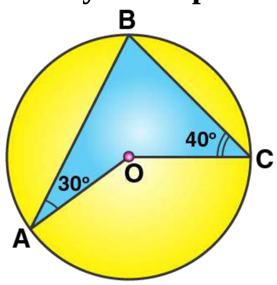
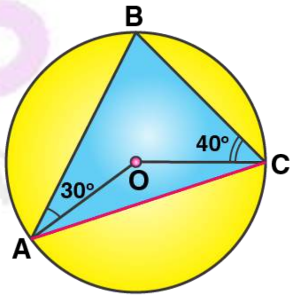
Exercise 17(A) Page No: 257

1. In the given figure, O is the center of the circle. ∠OAB and ∠OCB are 30º and 40º respectively. Find ∠AOC Show your steps of working.

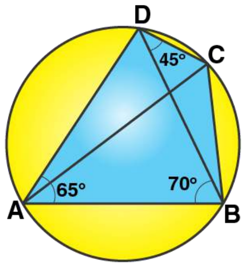


Solution:



Firstly, let's join AC.
And, let ∠OAC = ∠OCA = x [Angles opposite to equal sides are equal]
So, ∠AOC = 180º – 2x
Also,
∠BAC = 30º + x
∠BCA = 40º + x
Now, in ΔABC
∠ABC = 180º - ∠BAC - ∠BCA [Angles sum property of a triangle]
= 180º – (30º + x) – (40º + x)
= 110º – 2x
And, ∠AOC = 2∠ABC
[Angle at the center is double the angle at the circumference subtend by the same chord]
180º – 2x = 2(110º – 2x)
2x = 40º
x = 20º
Thus, ∠AOC = 180º – 2x20º = 140º

2. In the given figure, ∠BAD = 65°, ∠ABD = 70°, ∠BDC = 45°



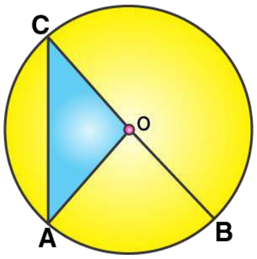
(i) Prove that AC is a diameter of the circle.
(ii) Find ∠ACB.
Solution:

(i) In ΔABD,
∠DAB + ∠ABD + ∠ADB = 180º

$65^\circ + 70^\circ + \angle ADB = 180^\circ$
$135^\circ + \angle ADB = 180^\circ$
$\angle ADB = 180^\circ - 135^\circ = 45^\circ$
Now,
$\angle ADC = \angle ADB + \angle BDC = 45^\circ + 45^\circ = 90^\circ$
As $\angle ADC$ is the angle of semi-circle for AC as the diameter of the circle.

(ii) $\angle ACB = \angle ADB$ [Angles in the same segment of a circle]
Hence, $\angle ACB = 45^\circ$

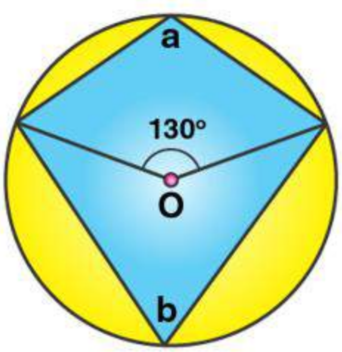
3. Given O is the centre of the circle and $\angle AOB = 70^\circ$.



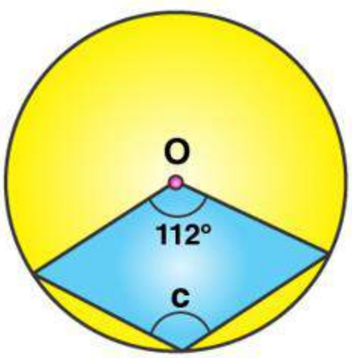
Calculate the value of:
(i) $\angle OCA$,
(ii) $\angle OAC$.
Solution:

Here, $\angle AOB = 2\angle ACB$
[Angle at the center is double the angle at the circumference subtend by the same chord]
$\angle ACB = 70^\circ/ 2 = 35^\circ$
Now, OC = OA [Radii of same circle]
Thus,
$\angle OCA = \angle OAC = 35^\circ$

4. In each of the following figures, O is the centre of the circle. Find the values of a, b and c.
Solution:



(i)



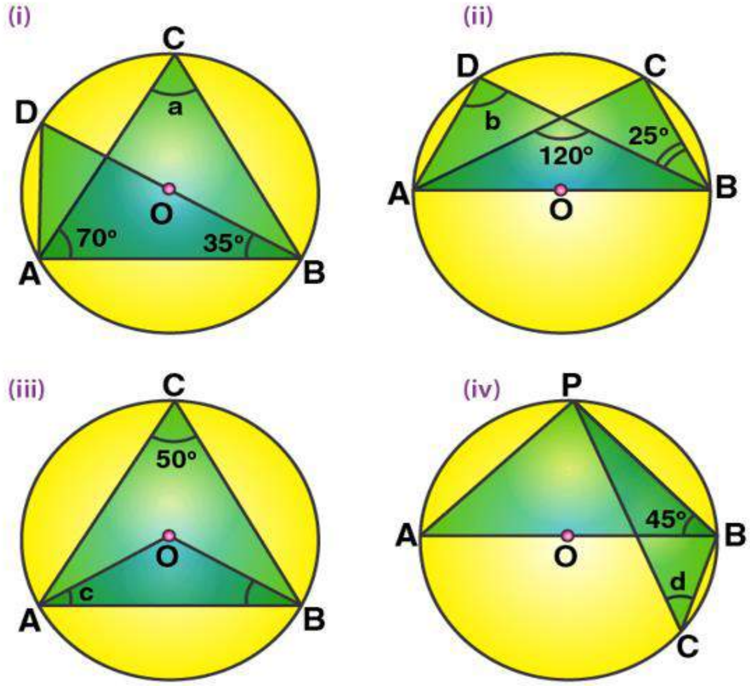
(ii)

(i) Here, $b = \frac{1}{2} \times 130^\circ$
[Angle at the center is double the angle at the circumference subtend by the same chord]
Thus, $b = 65^\circ$

Now,
a + b = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
a = 180° – 65° = 115°

(ii) Here, c = ½ x Reflex (112°)
[Angle at the center is double the angle at the circumference subtend by the same chord]
Thus, c = ½ x (360° – 112°) = 124°

5. In each of the following figures, O is the center of the circle. Find the values of a, b, c and d.



Solution:

(i) Here, ∠BAD = 90° [Angle in a semi-circle]
So, ∠BDA = 90° – 35° = 55°
And,
a = ∠ACB = ∠BDA = 55°
[Angles subtended by the same chord on the circle are equal]

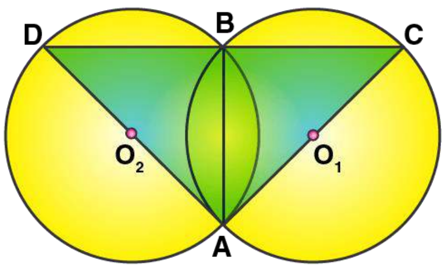
(ii) Here, ∠DAC = ∠CBD = 25°
[Angles subtended by the same chord on the circle are equal]
And, we have
120° = b + 25°
[Exterior angle property of a triangle]
b = 95°

(iii) ∠AOB = 2∠AOB = 2 x 50° = 100°
[Angle at the center is double the angle at the circumference subtend by the same chord]
Also, OA = OB

∠OBA = ∠OAB = c
c = (180° - 100°)/ 2 = 40°

(iv) We have, ∠APB = 90° [Angle in a semicircle]
∠BAP = 90° – 45° = 45°
Now, d = ∠BCP = ∠BAP = 45°
[Angles subtended by the same chord on the circle are equal]

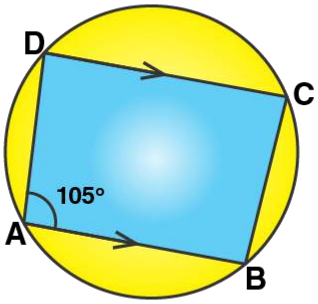
6. In the figure, AB is common chord of the two circles. If AC and AD are diameters; prove that D, B and C are in a straight line. O_1 and O_2 are the centers of two circles.



