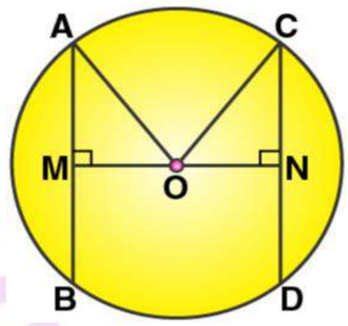
Exercise 18(C) Page No: 285

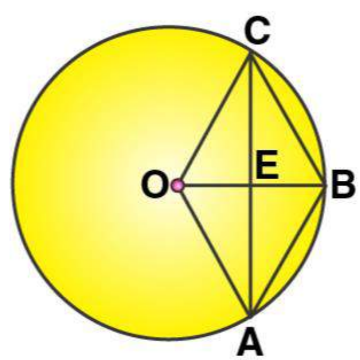
1. Prove that, of any two chords of a circle, the greater chord is nearer to the center.
Solution:

Given: A circle with center O and radius r. AB and CD are two chords such that AB > CD. Also, OM ⊥ AB and ON ⊥ CD.



Required to prove: OM < ON
Proof:
Join OA and OC.
Then in right ΔAOM, we have
$AO^2 = AM^2 + OM^2$
$r^2 = (½AB)^2 + OM^2$
$r^2 = ¼\ AB^2 + OM^2$ ….. (i)
Again, in right ΔONC, we have
$OC^2 = NC^2 + ON^2$
$r^2 = (½CD)^2 + ON^2$
$r^2 = ¼\ CD^2 + ON^2$ ….. (ii)
On equating (i) and (ii), we get
$¼\ AB^2 + OM^2 = ¼\ CD^2 + ON^2$
But, AB > CD [Given]
So, ON will be greater than OM to be equal on both sides.
Thus,
OM < ON
Hence, AB is nearer to the centre than CD.

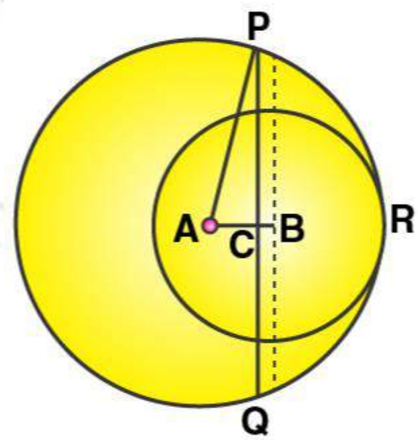
2. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O.
i) If the radius of the circle is 10 cm, find the area of the rhombus.
ii) If the area of the rhombus is 32√3 cm², find the radius of the circle.
Solution:



(i) Given, radius = 10 cm
In rhombus OABC,
OC = 10 cm
So,
OE = ½ x OB = ½ x 10 = 5 cm
Now, in right ΔOCE
$OC^2 = OE^2 + EC^2$
$10^2 = 5^2 + EC^2$
$EC^2 = 100 – 25 = 75$
EC = √75 = 5√3
Hence, AC = 2 x EC = 2 x 5√3 = 10√3
We know that,
Area of rhombus = ½ x OB x AC
= ½ x 10 x 10√3
= 50√3 cm² ≈ 86.6 cm²

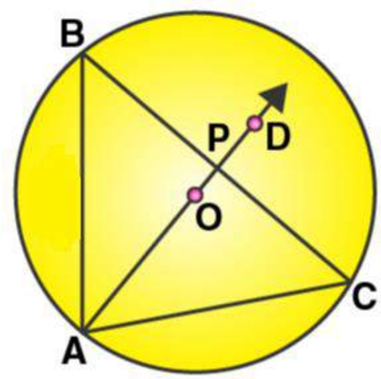
(ii) We have the area of rhombus = $32\sqrt{3}$ cm^2
But area of rhombus OABC = 2 x area of ΔOAB
Area of rhombus OABC = 2 x $(\sqrt{3}/4)$ r^2
Where r is the side of the equilateral triangle OAB.
2 x $(\sqrt{3}/4)$ r^2 = $32\sqrt{3}$
$\sqrt{3}/2$ r^2 = $32\sqrt{3}$
$r^2 = 64$
r = 8
Therefore, the radius of the circle is 8 cm.

