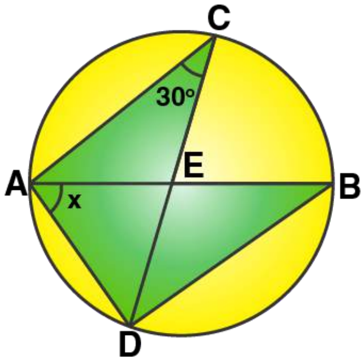
Exercise 17(C)

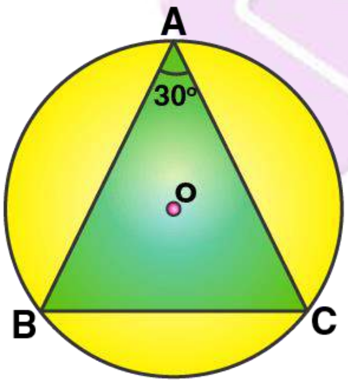
1. In the given circle with diameter AB, find the value of x.



Solution:

Now,
$\angle ABD = \angle ACD = 30^o$ [Angles in the same segment]
In ΔADB, by angle sum property we have
$\angle BAD + \angle ADB + \angle ABD = 180^o$
But, we know that angle in a semi-circle is 90^o
$\angle ADB = 90^o$
So,
$x + 90^o + 30^o = 180^o$
$x = 180^o - 120^o$
Hence, $x = 60^o$

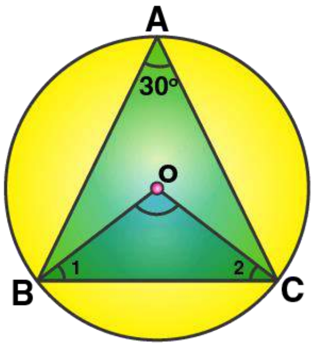
2. In the given figure, ABC is a triangle in which $\angle BAC = 30^o$. Show that BC is equal to the radius of the circum-circle of the triangle ABC, whose center is O.



Solution:

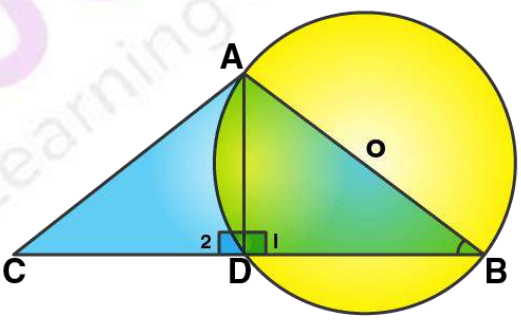
Firstly, join OB and OC.
Proof:
$\angle BOC = 2\angle BAC = 2 \times 30^o = 60^o$
Now, in ΔOBC
OB = OC [Radii of same circle]

So, ∠OBC = ∠OCB [Angles opposite to equal sides]
And in ΔOBC, by angle sum property we have
∠OBC + ∠OCB + ∠BOC = 180°
∠OBC + ∠OBC + 60° = 180°
2 ∠OBC = 180° – 60° = 120°
∠OBC = 120°/ 2 = 60°
So, ∠OBC = ∠OCB = ∠BOC = 60°
Thus, ΔOBC is an equilateral triangle.
So,
BC = OB = OC
But, OB and OC are the radii of the circum-circle.
Therefore, BC is also the radius of the circum-circle.