Solution:

It's seen that,
∠DBA = ∠CBA = 90° [Angle in a semi-circle is a right angle]
So, adding both
∠DBA + ∠CBA = 180°
Thus, DBC is a straight line i.e. D, B and C form a straight line.

7. In the figure, given below, find:
(i) ∠BCD,
(ii) ∠ADC,
(iii) ∠ABC.
Show steps of your working.



Solution:

From the given fig, it's seen that
In cyclic quadrilateral ABCD, DC || AB
And given, ∠DAB = 105°

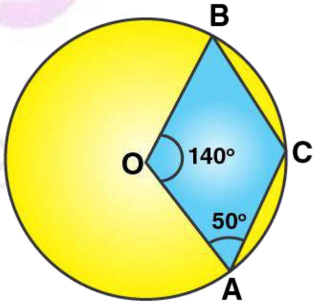
(i) So,
∠BCD = 180° – 105° = 75°
[Sum of opposite angles in a cyclic quadrilateral is 180°]

(ii) Now,
∠ADC and ∠DAB are corresponding angles.

So,
∠ADC + ∠DAB = 180°
∠ADC = 180° – 105°
Thus,
∠ADC = 75°

(iii) We know that, the sum of angles in a quadrilateral is 360°
So,
∠ADC + ∠DAB +∠BCD + ∠ABC = 360°
75° + 105° + 75° + ∠ABC = 360°
∠ABC = 360° – 255°
Thus,
∠ABC = 105°

8. In the figure, given below, O is the centre of the circle. If ∠AOB = 140° and ∠OAC = 50°; find:
(i) ∠ACB,
(ii) ∠OBC,
(iii) ∠OAB,
(iv) ∠CBA.

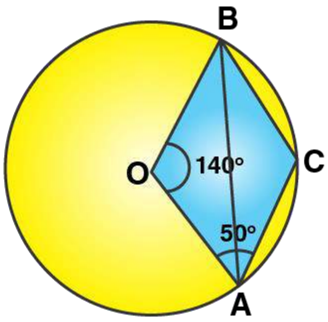


Solution:

Given, ∠AOB = 140° and ∠OAC = 50°

(i) Now,
∠ACB = ½ Reflex (∠AOB) = ½ (360° – 140°) = 110°
[Angle at the center is double the angle at the circumference subtend by the same chord]

(ii) In quadrilateral OBCA,
∠OBC + ∠ACB + ∠OCA + ∠AOB = 360° [Angle sum property of a quadrilateral]
∠OBC + 110° + 50° + 140° = 360°
Thus, ∠OBC = 360° – 300° = 60°

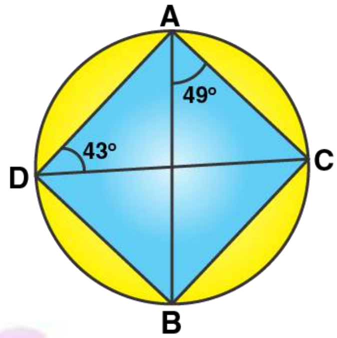


(iii) In ΔAOB, we have
OA = OB (radii)
So, ∠OBA = ∠OAB
Hence, by angle sum property of a triangle
∠OBA + ∠OAB + ∠AOB = 180°
2∠OBA + 140° = 180°
2∠OBA = 40°
∠OBA = 20°

(iv) We already found, ∠OBC = 60°
And, ∠OBC = ∠CBA + ∠OBA
60° = ∠CBA + 20°

Therefore,
∠CBA = 40°

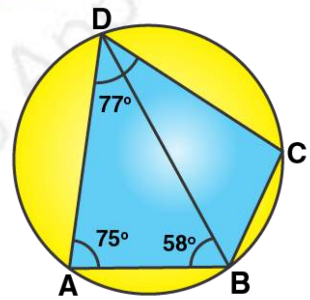
9. Calculate:
(i) ∠CDB,
(ii) ∠ABC,
(iii) ∠ACB.



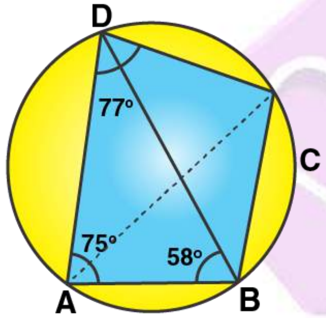
Solution:

Here, we have
∠CDB = ∠BAC = 49°
∠ABC = ∠ADC = 43°
[Angles subtended by the same chord on the circle are equal]
Now, by angle sum property of a triangle we have
∠ACB = 180° – 49° – 43° = 88°

10. In the figure given below, ABCD is a cyclic quadrilateral in which ∠BAD = 75°; ∠ABD = 58° and ∠ADC = 77°.
Find:
(i) ∠BDC,
(ii) ∠BCD,
(iii) ∠BCA.



Solution:

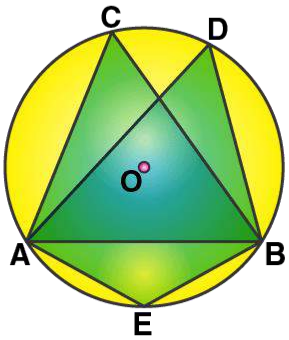


(i) By angle sum property of triangle ABD,
∠ADB = 180° – 75° – 58° = 47°
Thus, ∠BDC = ∠ADC - ∠ADB = 77° – 47° = 30°

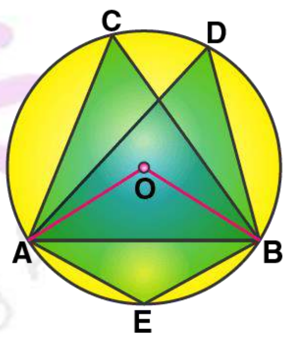
(ii) ∠BAD + ∠BCD = 180°
[Sum of opposite angles of a cyclic quadrilateral is 180°]
Thus, ∠BCD = 180° - 75° = 105°

(iii) ∠BCA = ∠ADB = 47°
[Angles subtended by the same chord on the circle are equal]

11. In the figure given below, O is the centre of the circle and triangle ABC is equilateral.



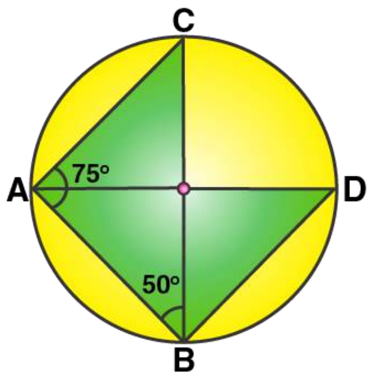
Find:
(i) ∠ADB, (ii) ∠AEB
Solution:



(i) As, it's seen that ∠ACB and ∠ADB are in the same segment,
So,
∠ADB = 2 ∠ACB = 60°

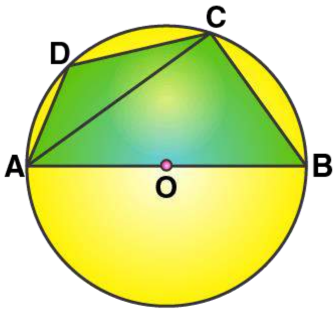
(ii) Now, join OA and OB.
And, we have
∠AEB = ½ Reflex (∠AOB) = ½ (360° – 120°) = 120°
[Angle at the center is double the angle at the circumference subtend by the same chord]

12. Given: ∠CAB = 75° and ∠CBA = 50°. Find the value of ∠DAB + ∠ABD.
Solution:



Given, ∠CAB = 75° and ∠CBA = 50°
In ΔABC, by angle sum property we have
∠ACB = 180° – (∠CBA + ∠CAB)
= 180° – (50° + 75°) = 180° – 125°
= 55°
And,
∠ADB = ∠ACB = 55°
[Angles subtended by the same chord on the circle are equal]
Now, taking ΔABD
∠DAB + ∠ABD + ∠ADB = 180°
∠DAB + ∠ABD + 55° = 180°
∠DAB + ∠ABD = 180° - 55°
∠DAB + ∠ABD = 125°

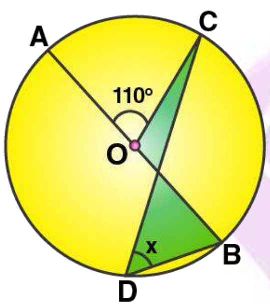
13. ABCD is a cyclic quadrilateral in a circle with centre O. If ∠ADC = 130°, find ∠BAC.



