now,
∠ADC and ∠DAB are corresponding angles.

So,
$\angle ADC + \angle DAB = 180^o$
$\angle ADC = 180^o - 105^o$
Thus,
$\angle ADC = 75^o$

(iii) We know that, the sum of angles in a quadrilateral is 360^o
So,
$\angle ADC + \angle DAB + \angle BCD + \angle ABC = 360^o$
$75^o + 105^o + 75^o + \angle ABC = 360^o$
$\angle ABC = 360^o - 255^o$
Thus,
$\angle ABC = 105^o$

8. In the figure, given below, O is the centre of the circle. If $\angle AOB = 140^o$ and $\angle OAC = 50^o$; find:
(i) $\angle ACB$,
(ii) $\angle OBC$,
(iii) $\angle OAB$,
(iv) $\angle CBA$.

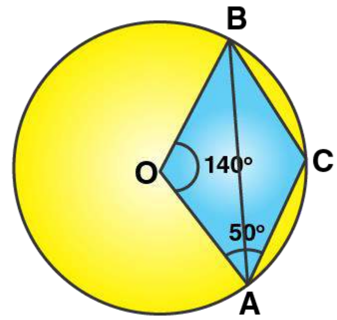


Solution:

Given, $\angle AOB = 140^o$ and $\angle OAC = 50^o$

(i) Now,
$\angle ACB = ½ \text{ Reflex } (\angle AOB) = ½ (360^o - 140^o) = 110^o$
[Angle at the center is double the angle at the circumference subtend by the same chord]

(ii) In quadrilateral OBCA,
$\angle OBC + \angle ACB + \angle OCA + \angle AOB = 360^o$ [Angle sum property of a quadrilateral]
$\angle OBC + 110^o + 50^o + 140^o = 360^o$
Thus, $\angle OBC = 360^o - 300^o = 60^o$

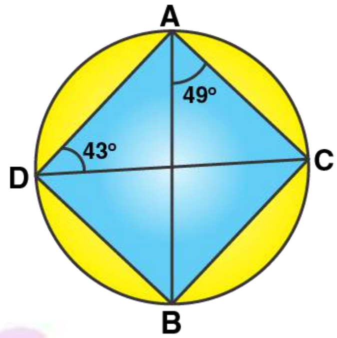


(iii) In ΔAOB, we have
OA = OB (radii)
So, $\angle OBA = \angle OAB$
Hence, by angle sum property of a triangle
$\angle OBA + \angle OAB + \angle AOB = 180^o$
$2\angle OBA + 140^o = 180^o$
$2\angle OBA = 40^o$
$\angle OBA = 20^o$

(iv) We already found, $\angle OBC = 60^o$
And, $\angle OBC = \angle CBA + \angle OBA$
$60^o = \angle CBA + 20^o$

Therefore,
∠CBA = 40°

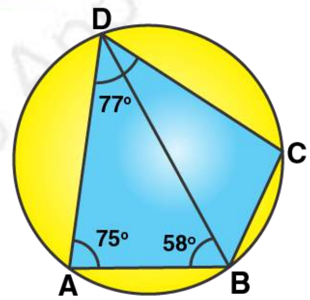
9. Calculate:
(i) ∠CDB,
(ii) ∠ABC,
(iii) ∠ACB.



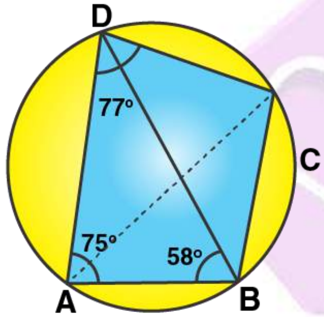
Solution:

Here, we have
∠CDB = ∠BAC = 49°
∠ABC = ∠ADC = 43°
[Angles subtended by the same chord on the circle are equal]
Now, by angle sum property of a triangle we have
∠ACB = 180° – 49° – 43° = 88°

10. In the figure given below, ABCD is a cyclic quadrilateral in which ∠BAD = 75°; ∠ABD = 58° and ∠ADC = 77°.
Find:
(i) ∠BDC,
(ii) ∠BCD,
(iii) ∠BCA.