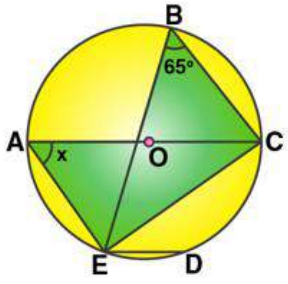


3. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.
Solution:

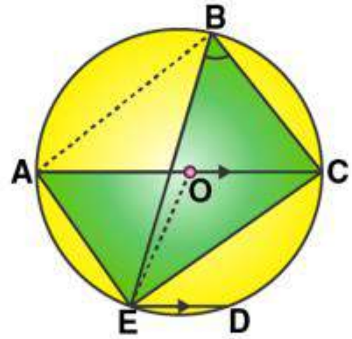
Let's consider ΔABC, AB = AC and circle with AB as diameter is drawn which intersects the side BC and D.
And, join AD
Proof:
It's seen that,
∠ADB = 90° [Angle in a semi-circle]
And,
∠ADC + ∠ADB = 180° [Linear pair]
Thus, ∠ADC = 90°
Now, in right ΔABD and ΔACD
AB = AC [Given]
AD = AD [Common]
∠ADB = ∠ADC = 90°
Hence, by R.H.S criterion of congruence.
ΔABD ≅ ΔACD
Now, by CPCT
BD = DC
Therefore, D is the mid-point of BC.



4. In the given figure, chord ED is parallel to diameter AC of the circle. Given ∠CBE = 65°, calculate ∠DEC.



Solution:



Join OE.

Arc EC subtends ∠EOC at the centre and ∠EBC at the remaining part of the circle.

∠EOC = 2∠EBC = 2 x 65° = 130°

Now, in ΔOEC

OE = OC [Radii of the same circle]

So, ∠OEC = ∠OCE

But, in ΔEOC by angle sum property

∠OEC + ∠OCE + ∠EOC = 180° [Angles of a triangle]

∠OCE + ∠OCE + ∠EOC = 180°

2 ∠OCE + 130° = 180°

2 ∠OCE = 180° - 130°

∠OCE = 50°/ 2 = 25°

And, AC || ED [Given]

∠DEC = ∠OCE [Alternate angles]

Thus,

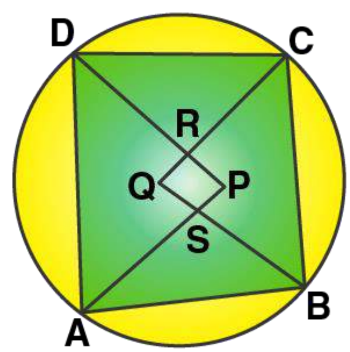
∠DEC = 25°

5. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic. Prove it.

Solution:

Let ABCD be a cyclic quadrilateral and PQRS be the quadrilateral formed by the angle bisectors of angle ∠A, ∠B, ∠C and ∠D.

Required to prove: PQRS is a cyclic quadrilateral.



Proof:

By angle sum property of a triangle

In ΔAPD,

∠PAD + ∠ADP + ∠APD = 180° …. (i)

And, in ΔBQC

∠QBC + ∠BCQ + ∠BQC = 180° …. (ii)

Adding (i) and (ii), we get

∠PAD + ∠ADP + ∠APD + ∠QBC + ∠BCQ + ∠BQC = 180° + 180° = 360° …… (iii)

But,

∠PAD + ∠ADP + ∠QBC + ∠BCQ = ½ [∠A + ∠B + ∠C + ∠D]

= ½ x 360° = 180°

Therefore,

∠APD + ∠BQC = 360° - 180° = 180° [From (iii)]

But, these are the sum of opposite angles of quadrilateral PRQS.

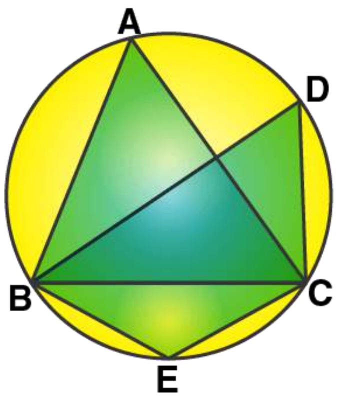
Therefore,

Quadrilateral PQRS is also a cyclic quadrilateral.