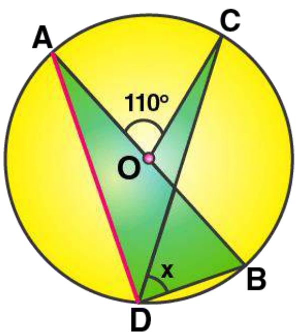
Solution:

From the fig. its seem that,
∠ACB = 90° [Angle in a semi-circle is 90°]
Also,
∠ABC = 180° - ∠ADC = 180° - 130° = 50°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
By angle sum property of the right triangle ACB, we have
∠BAC = 90° - ∠ABC
= 90° – 50°
Thus, ∠BAC = 40°

14. In the figure given alongside, AOB is a diameter of the circle and ∠AOC = 110°, find ∠BDC.

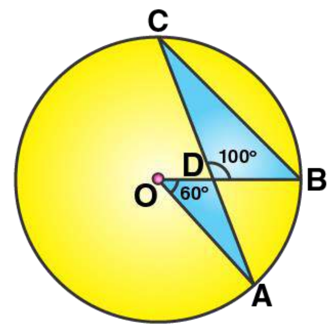


Solution:



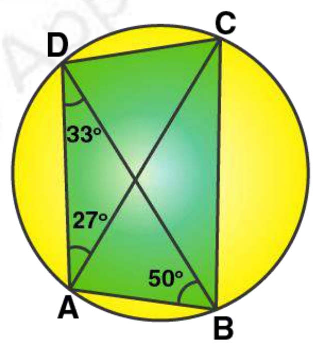
Let's join AD first.
So, we have
∠ADC = ½ ∠AOC = ½ x 110° = 55°
[Angle at the centre is double the angle at the circumference subtended by the same chord]
Also, we know that
∠ADB = 90°
[Angle in the semi-circle is a right angle]
Therefore,
∠BDC = 90° - ∠ADC = 90° – 55°
∠BDC = 35°

15. In the following figure, O is the centre of the circle; ∠AOB = 60º and ∠BDC = 100º, find ∠OBC.
Solution:



Form the figure, we have
∠ACB = ½ ∠AOB = ½ x 60º = 30º
[Angle at the centre is double the angle at the circumference subtended by the same chord]
Now, by applying angle sum property in ΔBDC,
∠DBC = 180º - 100º – 30º = 50º
Therefore,
∠OBC = 50º

16. In ABCD is a cyclic quadrilateral in which ∠DAC = 27º, ∠DBA = 50º and ∠ADB = 33º. Calculate (i) ∠DBC, (ii) ∠DCB, (iii) ∠CAB.
Solution:

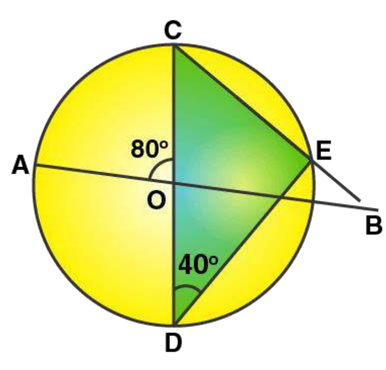


(i) It's seen that,
∠DBC = ∠DAC = 27º
[Angles subtended by the same chord on the circle are equal]

(ii) It's seen that,
∠ACB = ∠ADB = 33º
And,
∠ACD = ∠ABD = 50º
[Angles subtended by the same chord on the circle are equal]
Thus,
∠DCB = ∠ACD + ∠ACB = 50º + 33º = 83º

(iii) In quad. ABCD,
∠DAB + ∠DCB = 180º
27º + ∠CAB + 83º = 180º
Thus,
∠CAB = 180º – 110º = 70º

17. In the figure given alongside, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 80º and ∠CDE = 40º. Find the number of degrees in: (i) ∠DCE; (ii) ∠ABC.
Solution:



(i) Form the fig. its seen that,
∠DCE = 90º - ∠CDE = 90º – 40º = 50º
Therefore,
∠DEC = ∠OCB = 50º

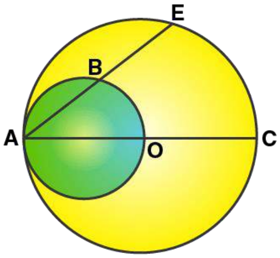
(ii) In ΔBOC, we have

∠AOC = ∠OCB + ∠OBC [Exterior angle property of a triangle]

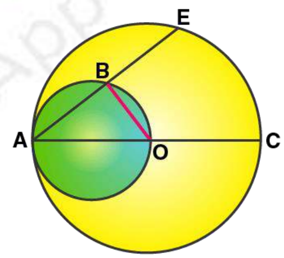
∠OBC = 80° - 50° = 30° [Given ∠AOC = 80°]

Therefore, ∠ABC = 30°

18. In the figure given below, AC is a diameter of a circle, whose centre is O. A circle is described on AO as diameter. AE, a chord of the larger circle, intersects the smaller circle at B. Prove that AB = BE.



Solution:



Firstly, join OB.

Then, ∠OBA = 90° [Angle in a semi-circle is a right angle]

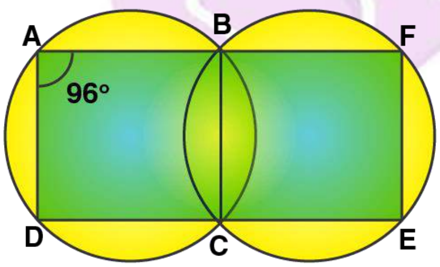
That is, OB is perpendicular to AE.

Now, we know that the perpendicular draw from the centre to a chord bisects the chord.

Therefore,

AB = BE

19. (a) In the following figure,



(i) if ∠BAD = 96°, find ∠BCD and ∠BFE.
(ii) Prove that AD is parallel to FE.
(b) ABCD is a parallelogram. A circle
Solution:

(i) ABCD is a cyclic quadrilateral

So, ∠BAD + ∠BCD = 180°

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

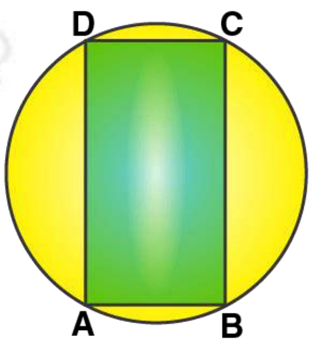
∠BCD = 180° - 96° = 84°

And, ∠BCE = 180° - 84° = 96° [Linear pair of angles]
Similarly, BCEF is a cyclic quadrilateral
So, ∠BCE + ∠BFE = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
∠BFE = 180° - 96° = 84°

(ii) Now, ∠BAD + ∠BFE = 96° + 84° = 180°
But these two are interior angles on the same side of a pair of lines AD and FE.
Therefore, AD || FE.