Solution:

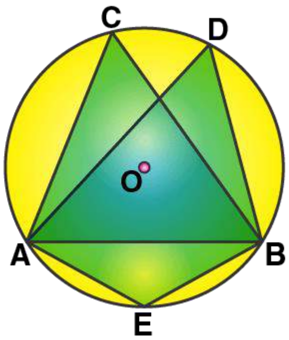


(i) By angle sum property of triangle ABD,
∠ADB = 180° – 75° – 58° = 47°
Thus, ∠BDC = ∠ADC - ∠ADB = 77° – 47° = 30°

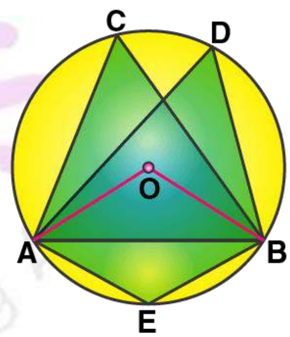
(ii) ∠BAD + ∠BCD = 180°
[Sum of opposite angles of a cyclic quadrilateral is 180°]
Thus, ∠BCD = 180° - 75° = 105°

(iii) ∠BCA = ∠ADB = 47°
[Angles subtended by the same chord on the circle are equal]

11. In the figure given below, O is the centre of the circle and triangle ABC is equilateral.



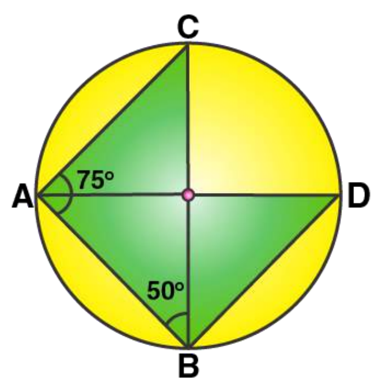
Find:
(i) ∠ADB, (ii) ∠AEB
Solution:



(i) As, it's seen that ∠ACB and ∠ADB are in the same segment,
So,
∠ADB = 2 ∠ACB = 60°

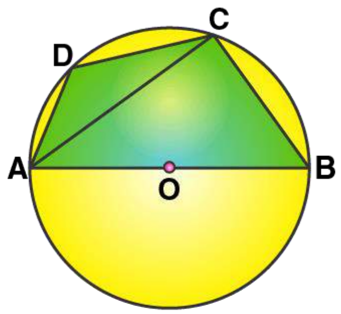
(ii) Now, join OA and OB.
And, we have
∠AEB = ½ Reflex (∠AOB) = ½ (360° – 120°) = 120°
[Angle at the center is double the angle at the circumference subtend by the same chord]

12. Given: ∠CAB = 75° and ∠CBA = 50°. Find the value of ∠DAB + ∠ABD.
Solution:



Given, ∠CAB = 75° and ∠CBA = 50°
In ΔABC, by angle sum property we have
∠ACB = 180° – (∠CBA + ∠CAB)
= 180° – (50° + 75°) = 180° – 125°
= 55°
And,
∠ADB = ∠ACB = 55°
[Angles subtended by the same chord on the circle are equal]
Now, taking ΔABD
∠DAB + ∠ABD + ∠ADB = 180°
∠DAB + ∠ABD + 55° = 180°
∠DAB + ∠ABD = 180° - 55°
∠DAB + ∠ABD = 125°

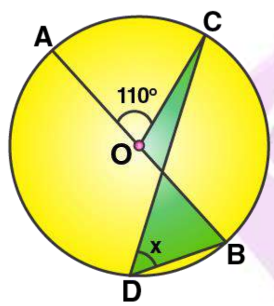
13. ABCD is a cyclic quadrilateral in a circle with centre O. If ∠ADC = 130°, find ∠BAC.



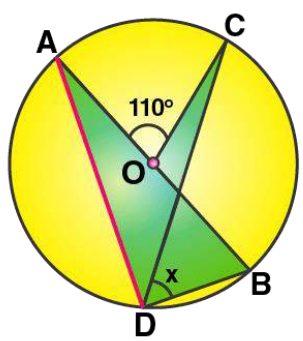
Solution:

From the fig. its seem that,
∠ACB = 90° [Angle in a semi-circle is 90°]
Also,
∠ABC = 180° - ∠ADC = 180° - 130° = 50°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
By angle sum property of the right triangle ACB, we have
∠BAC = 90° - ∠ABC
= 90° – 50°
Thus, ∠BAC = 40°