6. In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate:

(i) ∠BDC

(ii) ∠BEC
(iii) ∠BAC



Solution:

(i) Given that BD is a diameter of the circle.
And, the angle in a semicircle is a right angle.
So, ∠BCD = 90°
Also given that,
∠DBC = 58°
In ΔBDC,
∠DBC + ∠BCD + ∠BDC = 180°
58° + 90° + ∠BDC = 180°
148° + ∠BDC = 180°
∠BDC = 180° - 148°
Thus, ∠BDC = 32°

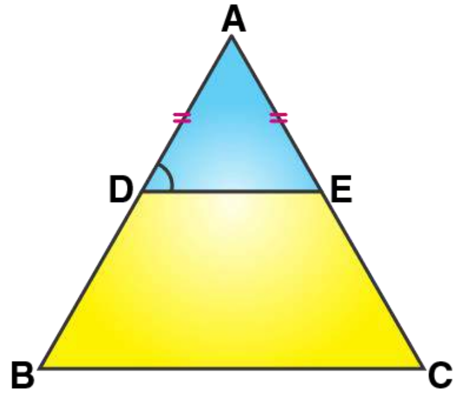
(ii) We know that, the opposite angles of a cyclic quadrilateral are supplementary.
So, in cyclic quadrilateral BECD
∠BEC + ∠BDC = 180°
∠BEC + 32° = 180°
∠BEC = 148°

(iii) In cyclic quadrilateral ABEC,
∠BAC + ∠BEC = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
∠BAC + 148° = 180°
∠BAC = 180° - 148°
Thus, ∠BAC = 32°

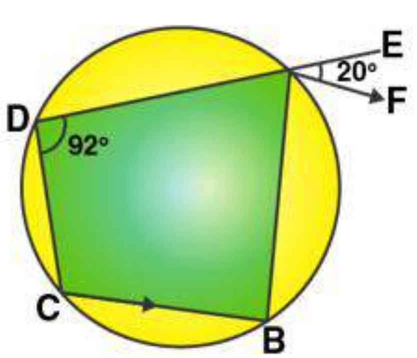
7. D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Prove that the points B, C, E and D are concyclic.
Solution:

Given,
ΔABC, AB = AC and D and E are points on AB and AC such that AD = AE.

And, DE is joined.
Required to prove: Points B, C, E and D are concyclic
Proof:
In ΔABC,
AB = AC [Given]
So, ∠B = ∠C [Angles opposite to equal sides]
Similarly,
In ΔADE,
AD = AE [Given]
So, ∠ADE = ∠AED [Angles opposite to equal sides]
Now, in ΔABC we have
AD/AB = AE/AC
Hence, DE || BC [Converse of BPT]
So,
∠ADE = ∠B [Corresponding angles]
$(180^o - \angle EDB) = \angle B$
$\angle B + \angle EDB = 180^o$
But, it's proved above that
∠B = ∠C
So,
$\angle C + \angle EDB = 180^o$
Thus, opposite angles are supplementary.
Similarly,
$\angle B + \angle CED = 180^o$
Hence, B, C, E and D are concyclic.



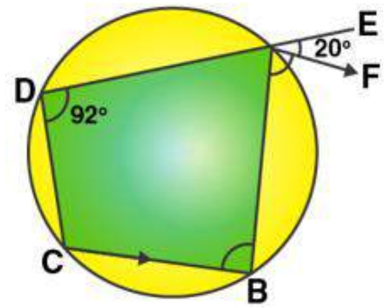
8. In the given figure, ABCD is a cyclic quadrilateral. AF is drawn parallel to CB and DA is produced to point E. If ∠ADC = 92º, ∠FAE = 20º; determine ∠BCD. Given reason in support of your answer.