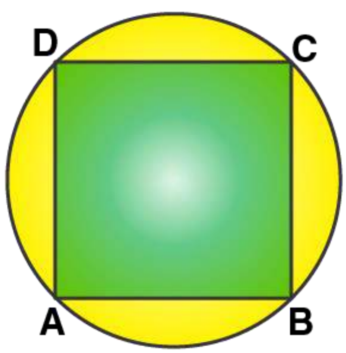
20. Prove that:
(i) the parallelogram, inscribed in a circle, is a rectangle.
(ii) the rhombus, inscribed in a circle, is a square.
Solution:

(i) Let's assume that ABCD is a parallelogram which is inscribed in a circle.



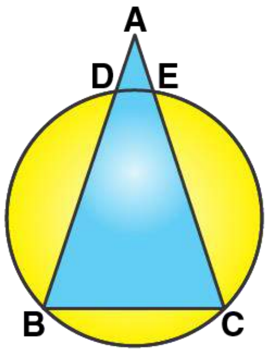
So, we have
∠BAD = ∠BCD [Opposite angles of a parallelogram are equal]
And ∠BAD + ∠BCD = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
So, 2∠BAD = 180°
Thus, ∠BAD = ∠BCD = 90°
Similarly, the remaining two angles are 90° each and pair of opposite sides are equal.
Therefore,
ABCD is a rectangle.
- Hence Proved

(ii) Let's assume that ABCD is a rhombus which is inscribed in a circle.



So, we have
∠BAD = ∠BCD [Opposite angles of a rhombus are equal]
And ∠BAD + ∠BCD = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
So, 2∠BAD = 180°
Thus, ∠BAD = ∠BCD = 90°
Similarly, the remaining two angles are 90° each and all the sides are equal.
Therefore,
ABCD is a square.
- Hence Proved

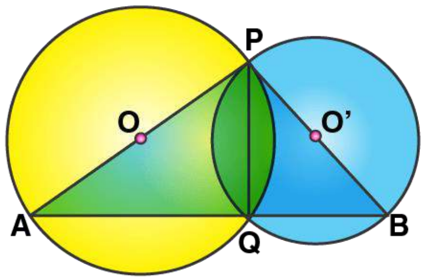
21. In the following figure, AB = AC. Prove that DECB is an isosceles trapezium.



Solution:

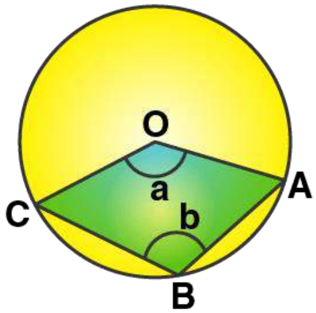
Give, AB = AC
So, ∠B = ∠C … (1)
[Angles opposite to equal sides are equal]
And, DECB is a cyclic quadrilateral.
So, ∠B + ∠DEC = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
∠C + ∠DEC = 180° …. (Using 1)
But this is the sum of interior angles on one side of a transversal.
DE || BC.
But, ∠ADE = ∠B and ∠AED = ∠C [Corresponding angles]
Thus, ∠ADE = ∠AED
AD = AE
AB – AD = AC = AE [As AB = AC]
BD = CE
Hence, we have DE || BC and BD = CE
Therefore,
DECB is an isosceles trapezium.

22. Two circles intersect at P and Q. Through P diameters PA and PB of the two circles are drawn. Show that the points A, Q and B are collinear.
Solution:



Let O and O' be the centres of two intersecting circles, where points of the intersection are P and Q and PA and PB are their diameters respectively.
Join PQ, AQ and QB.
Thus, ∠AQP = 90° and ∠BQP = 90°
[Angle in a semicircle is a right angle]
Now, adding both these angles we get
∠AQP + ∠BQP = 180°
∠AQB = 180°
Therefore, the points A, Q and B are collinear.

23. The figure given below, shows a circle with centre O. Given: ∠AOC = a and ∠ABC = b.
(i) Find the relationship between a and b
(ii) Find the measure of angle OAB, if OABC is a parallelogram.

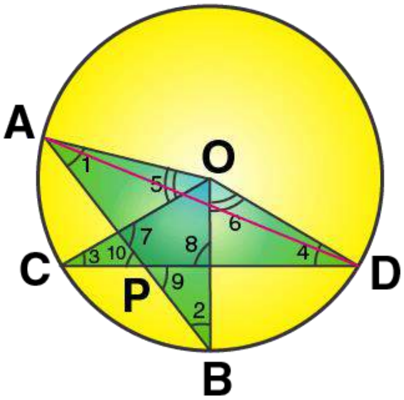


Solution:

(i) It's seen that,
∠ABC = ½ Reflex (∠COA)
[Angle at the centre is double the angle at the circumference subtended by the same chord]
So, b = ½ (360° - a)
a + 2b = 180° ….. (1)

(ii) As OABC is a parallelogram, the opposite angles are equal.
So, a = b
Now, using the above relationship in (1)
3a = 180°
a = 60°
Also, OC || BA
∠COA + ∠OAB = 180°
60° + ∠OAB = 180°
Therefore,
∠OAB = 120°

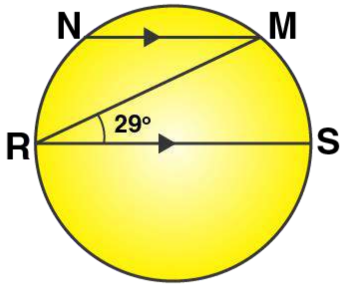
24. Two chords AB and CD intersect at P inside the circle. Prove that the sum of the angles subtended by the arcs AC and BD as the center O is equal to twice the angle APC
Solution:



Required to prove: ∠AOC + ∠BOD = 2∠APC
OA, OB, OC and OD are joined.
Also, AD is joined.
Now, it's seen that
∠AOC = 2∠ADC …. (1)
[Angle at the centre is double the angle at the circumference subtended by the same chord]
Similarly,
∠BOD = 2∠BAD …. (2)
Adding (1) and (2), we have
∠AOC + ∠BOD = 2∠ADC + 2∠BAD
= 2(∠ADC + ∠BAD) ….. (3)

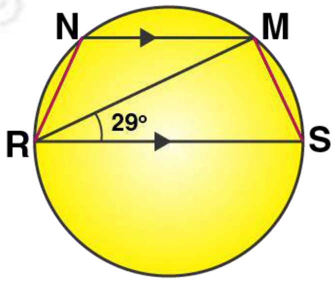
And in ΔPAD,
Ext. ∠APC = ∠PAD + ∠ADC
= ∠BAD + ∠ADC …. (4)
So, from (3) and (4) we have
∠AOC + ∠BOD = 2∠APC

25. In the figure given RS is a diameter of the circle. NM is parallel to RS and ∠MRS = 29° Calculate: (i) ∠RNM; (ii) ∠NRM.



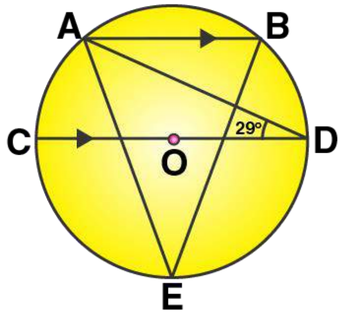
Solution:

(i) Join RN and MS
∠RMS = 90° [Angle in a semi-circle is a right angle]
So, by angle sum property of ΔRMS
∠RMS = 90° – 29° = 61°
And,
∠RNM = 180° - ∠RSM = 180° – 61° = 119°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]

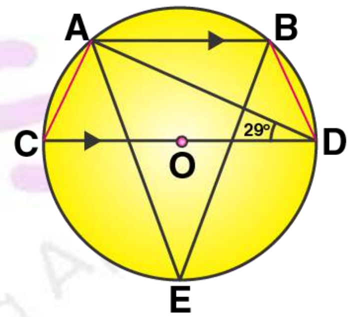


(ii) Now as RS || NM,
∠NMR = ∠MRS = 29° [Alternate angles]
∠NMS = 90° + 29° = 119°
Also, we know that
∠NRS + ∠NMS = 180°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
∠NRM + 29° + 119° = 180°
∠NRM = 180° – 148°
Therefore,
∠NRM = 32°

26. In the figure given alongside, AB || CD and O is the center of the circle. If ∠ADC = 25°; find the angle AEB. Give reasons in support of your answer.