14. In the figure given alongside, AOB is a diameter of the circle and ∠AOC = 110°, find ∠BDC.

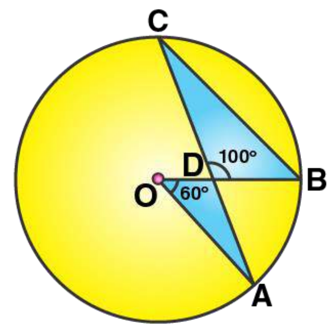


Solution:



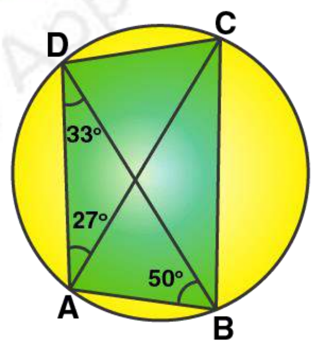
Let's join AD first.
So, we have
∠ADC = ½ ∠AOC = ½ x 110° = 55°
[Angle at the centre is double the angle at the circumference subtended by the same chord]
Also, we know that
∠ADB = 90°
[Angle in the semi-circle is a right angle]
Therefore,
∠BDC = 90° - ∠ADC = 90° – 55°
∠BDC = 35°

15. In the following figure, O is the centre of the circle; ∠AOB = 60º and ∠BDC = 100º, find ∠OBC.
Solution:



Form the figure, we have
∠ACB = ½ ∠AOB = ½ x 60º = 30º
[Angle at the centre is double the angle at the circumference subtended by the same chord]
Now, by applying angle sum property in ΔBDC,
∠DBC = 180º - 100º – 30º = 50º
Therefore,
∠OBC = 50º

16. In ABCD is a cyclic quadrilateral in which ∠DAC = 27º, ∠DBA = 50º and ∠ADB = 33º. Calculate (i) ∠DBC, (ii) ∠DCB, (iii) ∠CAB.
Solution:

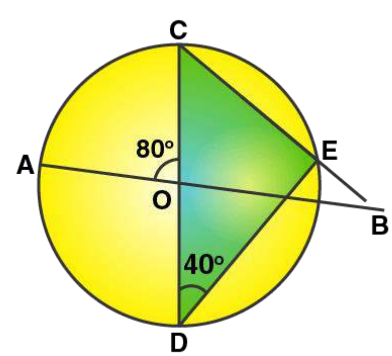


(i) It's seen that,
∠DBC = ∠DAC = 27º
[Angles subtended by the same chord on the circle are equal]

(ii) It's seen that,
∠ACB = ∠ADB = 33º
And,
∠ACD = ∠ABD = 50º
[Angles subtended by the same chord on the circle are equal]
Thus,
∠DCB = ∠ACD + ∠ACB = 50º + 33º = 83º

(iii) In quad. ABCD,
∠DAB + ∠DCB = 180º
27º + ∠CAB + 83º = 180º
Thus,
∠CAB = 180º – 110º = 70º

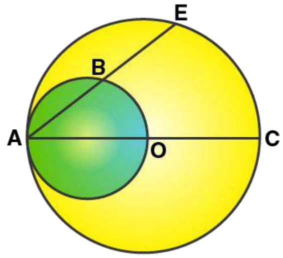
17. In the figure given alongside, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 80º and ∠CDE = 40º. Find the number of degrees in: (i) ∠DCE; (ii) ∠ABC.
Solution:



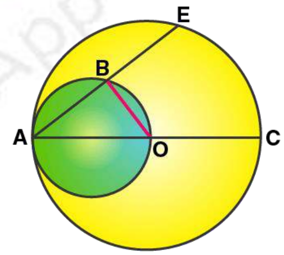
(i) Form the fig. its seen that,
∠DCE = 90º - ∠CDE = 90º – 40º = 50º
Therefore,
∠DEC = ∠OCB = 50º

(ii) In ΔBOC, we have
∠AOC = ∠OCB + ∠OBC [Exterior angle property of a triangle]
∠OBC = 80° - 50° = 30° [Given ∠AOC = 80°]
Therefore, ∠ABC = 30°

18. In the figure given below, AC is a diameter of a circle, whose centre is O. A circle is described on AO as diameter. AE, a chord of the larger circle, intersects the smaller circle at B. Prove that AB = BE.