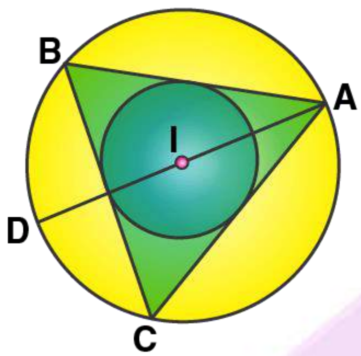
Solution:

Given,
In cyclic quad. ABCD
AF || CB and DA is produced to E such that $\angle ADC = 92^o$ and $\angle FAE = 20^o$
So,
$\angle B + \angle D = 180^o$

$\angle B + 92^o = 180^o$
$\angle B = 88^o$
As AF || CB, $\angle FAB = \angle B = 88^o$
But, $\angle FAD = 20^o$ [Given]
Ext. $\angle BAE = \angle BAF + \angle FAE$
$= 88^o + 22^o = 108^o$
But, Ext. $\angle BAE = \angle BCD$
Therefore,
$\angle BCD = 108^o$

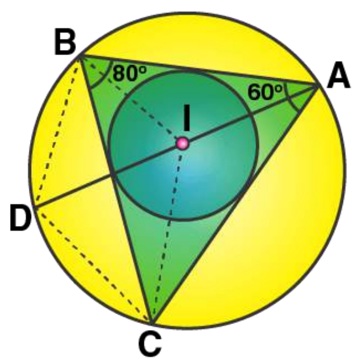


9. If I is the incentre of triangle ABC and AI when produced meets the circumcircle of triangle ABC in point D. If ∠BAC = 66° and ∠ABC = 80°. Calculate:
(i) ∠DBC,
(ii) ∠IBC,
(iii) ∠BIC



Solution:

Join DB and DC, IB and IC.
Given, if $\angle BAC = 66^o$ and $\angle ABC = 80^o$, I is the incentre of the ΔABC.



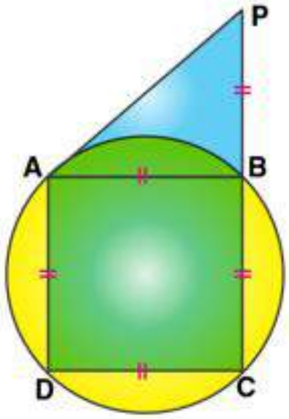
(i) As it's seen that ∠DBC and ∠DAC are in the same segment,
So, $\angle DBC = \angle DAC$
But, $\angle DAC = ½ \angle BAC = ½ \times 66^o = 33^o$
Thus, $\angle DBC = 33^o$

(ii) And, as I is the incentre of ΔABC, IB bisects ∠ABC.
Therefore,
$\angle IBC = ½ \angle ABC = ½ \times 80^o = 40^o$

(iii) In ΔABC, by angle sum property
$\angle ACB = 180^o – (\angle ABC + \angle BAC)$
$\angle ACB = 180^o – (80^o + 66^o)$
$\angle ACB = 180^o – 156^o$
$\angle ACB = 34^o$
And since, IC bisects ∠C
Thus, $\angle ICB = ½ \angle C = ½ \times 34^o = 17^o$

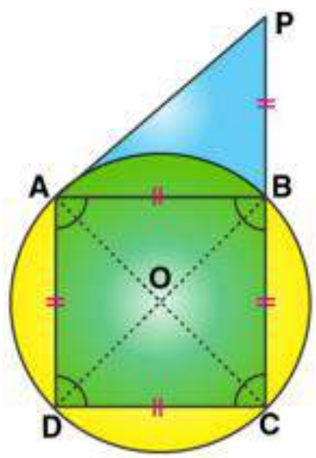
Now, in ΔIBC
∠IBC + ∠ICB + ∠BIC = 180°
40° + 17° + ∠BIC = 180°
57° + ∠BIC = 180°
∠BIC = 180° – 57°
Therefore, ∠BIC = 123°

10. In the given figure, AB = AD = DC = PB and ∠DBC = x°. Determine, in terms of x:
(i) ∠ABD, (ii) ∠APB.
Hence or otherwise, prove that AP is parallel to DB.