3. Two circles with centers A and B, and radii 5 cm and 3 cm, touch each other internally. If the perpendicular bisector of the segment AB meets the bigger circle in P and Q; find the length of PQ.
Solution:



We know that,
If two circles touch internally, then distance between their centres is equal to the difference of their radii. So, AB = (5 - 3) cm = 2 cm.
Also, the common chord PQ is the perpendicular bisector of AB.
Thus, AC = CB = ½ AB = 1 cm
In right ΔACP, we have
$AP^2 = AC^2 + CP^2$ [Pythagoras Theorem]
$5^2 = 1^2 + CP^2$
$CP^2 = 25 - 1 = 24$
CP = $\sqrt{24}$ cm = $2\sqrt{6}$ cm
Now,
PQ = 2 CP
= 2 x $2\sqrt{6}$ cm
= $4\sqrt{6}$ cm
Therefore, the length of PQ is $4\sqrt{6}$ cm.

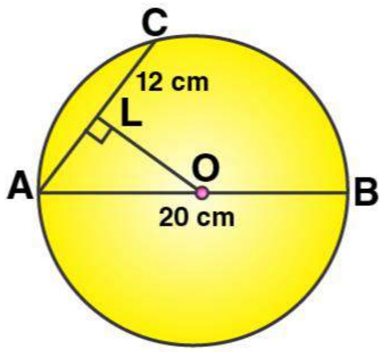
4. Two chords AB and AC of a circle are equal. Prove that the center of the circle, lies on the bisector of the angle BAC.
Solution:



Given: AB and AC are two equal chords of C (O, r).
Required to prove: Centre, O lies on the bisector of $\angle BAC$.
Construction: Join BC. Let the bisector of $\angle BAC$ intersects BC in P.
Proof:
In ΔAPB and ΔAPC,
AB = AC [Given]
$\angle BAP = \angle CAP$ [Given]
AP = AP [Common]
Hence, $\Delta APB \cong \Delta APC$ by SAA congruence criterion
So, by CPCT we have
BP = CP and $\angle APB = \angle APC$

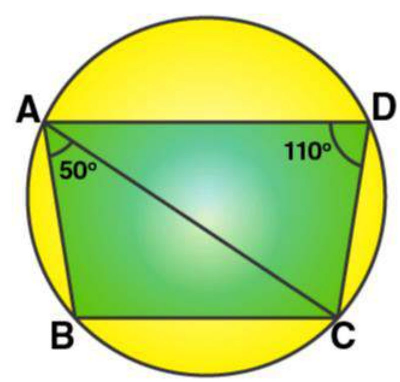
And,
$\angle APB + \angle APC = 180^\circ$ [Linear pair]
$2\angle APB = 180^\circ$ [$\angle APB = \angle APC$]
$\angle APB = 90^\circ$
Now, BP = CP and $\angle APB = 90^\circ$
Therefore, AP is the perpendicular bisector of chord BC.
Hence, AP passes through the centre, O of the circle.

5. The diameter and a chord of circle have a common end-point. If the length of the diameter is 20 cm and the length of the chord is 12 cm, how far is the chord from the center of the circle?
Solution:



We have, AB as the diameter and AC as the chord.
Now, draw OL ⊥ AC
Since OL ⊥ AC and hence it bisects AC, O is the centre of the circle.
Therefore, OA = 10 cm and AL = 6 cm
Now, in right ΔOLA
$AO^2 = AL^2 + OL^2$ [By Pythagoras Theorem]
$10^2 = 6^2 + OL^2$
$OL^2 = 100 – 36 = 64$
OL = 8 cm
Therefore, the chord is at a distance of 8 cm from the centre of the circle.

