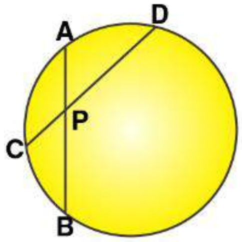
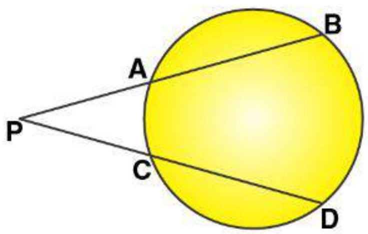
Exercise 18(B)

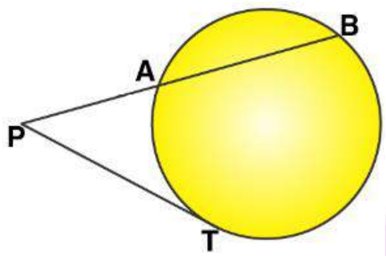
1. (i) In the given figure, 3 x CP = PD = 9 cm and AP = 4.5 cm. Find BP.



(ii) In the given figure, 5 x PA = 3 x AB = 30 cm and PC = 4cm. Find CD.



(iii) In the given figure, tangent PT = 12.5 cm and PA = 10 cm; find AB.



Solution:

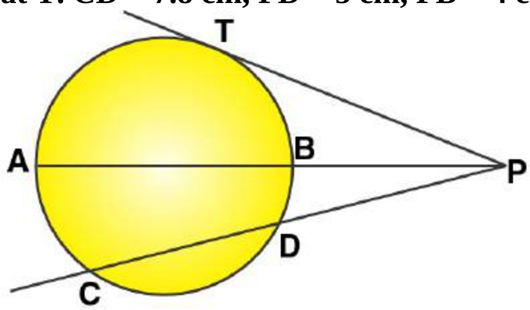
(i) As the two chords AB and CD intersect each other at P, we have
AP x PB = CP x PD
4.5 x PB = 3 x 9 [3CP = 9 cm so, CP = 3 cm]
PB = (3 x 9)/ 4.5 = 6 cm

(ii) As the two chords AB and CD intersect each other at P, we have
AP x PB = CP x PD
But, 5 x PA = 3 x AB = 30 cm
So, PA = 30/5 = 6 cm and AB = 30/3 = 10 cm
And, BP = PA + AB = 6 + 10 = 16 cm
Now, as
AP x PB = CP x PD
6 x 16 = 4 x PD
PD = (6 x 16)/ 4 = 24 cm
CD = PD – PC = 24 – 4 = 20 cm

(iii) As PAB is the secant and PT is the tangent, we have
PT^2 = PA x PB

$12.5^2 = 10 \times PB$
PB = (12.5 x 12.5)/ 10 = 15.625 cm
AB = PB – PA = 15.625 – 10 = 5.625 cm

2. In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm.

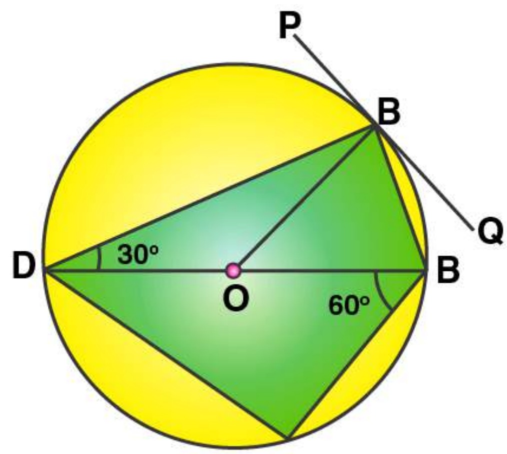


Find
(i) AB.
(ii) the length of tangent PT.
Solution:

(i) PA = AB + BP = (AB + 4) cm
PC = PD + CD = 5 + 7.8 = 12.8 cm
As PA x PB = PC x PD
(AB + 4) x 4 = 12.8 x 5
AB + 4 = (12.8 x 5)/ 4
AB + 4 = 16
Hence, AB = 12 cm

(ii) As we know,
$PT^2 = PC \times PD$
$PT^2 = 12.8 \times 5 = 64$
Thus, PT = 8 cm