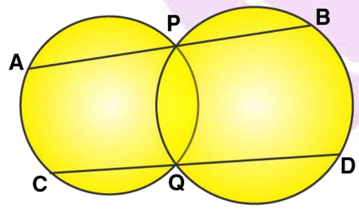


Solution:

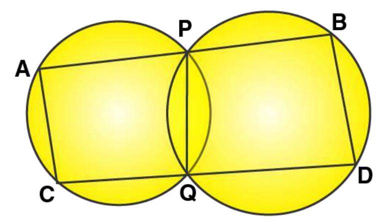


Join AC and BD.
So, we have
∠CAD = 90° and ∠CBD = 90°
[Angle is a semicircle is a right angle]
And, AB || CD
So, ∠BAD = ∠ADC = 25° [Alternate angles]
∠BAC = ∠BAD + ∠CAD = 25° + 90° = 115°
Thus,
∠ADB = 180° – 25° - ∠BAC = 180° – 25° – 115° = 40°
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
Finally,
∠AEB = ∠ADB = 40°
[Angles subtended by the same chord on the circle are equal]

27. Two circles intersect at P and Q. Through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. Prove that AC is parallel to BD.



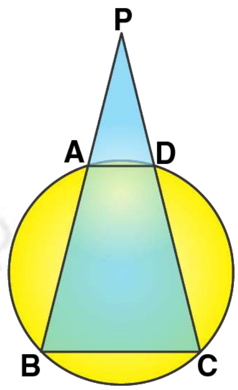
Solution:



Let's join AC, PQ and BD.
As ACQP is a cyclic quadrilateral
∠CAP + ∠PQC = 180° ……. (i)
[Pair of opposite angles in a cyclic quadrilateral are supplementary]
Similarly, as PQDB is a cyclic quadrilateral
∠PQD + ∠DBP = 180° ……. (ii)
Again, ∠PQC + ∠PQD = 180° …… (iii) [Linear pair of angles]
Using (i), (ii) and (iii) we have

∠CAP + ∠DBP = 180°
Or ∠CAB + ∠DBA = 180°
We know that, if the sum of interior angles between two lines when intersected by a transversal are supplementary.
Then, AC || BD.

28. ABCD is a cyclic quadrilateral in which AB and DC on being produced, meet at P such that PA = PD. Prove that AD is parallel to BC.
Solution:

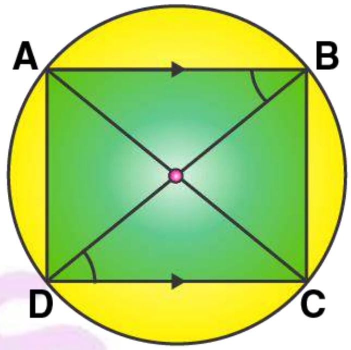


Let's assume that ABCD be the given cyclic quadrilateral.
Also, PA = PD [Given]
So, ∠PAD = ∠PDA …… (1)
[Angles opposite to equal sides are equal]
And,
∠BAD = 180° - ∠PAD [Linear pair of angles]
Similarly,
∠CDA = 180° - ∠PDA = 180° - ∠PAD [From (1)]
As the opposite angles of a cyclic quadrilateral are supplementary,
∠ABC = 180° - ∠CDA = 180° – (180° - ∠PAD) = ∠PAD
And, ∠DCB = 180° - ∠BAD = 180° – (180° - ∠PAD) = ∠PAD
Thus,
∠ABC = ∠DCB = ∠PAD = ∠PDA
Which is only possible when AD || BC.

Exercise 17(B) Page No: 265

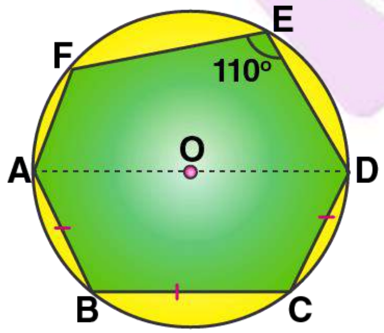
1. In a cyclic-trapezium, the non-parallel sides are equal and the diagonals are also equal. Prove it.

Solution:



Let ABCD be the cyclic trapezium in which AB || DC, AC and BD are the diagonals.
Required to prove:
(i) AD = BC
(ii) AC = BD
Proof:
It's seen that chord AD subtends ∠ABD and chord BC subtends ∠BDC at the circumference of the circle.
But, ∠ABD = ∠BDC [Alternate angles, as AB || DC with BD as the transversal]
So, Chord AD must be equal to chord BC
AD = BC
Now, in ΔADC and ΔBCD
DC = DC [Common]
∠CAD = ∠CBD [Angles in the same segment are equal]
AD = BC [Proved above]
Hence, by SAS criterion of congruence
ΔADC ≅ ΔBCD
Therefore, by CPCT
AC = BD

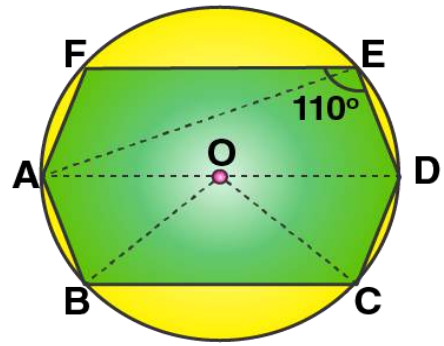
2. In the following figure, AD is the diameter of the circle with centre O. Chords AB, BC and CD are equal. If ∠DEF = 110°, calculate:
(i) ∠AFE, (ii) ∠FAB.



Solution:

Join AE, OB and OC.
(i) As AOD is the diameter
∠AED = 90° [Angle in a semi-circle is a right angle]
But, given ∠DEF = 110°

So,
∠AEF = ∠DEF - ∠AED = 110° – 90° = 20°

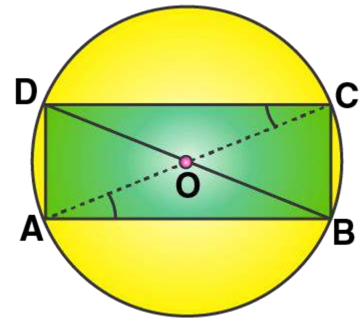


(ii) Also given, Chord AB = Chord BC = Chord CD
So,
∠AOB = ∠BOC = ∠COD [Equal chords subtends equal angles at the centre]
But,
∠AOB + ∠BOC + ∠COD = 180° [Since, AOD is a straight line]
Thus,
∠AOB = ∠BOC = ∠COD = 60°
Now, in ΔOAB we have
OA = OB [Radii of same circle]

So, ∠OAB = ∠OBA [Angles opposite to equal sides]
But, by angle sum property of ΔOAB
∠OAB + ∠OBA = 180° - ∠AOB
= 180° – 60°
= 120°
Therefore, ∠OAB = ∠OBA = 60°
Now, in cyclic quadrilateral ADEF
∠DEF + ∠DAF = 180°
∠DAF = 180° - ∠DEF
= 180° – 110°
= 70°
Thus,
∠FAB = ∠DAF + ∠OAB
= 70° + 60° = 130°

3. If two sides of a cycli-quadrilateral are parallel; prove that:
(i) its other two sides are equal.
(ii) its diagonals are equal.
Solution:

Let ABCD is a cyclic quadrilateral in which AB || DC. AC and BD are its diagonals.
Required to prove:
(i) AD = BC
(ii) AC = BD
Proof:

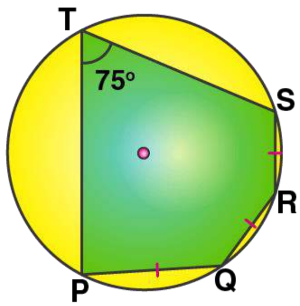


(i) As AB || DC (given)
∠DCA = ∠CAB [Alternate angles]
Now, chord AD subtends ∠DCA and chord BC subtends ∠CAB at the circumference of the circle.
So,
∠DCA = ∠CAB

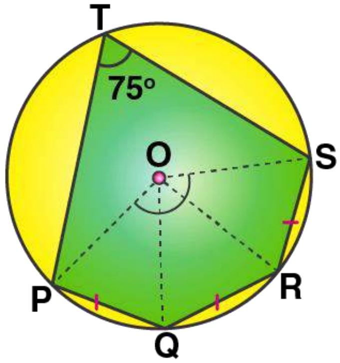
Hence, chord AD = chord BC or AD = BC.