6. ABCD is a cyclic quadrilateral in which BC is parallel to AD, angle ADC = 110º and angle BAC = 50º. Find angle DAC and angle DCA.
Solution:

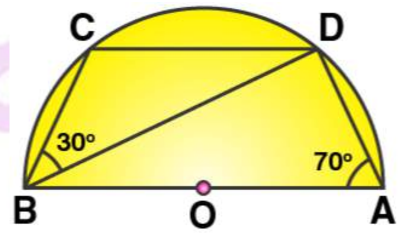


Given, ABCD is a cyclic quadrilateral in which AD || BC
And, $\angle ADC = 110^o$, $\angle BAC = 50^o$
We know that,
$\angle B + \angle D = 180^o$ [Sum of opposite angles of a quadrilateral]
$\angle B + 110^o = 180^o$
So, $\angle B = 70^o$
Now in ΔADC, we have
$\angle BAC + \angle ABC + \angle ACB = 180^o$
$50^o + 70^o + \angle ACB = 180^o$
$\angle ACB = 180^o – 120^o = 60^o$

And, as AD || BC we have
∠DAC = ∠ACB = 60° [Alternate angles]
Now in ΔADC,
∠DAC + ∠ADC + ∠DCA = 180°
60° + 110° + ∠DCA = 180°
Thus,
∠DCA = 180° – 170° = 10°

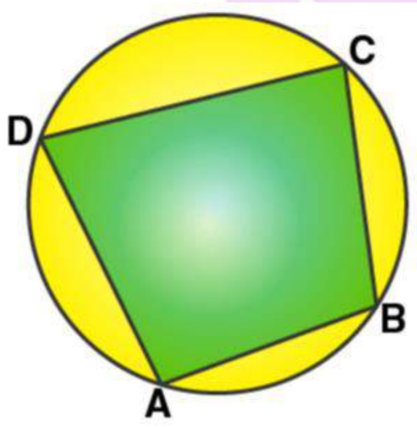
7. In the given figure, C and D are points on the semi-circle described on AB as diameter. Given angle BAD = 70° and angle DBC = 30°, calculate angle BDC.

Solution:



As ABCD is a cyclic quadrilateral, we have
∠BCD + ∠BAD = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
∠BCD + 70° = 180°
∠BCD = 180° – 70° = 110°
And, by angle sum property of ΔBCD we have
∠CBD + ∠BCD + ∠BDC = 180°
30° + 110° + ∠BDC = 180°
∠BDC = 180° - 140°
Thus,
∠BDC = 40°

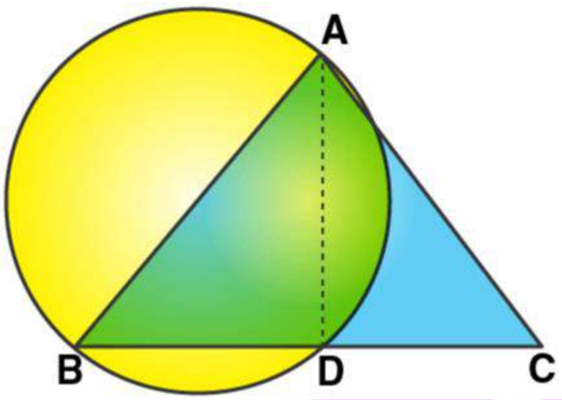
8. In cyclic quadrilateral ABCD, ∠A = 3 ∠C and ∠D = 5 ∠B. Find the measure of each angle of the quadrilateral.
Solution:



Given, cyclic quadrilateral ABCD
So, ∠A + ∠C = 180° [Opposite angles in a cyclic quadrilateral is supplementary]
3∠C + ∠C = 180° [As ∠A = 3 ∠C]
∠C = 45°
Now,
∠A = 3 ∠C = 3 x 45°

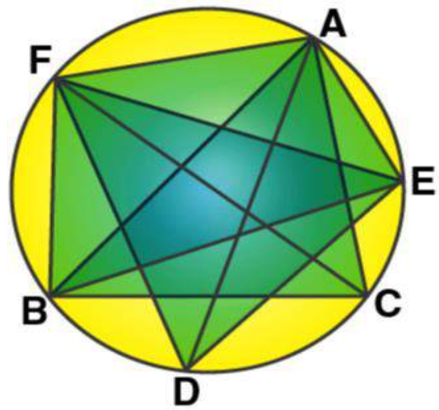
$\angle A = 135^o$
Similarly,
$\angle B + \angle D = 180^o$ [As $\angle D = 5\angle B$]
$\angle B + 5\angle B = 180^o$
$6\angle B = 180^o$
$\angle B = 30^o$
Now,
$\angle D = 5\angle B = 5 \times 30^o$
$\angle D = 150^o$
Therefore,
$\angle A = 135^o$, $\angle B = 30^o$, $\angle C = 45^o$, $\angle D = 150^o$