3. In the following figure, PQ is the tangent to the circle at A, DB is a diameter and O is the centre of the circle. If ∠ADB = 30° and ∠CBD = 60°; calculate:
i) ∠QAB
ii) ∠PAD
iii) ∠CDB
Solution:

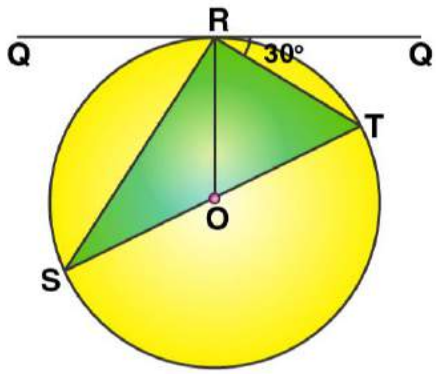


(i) Given, PAQ is a tangent and AB is the chord
∠QAB = ∠ADB = 30° [Angles in the alternate segment]

(ii) OA = OD [radii of the same circle]
So, ∠OAD = ∠ODA = 30°
But, as OA ⊥ PQ
∠PAD = ∠OAP - ∠OAD = 90° – 30° = 60°

(iii) As BD is the diameter, we have
$\angle BCD = 90^o$ [Angle in a semi-circle]
Now in ΔBCD,
$\angle CDB + \angle CBD + \angle BCD = 180^o$
$\angle CDB + 60^o + 90^o = 180^o$
Thus, $\angle CDB = 180^o - 150^o = 30^o$

4. If PQ is a tangent to the circle at R; calculate:
i) ∠PRS
ii) ∠ROT



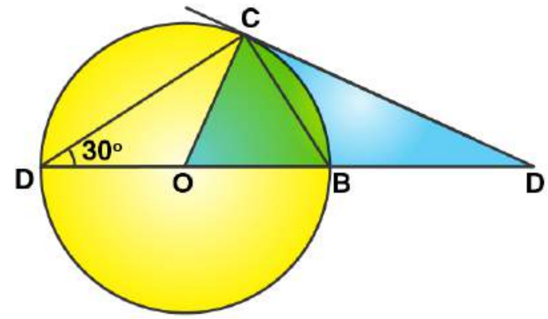
Given: O is the centre of the circle and $\angle TRQ = 30^o$
Solution:

(i) As PQ is the tangent and OR is the radius.
So, $OR \perp PQ$
$\angle ORT = 90^o$
$\angle TRQ = 90^o – 30^o = 60^o$
But in ΔOTR, we have
OT = OR [Radii of same circle]
$\angle OTR = 60^o$ or $\angle STR = 60^o$
But,
$\angle PRS = \angle STR = 60^o$ [Angles in the alternate segment]

(ii) In ΔOTR,
$\angle ORT = 60^o$
$\angle OTR = 60^o$
Thus,
$\angle ROT = 180^o – (60^o + 60^o) = 180^o – 120^o = 60^o$

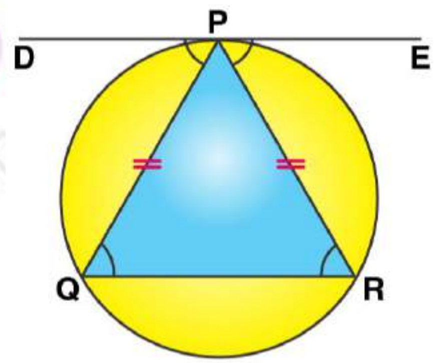
5. AB is diameter and AC is a chord of a circle with centre O such that angle BAC=30º. The tangent to the circle at C intersects AB produced in D. Show that BC = BD.
Solution:

Join OC.



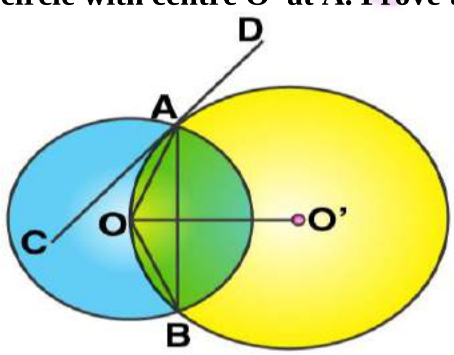
$\angle BCD = \angle BAC = 30^o$ [Angles in the alternate segment]
It's seen that, arc BC subtends $\angle DOC$ at the center of the circle and $\angle BAC$ at the remaining part of the circle.
So, $\angle BOC = 2\angle BAC = 2 \times 30^o = 60^o$
Now, in ΔOCD
$\angle BOC$ or $\angle DOC = 60^o$
$\angle OCD = 90^o$ [$OC \perp CD$]
$\angle DOC + \angle ODC = 90^o$
$\angle ODC = 90^o – 60^o = 30^o$
Now, in ΔBCD
As $\angle ODC$ or $\angle BDC = \angle BCD = 30^o$
Therefore, BC = BD