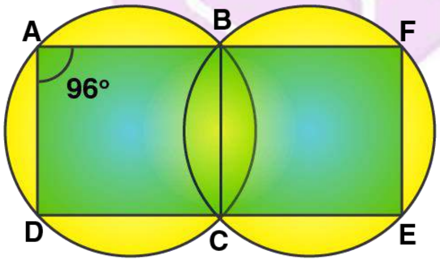


Solution:



Firstly, join OB.
Then, ∠OBA = 90° [Angle in a semi-circle is a right angle]
That is, OB is perpendicular to AE.
Now, we know that the perpendicular draw from the centre to a chord bisects the chord.
Therefore,
AB = BE

19. (a) In the following figure,



(i) if ∠BAD = 96°, find ∠BCD and ∠BFE.
(ii) Prove that AD is parallel to FE.
(b) ABCD is a parallelogram. A circle
Solution:

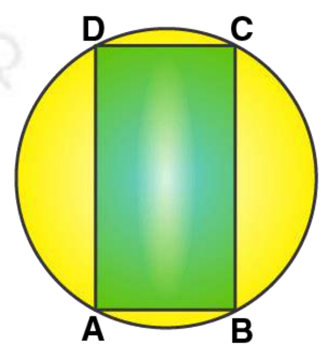
(i) ABCD is a cyclic quadrilateral
So, ∠BAD + ∠BCD = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
∠BCD = 180° - 96° = 84°

And, ∠BCE = 180° - 84° = 96° [Linear pair of angles]
Similarly, BCEF is a cyclic quadrilateral
So, ∠BCE + ∠BFE = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
∠BFE = 180° - 96° = 84°

(ii) Now, ∠BAD + ∠BFE = 96° + 84° = 180°
But these two are interior angles on the same side of a pair of lines AD and FE.
Therefore, AD || FE.

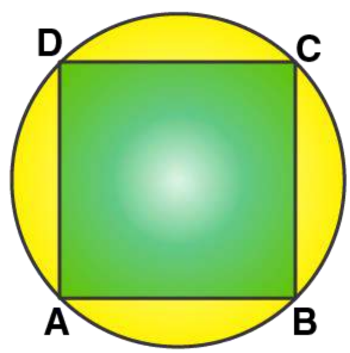
20. Prove that:
(i) the parallelogram, inscribed in a circle, is a rectangle.
(ii) the rhombus, inscribed in a circle, is a square.
Solution:

(i) Let's assume that ABCD is a parallelogram which is inscribed in a circle.



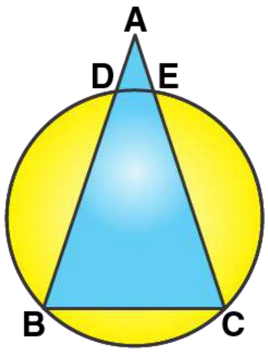
So, we have
∠BAD = ∠BCD [Opposite angles of a parallelogram are equal]
And ∠BAD + ∠BCD = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
So, 2∠BAD = 180°
Thus, ∠BAD = ∠BCD = 90°
Similarly, the remaining two angles are 90° each and pair of opposite sides are equal.
Therefore,
ABCD is a rectangle.
- Hence Proved

(ii) Let's assume that ABCD is a rhombus which is inscribed in a circle.



So, we have
∠BAD = ∠BCD [Opposite angles of a rhombus are equal]
And ∠BAD + ∠BCD = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
So, 2∠BAD = 180°
Thus, ∠BAD = ∠BCD = 90°
Similarly, the remaining two angles are 90° each and all the sides are equal.
Therefore,
ABCD is a square.
- Hence Proved

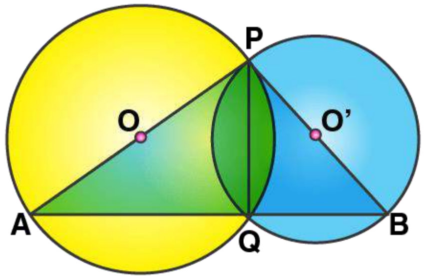
21. In the following figure, AB = AC. Prove that DECB is an isosceles trapezium.



Solution:

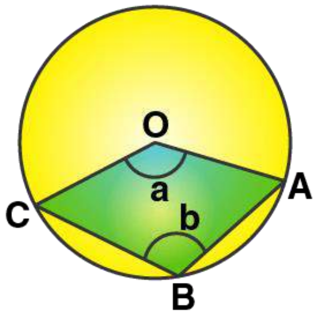
Give, AB = AC
So, ∠B = ∠C … (1)
[Angles opposite to equal sides are equal]
And, DECB is a cyclic quadrilateral.
So, ∠B + ∠DEC = 180^o
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
∠C + ∠DEC = 180^o …. (Using 1)
But this is the sum of interior angles on one side of a transversal.
DE || BC.
But, ∠ADE = ∠B and ∠AED = ∠C [Corresponding angles]
Thus, ∠ADE = ∠AED
AD = AE
AB – AD = AC = AE [As AB = AC]
BD = CE
Hence, we have DE || BC and BD = CE
Therefore,
DECB is an isosceles trapezium.