9. Show that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.
Solution:



Let's join AD.
And. AB is the diameter.
We have $\angle ADB = 90^o$ [Angle in a semi-circle]
But,
$\angle ADB + \angle ADC = 180^o$ [Linear pair]
So, $\angle ADC = 90^o$
Now, in ΔABD and ΔACD we have
$\angle ADB = \angle ADC$ [each 90^o]
AB = AC [Given]
AD = AD [Common]
Hence, $\Delta ABD \cong \Delta ACD$ by RHS congruence criterion
So, by C.P.C.T
BD = DC
Therefore, the circle bisects base BC at D.

10. Bisectors of vertex angles A, B and C of a triangle ABC intersect its circumcircle at points D, E and F respectively. Prove that angle EDF = 90º – ½ ∠A
Solution:



Join ED, EF and DF. Also join BF, FA, AE and EC.

∠EBF = ∠ECF = ∠EDF ….. (i) [Angle in the same segment]

In cyclic quadrilateral AFBE,

∠EBF + ∠EAF = 180° …… (ii)

[Sum of opposite angles in a cyclic quadrilateral is supplementary]

Similarly in cyclic quadrilateral CEAF,

∠EAF + ∠ECF = 180° ……. (iii)

Adding (ii) and (iii) we get,

∠EBF + ∠ECF + 2∠EAF = 360°

∠EDF + ∠EDF + 2∠EAF = 360° [From (i)]

∠EDF + ∠EAF = 180°

∠EDF + ∠1 + ∠BAC + ∠2 = 180°

But, ∠1 = ∠3 and ∠2 and ∠4 [Angles in the same segment]

∠EDF + ∠3 + ∠BAC + ∠4 = 180°

But, ∠4 = ½ ∠C, ∠3 = ½ ∠B

Thus, ∠EDF + ½ ∠B + ∠BAC + ½ ∠C = 180°

∠EDF + ½ ∠B + 2 x ½ ∠A + ½ ∠C = 180°

∠EDF + ½ (∠A + ∠B + ∠C) + ½ ∠A = 180°

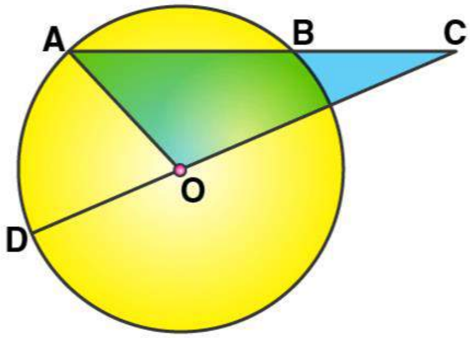
∠EDF + ½ (180°) + ½ ∠A = 180°

∠EDF + 90° + ½ ∠A = 180°

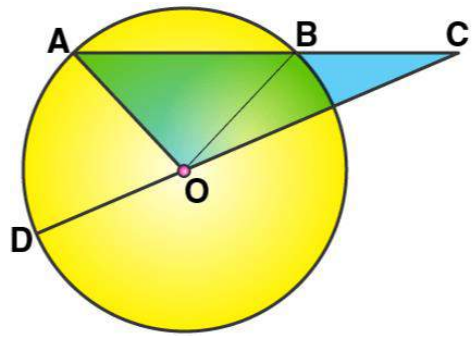
∠EDF = 180° – (90° + ½ ∠A)

∠EDF = 90° – ½ ∠A

11. In the figure, AB is the chord of a circle with centre O and DOC is a line segment such that BC = DO. If ∠C = 20°, find angle AOD.



Solution:



Join OB.