(ii) Now, in ΔABC and ΔADB
AB = AB [Common]
∠ACB = ∠ADB [Angles in the same segment are equal]
BC = AD [Proved above]
Hence, by SAS criterion of congruence
ΔACB ≅ ΔADB
Therefore, by CPCT
AC = BD

4. The given figure show a circle with centre O. Also, PQ = QR = RS and ∠PTS = 75°.



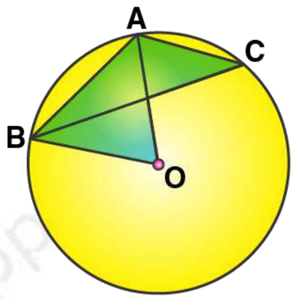
Calculate:
(i) ∠POS,
(ii) ∠QOR,
(iii) ∠PQR.
Solution:



Join OP, OQ, OR and OS.
Given, PQ = QR = RS
So, ∠POQ = ∠QOR = ∠ROS [Equal chords subtends equal angles at the centre]
Arc PQRS subtends ∠POS at the centre and ∠PTS at the remaining part of the circle.
Thus,
$\angle POS = 2 \times \angle PTS = 2 \times 75^o = 150^o$
$\angle POQ + \angle QOR + \angle ROS = 150^o$
$\angle POQ = \angle QOR = \angle ROS = 150^o/ 3 = 50^o$
In ΔOPQ we have,
OP = OQ [Radii of the same circle]
So, ∠OPQ = ∠OQP [Angles opposite to equal sides are equal]
But, by angle sum property of ΔOPQ
$\angle OPQ + \angle OQP + \angle POQ = 180^o$
$\angle OPQ + \angle OQP + 50^o = 180^o$
$\angle OPQ + \angle OQP = 130^o$
$2 \angle OPQ = 130^o$
$\angle OPQ = \angle OPQ = 130^o/ 2 = 65^o$

Similarly, we can prove that
In ΔOQR,
∠OQR = ∠ORQ = 65°
And in ΔORS,
∠ORS = ∠OSR = 65°
Hence,
(i) ∠POS = 150°
(ii) ∠QOR = 50° and
(iii) ∠PQR = ∠PQO + ∠OQR = 65° + 65° = 130°

5. In the given figure, AB is a side of a regular six-sided polygon and AC is a side of a regular eight-sided polygon inscribed in the circle with centre O. calculate the sizes of:
(i) ∠AOB,
(ii) ∠ACB,
(iii) ∠ABC.



Solution:

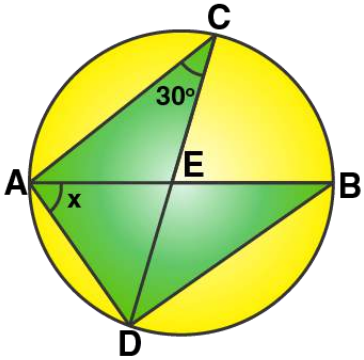
(i) Arc AB subtends ∠AOB at the centre and ∠ACB at the remaining part of the circle.
∠ACB = ½ ∠AOB
And as AB is the side of a regular hexagon, we have
∠AOB = 60°

(ii) Now,
∠ACB = ½ (60°) = 30°

(iii) Since AC is the side of a regular octagon,
∠AOC = 360°/ 8 = 45°
Again, arc AC subtends ∠AOC at the centre and ∠ABC at the remaining part of the circle.
∠ABC = ½ ∠AOC
∠ABC = 45°/ 2 = 22.5°

Exercise 17(C)

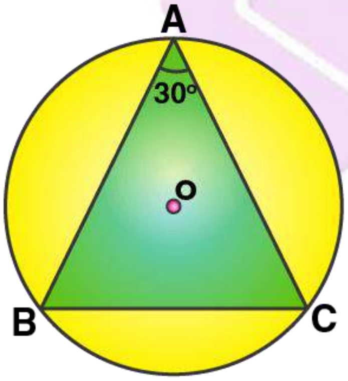
1. In the given circle with diameter AB, find the value of x.



Solution:

Now,
∠ABD = ∠ACD = 30° [Angles in the same segment]
In ΔADB, by angle sum property we have
∠BAD + ∠ADB + ∠ABD = 180°
But, we know that angle in a semi-circle is 90°
∠ADB = 90°
So,
x + 90° + 30° = 180°
x = 180° - 120°
Hence, x = 60°

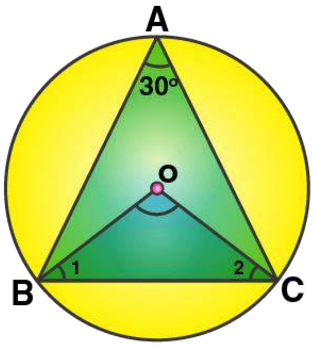
2. In the given figure, ABC is a triangle in which ∠BAC = 30°. Show that BC is equal to the radius of the circum-circle of the triangle ABC, whose center is O.



Solution:

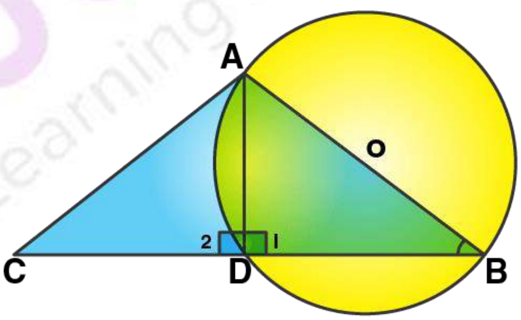
Firstly, join OB and OC.
Proof:
∠BOC = 2∠BAC = 2 x 30° = 60°
Now, in ΔOBC
OB = OC [Radii of same circle]

So, ∠OBC = ∠OCB [Angles opposite to equal sides]
And in ΔOBC, by angle sum property we have
∠OBC + ∠OCB + ∠BOC = 180°
∠OBC + ∠OBC + 60° = 180°
2 ∠OBC = 180° – 60° = 120°
∠OBC = 120°/ 2 = 60°
So, ∠OBC = ∠OCB = ∠BOC = 60°
Thus, ΔOBC is an equilateral triangle.
So,
BC = OB = OC
But, OB and OC are the radii of the circum-circle.
Therefore, BC is also the radius of the circum-circle.