22. Two circles intersect at P and Q. Through P diameters PA and PB of the two circles are drawn. Show that the points A, Q and B are collinear.
Solution:



Let O and O' be the centres of two intersecting circles, where points of the intersection are P and Q and PA and PB are their diameters respectively.
Join PQ, AQ and QB.
Thus, ∠AQP = 90^o and ∠BQP = 90^o
[Angle in a semicircle is a right angle]
Now, adding both these angles we get
∠AQP + ∠BQP = 180^o
∠AQB = 180^o
Therefore, the points A, Q and B are collinear.

23. The figure given below, shows a circle with centre O. Given: ∠AOC = a and ∠ABC = b.
(i) Find the relationship between a and b
(ii) Find the measure of angle OAB, if OABC is a parallelogram.

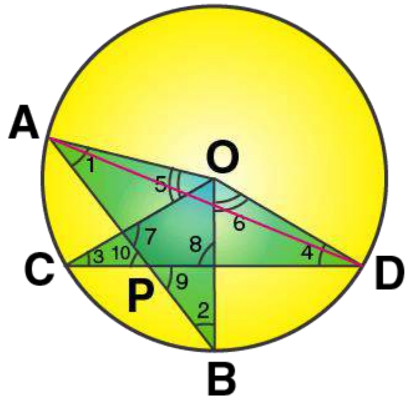


Solution:

(i) It's seen that,
∠ABC = ½ Reflex (∠COA)
[Angle at the centre is double the angle at the circumference subtended by the same chord]
So, b = ½ (360° - a)
a + 2b = 180° ….. (1)

(ii) As OABC is a parallelogram, the opposite angles are equal.
So, a = b
Now, using the above relationship in (1)
3a = 180°
a = 60°
Also, OC || BA
∠COA + ∠OAB = 180°
60° + ∠OAB = 180°
Therefore,
∠OAB = 120°

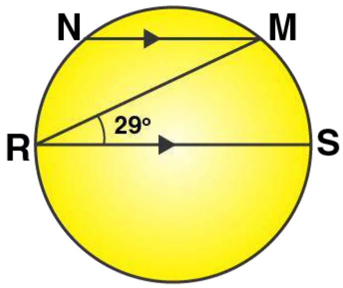
24. Two chords AB and CD intersect at P inside the circle. Prove that the sum of the angles subtended by the arcs AC and BD as the center O is equal to twice the angle APC
Solution:



Required to prove: ∠AOC + ∠BOD = 2∠APC
OA, OB, OC and OD are joined.
Also, AD is joined.
Now, it's seen that
∠AOC = 2∠ADC …. (1)
[Angle at the centre is double the angle at the circumference subtended by the same chord]
Similarly,
∠BOD = 2∠BAD …. (2)
Adding (1) and (2), we have
∠AOC + ∠BOD = 2∠ADC + 2∠BAD
= 2(∠ADC + ∠BAD) ….. (3)

And in ΔPAD,
Ext. ∠APC = ∠PAD + ∠ADC
= ∠BAD + ∠ADC …. (4)
So, from (3) and (4) we have
∠AOC + ∠BOD = 2∠APC

25. In the figure given RS is a diameter of the circle. NM is parallel to RS and ∠MRS = 29º Calculate: (i) ∠RNM; (ii) ∠NRM.