6. Tangent at P to the circumcircle of triangle PQR is drawn. If this tangent is parallel to side QR, show that triangle PQR is isosceles.
Solution:



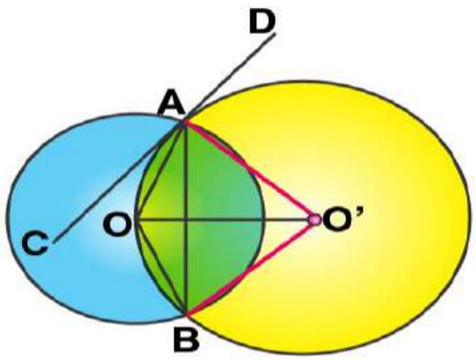
Let DE be the tangent to the circle at P.
And, DE || QR [Given]
$\angle EPR = \angle PRQ$ [Alternate angles are equal]
$\angle DPQ = \angle PQR$ [Alternate angles are equal] ….. (i)
Let $\angle DPQ = x$ and $\angle EPR = y$
As the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment, we have
$\angle DPQ = \angle PRQ$ …… (ii) [DE is tangent and PQ is chord]
So, from (i) and (ii),
$\angle PQR = \angle PRQ$
PQ = PR
Therefore, triangle PQR is an isosceles triangle.

7. Two circles with centres O and O' are drawn to intersect each other at points A and B. Centre O of one circle lies on the circumference of the other circle and CD is drawn tangent to the circle with centre O' at A. Prove that OA bisects angle BAC.



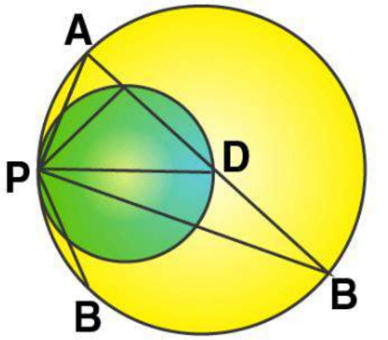
Solution:

Join OA, OB, O'A, O'B and O'O.

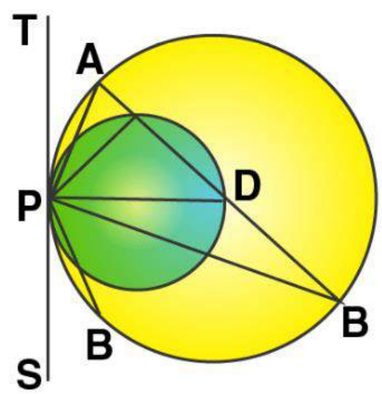


CD is the tangent and AO is the chord.
∠OAC = ∠OBA … (i) [Angles in alternate segment]
In ΔOAB,
OA = OB [Radii of the same circle]
∠OAB = ∠OBA ….. (ii)
From (i) and (ii), we have
∠OAC = ∠OAB
Thus, OA is the bisector of ∠BAC.

8. Two circles touch each other internally at a point P. A chord AB of the bigger circle intersects the other circle in C and D. Prove that: ∠CPA = ∠DPB



Solution:



Let's draw a tangent TS at P to the circles given.
As TPS is the tangent and PD is the chord, we have
∠PAB = ∠BPS …. (i) [Angles in alternate segment]
Similarly,
∠PCD = ∠DPS …. (ii)
Now, subtracting (i) from (ii) we have
∠PCD - ∠PAB = ∠DPS - ∠BPS
But in ΔPAC,
Ext. ∠PCD = ∠PAB + ∠CPA
∠PAB + ∠CPA - ∠PAB = ∠DPS - ∠BPS
Thus,
∠CPA = ∠DPB