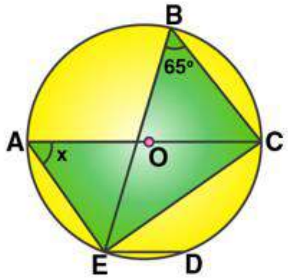


3. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.
Solution:

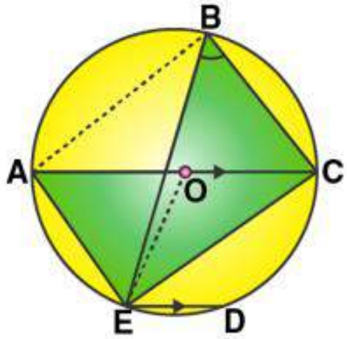
Let's consider ΔABC, AB = AC and circle with AB as diameter is drawn which intersects the side BC and D.
And, join AD
Proof:
It's seen that,
∠ADB = 90° [Angle in a semi-circle]
And,
∠ADC + ∠ADB = 180° [Linear pair]
Thus, ∠ADC = 90°
Now, in right ΔABD and ΔACD
AB = AC [Given]
AD = AD [Common]
∠ADB = ∠ADC = 90°
Hence, by R.H.S criterion of congruence.
ΔABD ≅ ΔACD
Now, by CPCT
BD = DC
Therefore, D is the mid-point of BC.



4. In the given figure, chord ED is parallel to diameter AC of the circle. Given ∠CBE = 65°, calculate ∠DEC.



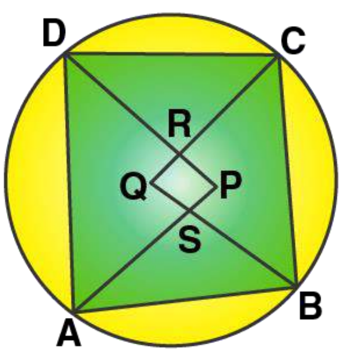
Solution:



Join OE.
Arc EC subtends ∠EOC at the centre and ∠EBC at the remaining part of the circle.
$\angle EOC = 2\angle EBC = 2 \times 65^o = 130^o$
Now, in ΔOEC
OE = OC [Radii of the same circle]
So, ∠OEC = ∠OCE
But, in ΔEOC by angle sum property
$\angle OEC + \angle OCE + \angle EOC = 180^o$ [Angles of a triangle]
$\angle OCE + \angle OCE + \angle EOC = 180^o$
$2\angle OCE + 130^o = 180^o$
$2\angle OCE = 180^o - 130^o$
$\angle OCE = 50^o/2 = 25^o$
And, AC || ED [Given]
∠DEC = ∠OCE [Alternate angles]
Thus,
$\angle DEC = 25^o$

5. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic. Prove it.
Solution:

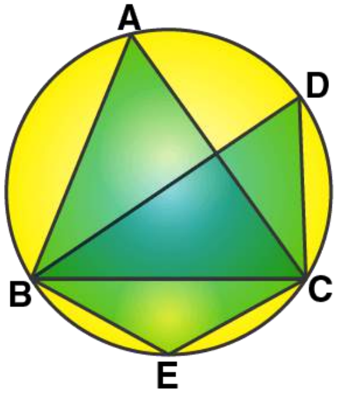
Let ABCD be a cyclic quadrilateral and PQRS be the quadrilateral formed by the angle bisectors of angle ∠A, ∠B, ∠C and ∠D.
Required to prove: PQRS is a cyclic quadrilateral.



Proof:
By angle sum property of a triangle
In ΔAPD,
$\angle PAD + \angle ADP + \angle APD = 180^o$ …. (i)
And, in ΔBQC
$\angle QBC + \angle BCQ + \angle BQC = 180^o$ …. (ii)
Adding (i) and (ii), we get
$\angle PAD + \angle ADP + \angle APD + \angle QBC + \angle BCQ + \angle BQC = 180^o + 180^o = 360^o$ …… (iii)
But,
$\angle PAD + \angle ADP + \angle QBC + \angle BCQ = ½ [\angle A + \angle B + \angle C + \angle D]$
$= ½ \times 360^o = 180^o$
Therefore,
$\angle APD + \angle BQC = 360^o - 180^o = 180^o$ [From (iii)]
But, these are the sum of opposite angles of quadrilateral PRQS.
Therefore,
Quadrilateral PQRS is also a cyclic quadrilateral.

6. In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate:
(i) ∠BDC

(ii) ∠BEC
(iii) ∠BAC



Solution:

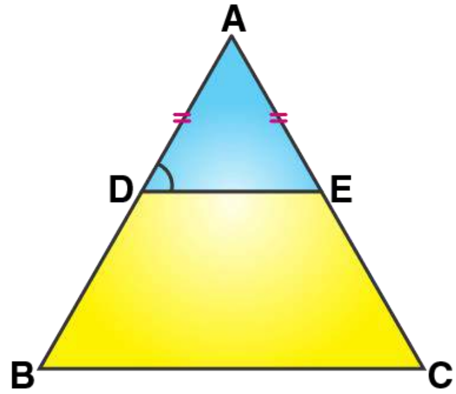
(i) Given that BD is a diameter of the circle.
And, the angle in a semicircle is a right angle.
So, ∠BCD = 90°
Also given that,
∠DBC = 58°
In ΔBDC,
∠DBC + ∠BCD + ∠BDC = 180°
58° + 90° + ∠BDC = 180°
148° + ∠BDC = 180°
∠BDC = 180° - 148°
Thus, ∠BDC = 32°

(ii) We know that, the opposite angles of a cyclic quadrilateral are supplementary.
So, in cyclic quadrilateral BECD
∠BEC + ∠BDC = 180°
∠BEC + 32° = 180°
∠BEC = 148°

(iii) In cyclic quadrilateral ABEC,
∠BAC + ∠BEC = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
∠BAC + 148° = 180°
∠BAC = 180° - 148°
Thus, ∠BAC = 32°

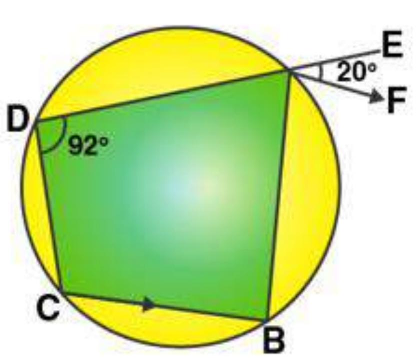
7. D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Prove that the points B, C, E and D are concyclic.
Solution:

Given,
ΔABC, AB = AC and D and E are points on AB and AC such that AD = AE.



And, DE is joined.
Required to prove: Points B, C, E and D are concyclic
Proof:
In ΔABC,
AB = AC [Given]
So, ∠B = ∠C [Angles opposite to equal sides]
Similarly,
In ΔADE,
AD = AE [Given]
So, ∠ADE = ∠AED [Angles opposite to equal sides]
Now, in ΔABC we have
AD/AB = AE/AC
Hence, DE || BC [Converse of BPT]
So,
∠ADE = ∠B [Corresponding angles]
$(180^o - \angle EDB) = \angle B$
$\angle B + \angle EDB = 180^o$
But, it's proved above that
∠B = ∠C
So,
$\angle C + \angle EDB = 180^o$
Thus, opposite angles are supplementary.
Similarly,
$\angle B + \angle CED = 180^o$
Hence, B, C, E and D are concyclic.

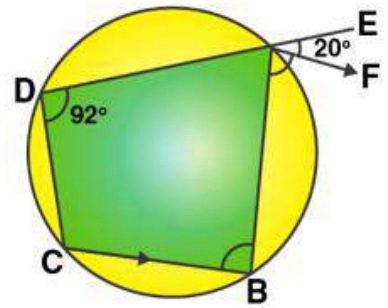
8. In the given figure, ABCD is a cyclic quadrilateral. AF is drawn parallel to CB and DA is produced to point E. If ∠ADC = 92º, ∠FAE = 20º; determine ∠BCD. Given reason in support of your answer.