In ΔOBC, we have

BC = OD = OB [Radii of the same circle]

∠BOC = ∠BCO = 20°

And ext. ∠ABO = ∠BCO + ∠BOC

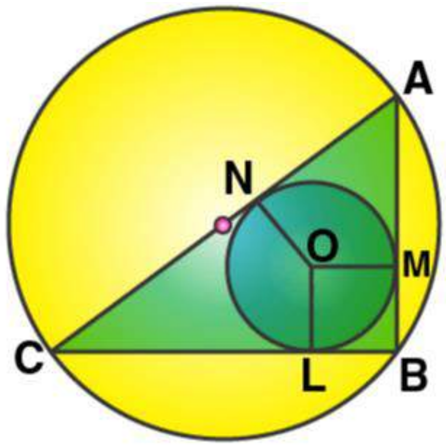
Ext. ∠ABO = 20° + 20° = 40° ….. (1)

Now in ΔOAB,

OA = OB [Radii of the same circle]

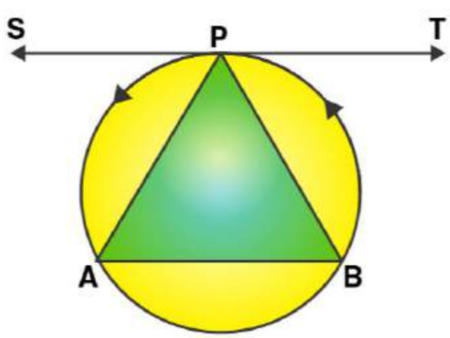
$\angle OAB = \angle OBA = 40^o$ [from (1)]
$\angle AOB = 180^o - 40^o - 40^o = 100^o$
As DOC is a straight line,
$\angle AOD + \angle AOB + \angle BOC = 180^o$
$\angle AOD + 100^o + 20^o = 180^o$
$\angle AOD = 180^o - 120^o$
Thus, $\angle AOD = 60^o$

12. Prove that the perimeter of a right triangle is equal to the sum of the diameter of its incircle and twice the diameter of its circumcircle.
Solution:



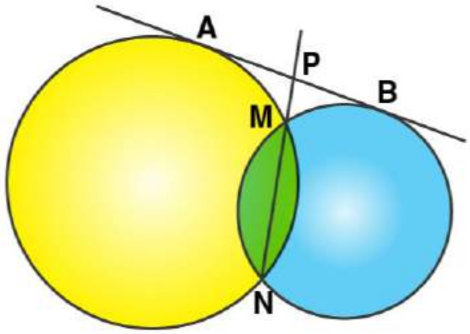
Let's join OL, OM and ON.
And, let D and d be the diameter of the circumcircle and incircle.
Also, let R and r be the radius of the circumcircle and incircle.
Now, in circumcircle of ΔABC,
$\angle B = 90^o$
Thus, AC is the diameter of the circumcircle i.e. AC = D
Let the radius of the incircle be 'r'
OL = OM = ON = r
Now, from B, BL and BM are the tangents to the incircle.
So, BL = BM = r
Similarly,
AM = AN and CL = CN = R
[Tangents from the point outside the circle]
Now,
AB + BC + CA = AM + BM + BL + CL + CA
= AN + r + r + CN + CA
= AN + CN + 2r + CA
= AC + AC + 2r
= 2AC + 2r
= 2D + d
- Hence Proved

13. P is the midpoint of an arc APB of a circle. Prove that the tangent drawn at P will be parallel to the chord AB.
Solution:



First join AP and BP.
As TPS is a tangent and PA is the chord of the circle.
$\angle BPT = \angle PAB$ [Angles in alternate segments]
But,
$\angle PBA = \angle PAB$ [Since PA = PB]
Thus, $\angle BPT = \angle PBA$
But these are alternate angles,
Hence, TPS || AB

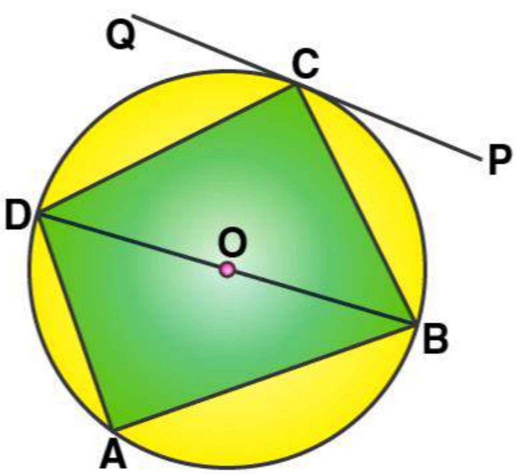
14. In the given figure, MN is the common chord of two intersecting circles and AB is their common tangent.
Prove that the line NM produced bisects AB at P.