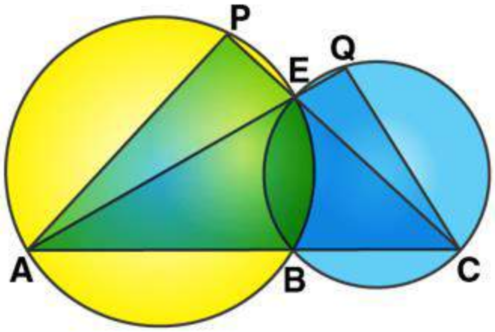


Solution:

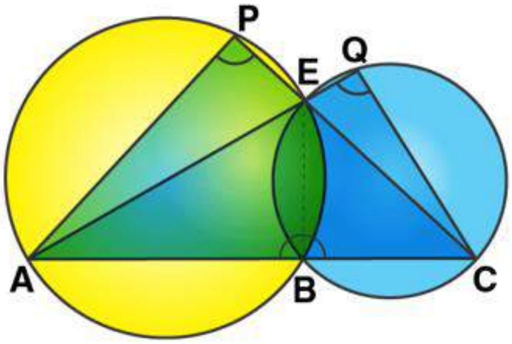
Given, AB = AD = DC = PB and ∠DBC = x°
Join AC and BD.
Proof:
∠DAC = ∠DBC = x° [Angles in the same segment]
And, ∠DCA = ∠DAC = x° [As AD = DC]
Also, we have
∠ABD = ∠DAC [Angles in the same segment]
And, in ΔABP
Ext. ∠ABD = ∠BAP + ∠APB
But, ∠BAP = ∠APB [Since, AB = BP]
2 x° = ∠APB + ∠APB = 2∠APB
2∠APB = 2x°
So, ∠APB = x°
Thus, ∠APB = ∠DBC = x°
But these are corresponding angles,
Therefore, AP || DB.



11. In the given figure; ABC, AEQ and CEP are straight lines. Show that ∠APE and ∠CQE are supplementary.

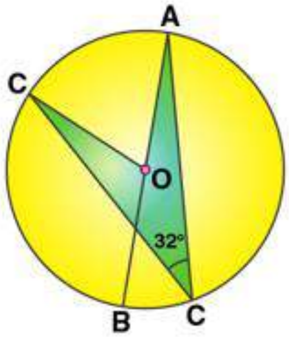


Solution:

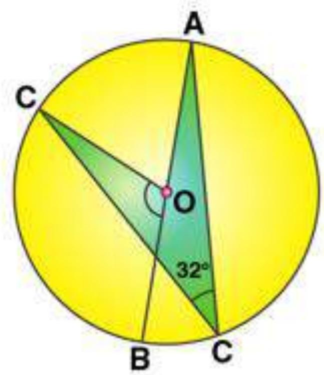


Join EB.
Then, in cyclic quad.ABEP
∠APE + ∠ABE = 180º ….. (i) [Opposite angles of a cyclic quad. are supplementary]
Similarly, in cyclic quad.BCQE
∠CQE + ∠CBE = 180º ….. (ii) [Opposite angles of a cyclic quad. are supplementary]
Adding (i) and (ii), we have
∠APE + ∠ABE + ∠CQE + ∠CBE = 180º + 180º = 360º
∠APE + ∠ABE + ∠CQE + ∠CBE = 360º
But, ∠ABE + ∠CBE = 180º [Linear pair]
∠APE + ∠CQE + 180º = 360º
∠APE + ∠CQE = 180º
Therefore, ∠APE and ∠CQE are supplementary.

12. In the given, AB is the diameter of the circle with centre O.
If ∠ADC = 32º, find angle BOC.



Solution:



Arc AC subtends ∠AOC at the centre and ∠ADC at the remaining part of the circle.
Thus, ∠AOC = 2∠ADC
∠AOC = 2 x 32º = 64º
As ∠AOC and ∠BOC are linear pair, we have
∠AOC + ∠BOC = 180º

$64^\circ + \angle BOC = 180^\circ$
$\angle BOC = 180^\circ - 64^\circ$
Therefore, $\angle BOC = 116^\circ$