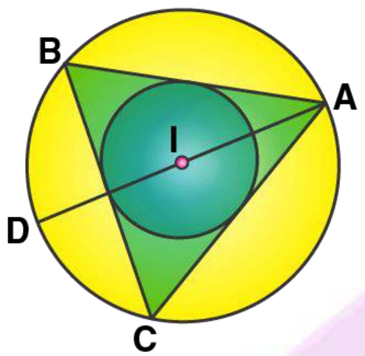
Solution:

Given,
In cyclic quad. ABCD
AF || CB and DA is produced to E such that $\angle ADC = 92^o$ and $\angle FAE = 20^o$
So,
$\angle B + \angle D = 180^o$

$\angle B + 92^o = 180^o$
$\angle B = 88^o$
As AF || CB, $\angle FAB = \angle B = 88^o$
But, $\angle FAD = 20^o$ [Given]
Ext. $\angle BAE = \angle BAF + \angle FAE$
$= 88^o + 22^o = 108^o$
But, Ext. $\angle BAE = \angle BCD$
Therefore,
$\angle BCD = 108^o$

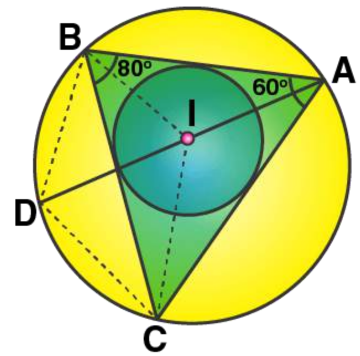


9. If I is the incentre of triangle ABC and AI when produced meets the circumcircle of triangle ABC in point D. If $\angle BAC = 66^o$ and $\angle ABC = 80^o$. Calculate:
(i) $\angle DBC$,
(ii) $\angle IBC$,
(iii) $\angle BIC$



Solution:

Join DB and DC, IB and IC.
Given, if $\angle BAC = 66^o$ and $\angle ABC = 80^o$, I is the incentre of the ΔABC.



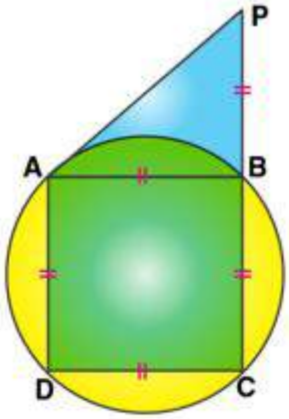
(i) As it's seen that $\angle DBC$ and $\angle DAC$ are in the same segment,
So, $\angle DBC = \angle DAC$
But, $\angle DAC = ½ \angle BAC = ½ \times 66^o = 33^o$
Thus, $\angle DBC = 33^o$

(ii) And, as I is the incentre of ΔABC, IB bisects $\angle ABC$.
Therefore,
$\angle IBC = ½ \angle ABC = ½ \times 80^o = 40^o$

(iii) In ΔABC, by angle sum property
$\angle ACB = 180^o – (\angle ABC + \angle BAC)$
$\angle ACB = 180^o – (80^o + 66^o)$
$\angle ACB = 180^o – 156^o$
$\angle ACB = 34^o$
And since, IC bisects $\angle C$
Thus, $\angle ICB = ½ \angle C = ½ \times 34^o = 17^o$

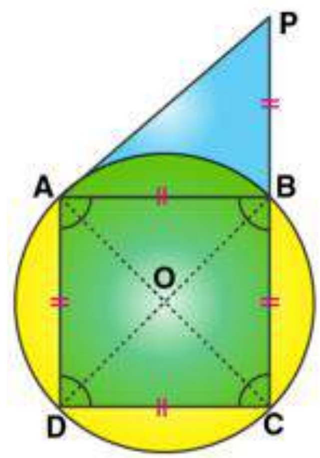
Now, in ΔIBC
∠IBC + ∠ICB + ∠BIC = 180°
40° + 17° + ∠BIC = 180°
57° + ∠BIC = 180°
∠BIC = 180° – 57°
Therefore, ∠BIC = 123°

10. In the given figure, AB = AD = DC = PB and ∠DBC = x°. Determine, in terms of x:
(i) ∠ABD, (ii) ∠APB.
Hence or otherwise, prove that AP is parallel to DB.

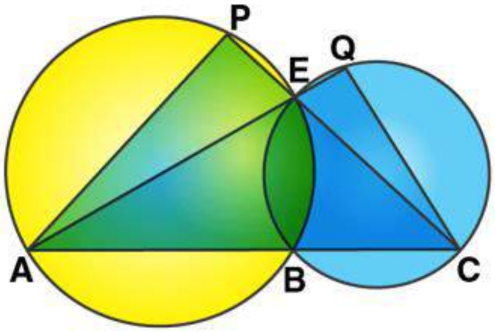


Solution:

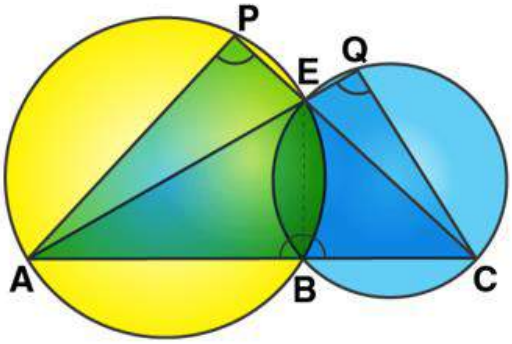
Given, AB = AD = DC = PB and ∠DBC = x°
Join AC and BD.
Proof:
∠DAC = ∠DBC = x° [Angles in the same segment]
And, ∠DCA = ∠DAC = x° [As AD = DC]
Also, we have
∠ABD = ∠DAC [Angles in the same segment]
And, in ΔABP
Ext. ∠ABD = ∠BAP + ∠APB
But, ∠BAP = ∠APB [Since, AB = BP]
2 x° = ∠APB + ∠APB = 2∠APB
2∠APB = 2x°
So, ∠APB = x°
Thus, ∠APB = ∠DBC = x°
But these are corresponding angles,
Therefore, AP || DB.



11. In the given figure; ABC, AEQ and CEP are straight lines. Show that ∠APE and ∠CQE are supplementary.

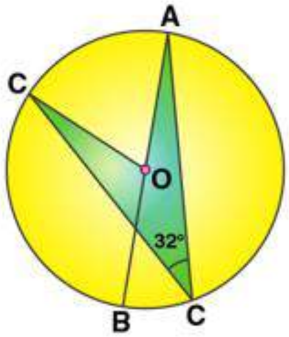


Solution:

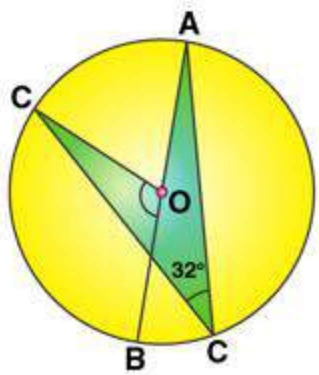


Join EB.
Then, in cyclic quad.ABEP
∠APE + ∠ABE = 180° ….. (i) [Opposite angles of a cyclic quad. are supplementary]
Similarly, in cyclic quad.BCQE
∠CQE + ∠CBE = 180° ….. (ii) [Opposite angles of a cyclic quad. are supplementary]
Adding (i) and (ii), we have
∠APE + ∠ABE + ∠CQE + ∠CBE = 180° + 180° = 360°
∠APE + ∠ABE + ∠CQE + ∠CBE = 360°
But, ∠ABE + ∠CBE = 180° [Linear pair]
∠APE + ∠CQE + 180° = 360°
∠APE + ∠CQE = 180°
Therefore, ∠APE and ∠CQE are supplementary.

12. In the given, AB is the diameter of the circle with centre O. If ∠ADC = 32°, find angle BOC.



Solution:



Arc AC subtends ∠AOC at the centre and ∠ADC at the remaining part of the circle.
Thus, ∠AOC = 2∠ADC
∠AOC = 2 x 32° = 64°
As ∠AOC and ∠BOC are linear pair, we have
∠AOC + ∠BOC = 180°

$64^o + \angle BOC = 180^o$
$\angle BOC = 180^o - 64^o$
Therefore, $\angle BOC = 116^o$