Solution:

(i) Join RN and MS

∠RMS = 90º [Angle in a semi-circle is a right angle]

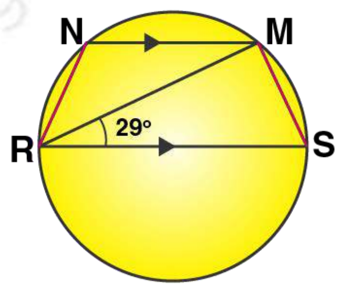
So, by angle sum property of ΔRMS

∠RMS = 90º – 29º = 61º

And,

∠RNM = 180º - ∠RSM = 180º – 61º = 119º

[Pair of opposite angles in a cyclic quadrilateral are supplementary]



(ii) Now as RS || NM,

∠NMR = ∠MRS = 29º [Alternate angles]

∠NMS = 90º + 29º = 119º

Also, we know that

∠NRS + ∠NMS = 180º

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

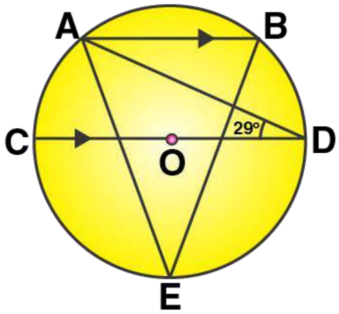
∠NRM + 29º + 119º = 180º

∠NRM = 180º – 148º

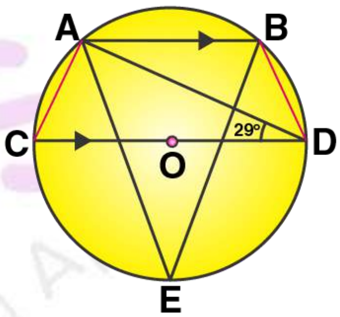
Therefore,

∠NRM = 32º

26. In the figure given alongside, AB || CD and O is the center of the circle. If ∠ADC = 25º; find the angle AEB. Give reasons in support of your answer.

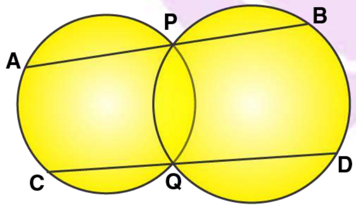


Solution:

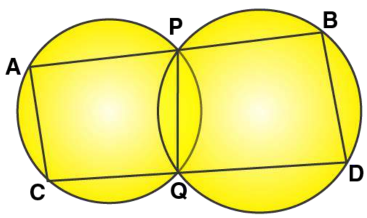


Join AC and BD.
So, we have
∠CAD = 90° and ∠CBD = 90°
[Angle is a semicircle is a right angle]
And, AB || CD
So, ∠BAD = ∠ADC = 25° [Alternate angles]
∠BAC = ∠BAD + ∠CAD = 25° + 90° = 115°
Thus,
∠ADB = 180° – 25° - ∠BAC = 180° – 25° – 115° = 40°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
Finally,
∠AEB = ∠ADB = 40°
[Angles subtended by the same chord on the circle are equal]

27. Two circles intersect at P and Q. Through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. Prove that AC is parallel to BD.



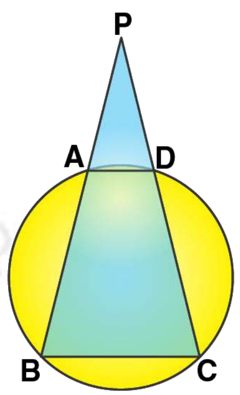
Solution:



Let's join AC, PQ and BD.
As ACQP is a cyclic quadrilateral
∠CAP + ∠PQC = 180° ……. (i)
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
Similarly, as PQDB is a cyclic quadrilateral
∠PQD + ∠DBP = 180° ……. (ii)
Again, ∠PQC + ∠PQD = 180° …… (iii) [Linear pair of angles]
Using (i), (ii) and (iii) we have

∠CAP + ∠DBP = 180^o
Or ∠CAB + ∠DBA = 180^o
We know that, if the sum of interior angles between two lines when intersected by a transversal are supplementary.
Then, AC || BD.

28. ABCD is a cyclic quadrilateral in which AB and DC on being produced, meet at P such that PA = PD. Prove that AD is parallel to BC.
Solution:



Let's assume that ABCD be the given cyclic quadrilateral.
Also, PA = PD [Given]
So, ∠PAD = ∠PDA …… (1)
[Angles opposite to equal sides are equal]
And,
∠BAD = 180^o - ∠PAD [Linear pair of angles]
Similarly,
∠CDA = 180^o - ∠PDA = 180^o - ∠PAD [From (1)]
As the opposite angles of a cyclic quadrilateral are supplementary,
∠ABC = 180^o - ∠CDA = 180^o – (180^o - ∠PAD) = ∠PAD
And, ∠DCB = 180^o - ∠BAD = 180^o – (180^o - ∠PAD) = ∠PAD
Thus,
∠ABC = ∠DCB = ∠PAD = ∠PDA
Which is only possible when AD || BC.