Solution:

From P, AP is the tangent and PMN is the secant for first circle.
$AP^2 = PM \times PN$ …. (1)
Again from P, PB is the tangent and PMN is the secant for second circle.
$PB^2 = PM \times PN$ …. (2)
From (i) and (ii), we have
$AP^2 = PB^2$
AP = PB
Thus, P is the midpoint of AB.

15. In the given figure, ABCD is a cyclic quadrilateral, PQ is tangent to the circle at point C and BD is its diameter. If ∠DCQ = 40º and ∠ABD = 60º, find:
i) ∠DBC
ii) ∠BCP
iii) ∠ADB
Solution:

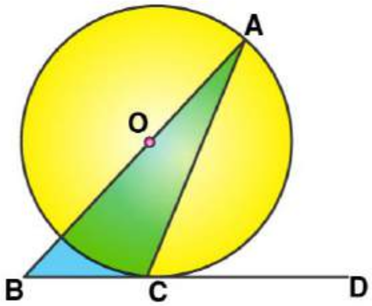


PQ is a tangent and CD is a chord.
$\angle DCQ = \angle DBC$ [Angles in the alternate segment]
$\angle DBC = 40^o$ [As $\angle DCQ = 40^o$]

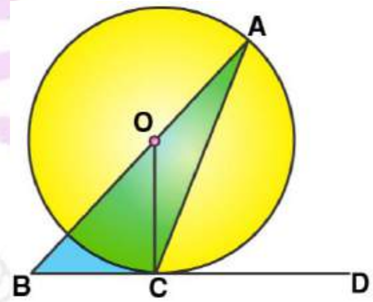
(ii) $\angle DCQ + \angle DCB + \angle BCP = 180^o$
$40^o + 90^o + \angle BCP = 180^o$ [As $\angle DCB = 90^o$]
$\angle BCP = 180^o - 130^o = 50^o$

(iii) In ΔABD,
$\angle BAD = 90^o$ [Angle in a semi-circle], $\angle ABD = 60^o$ [Given]
$\angle ADB = 180^o - (90^o + 60^o)$
$\angle ADB = 180^o - 150^o = 30^o$

16. The given figure shows a circle with centre O and BCD is a tangent to it at C. Show that: ∠ACD + ∠BAC = 90°

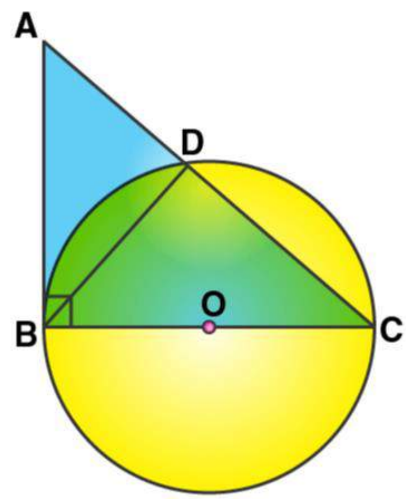


Solution:



Let's join OC.
BCD is the tangent and OC is the radius.
As, OC ⊥ BD
∠OCD = 90°
∠OCD + ∠ACD = 90° …. (i)
But, in ΔOCA
OA = OC   [Radii of the same circle]
Thus, ∠OCA = ∠OAC
Substituting in (i), we get
∠OAC + ∠ACD = 90°
Hence, ∠BAC + ∠ACD = 90°

17. ABC is a right triangle with angle B = 90°. A circle with BC as diameter meets by hypotenuse AC at point D. Prove that:
i) AC x AD = AB^2
ii) BD^2 = AD x DC.
Solution:

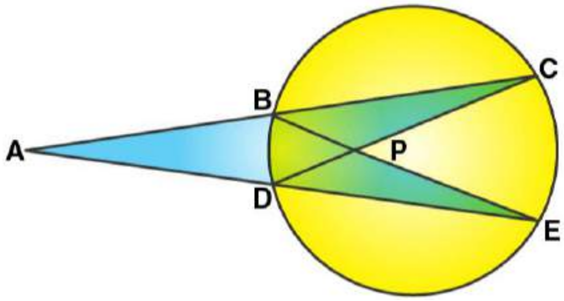


i) In ΔABC, we have
∠B = 90° and BC is the diameter of the circle.
Hence, AB is the tangent to the circle at B.
Now, as AB is tangent and ADC is the secant we have
AB^2 = AD x AC

ii) In ΔADB,
∠D = 90°
So, ∠A + ∠ABD = 90° …… (i)
But in ΔABC, ∠B = 90°
∠A + ∠C = 90° ……. (ii)
From (i) and (ii),
∠C = ∠ABD
Now in ΔABD and ΔCBD, we have
∠BDA = ∠BDC = 90°

∠ABD = ∠BCD
Hence, ΔABD ~ ΔCBD by AA postulate
So, we have
BD/DC = AD/BD
Therefore,
BD^2 = AD x DC

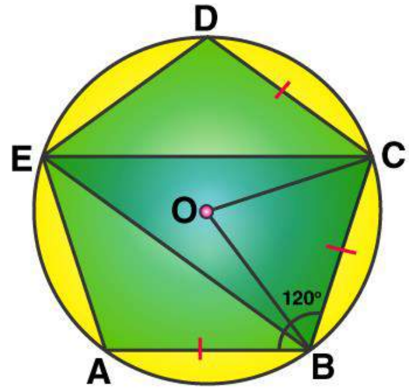
18. In the given figure, AC = AE.



Show that:
i) CP = EP
ii) BP = DP
Solution:

In ΔADC and ΔABE,
∠ACD = ∠AEB [Angles in the same segment]
AC = AE [Given]
∠A = ∠A [Common]
Hence, ΔADC ≅ ΔABE by ASA postulate
So, by C.P.C.T we have
AB = AD
But, AC = AE [Given]
So, AC – AB = AE – AD
BC = DE
In ΔBPC and ΔDPE,
∠C = ∠E [Angles in the same segment]
BC = DE
∠CBP = ∠CDE [Angles in the same segment]
Hence, ΔBPC ≅ ΔDPE by ASA postulate
So, by C.P.C.T we have
BP = DP and CP = PE

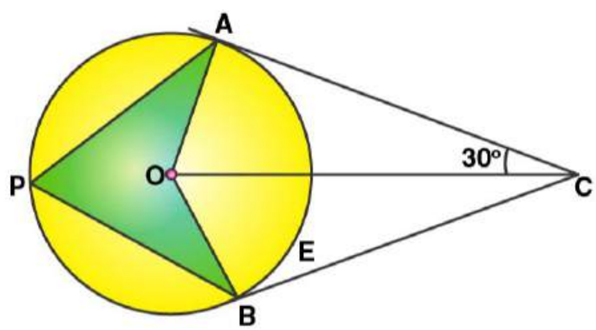
19. ABCDE is a cyclic pentagon with centre of its circumcircle at point O such that AB = BC = CD and angle ABC = 120º
Calculate:
i) ∠BEC
ii) ∠BED
Solution:



i) Join OC and OB.
AB = BC = CD and ∠ABC = 120° [Given]
So, ∠BCD = ∠ABC = 120°
OB and OC are the bisectors of ∠ABC and ∠BCD and respectively.
So, ∠OBC = ∠BCO = 60°
In ΔBOC,
∠BOC = 180° – (∠OBC + ∠BOC)
∠BOC = 180° – (60° + 60°) = 180° – 120°
∠BOC = 60°
Arc BC subtends ∠BOC at the centre and ∠BEC at the remaining part of the circle.
∠BEC = ½ ∠BOC = ½ x 60° = 30°

ii) In cyclic quadrilateral BCDE, we have
∠BED + ∠BCD = 180°
∠BED + 120° = 180°
Thus, ∠BED = 60°

20. In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If angle ACO = 30°, find:
(i) angle BCO
(ii) angle AOB
(iii) angle APB
Solution:



In the given fig, O is the centre of the circle and, CA and CB are the tangents to the circle from C.
Also, ∠ACO = 30°
P is any point on the circle. P and B are joined.
To find:
(i) ∠BCO
(ii) ∠AOB
(iii) ∠APB
Proof:

(i) In ΔOAC and ΔOBC, we have
OC = OC [Common]
OA = OB [Radii of the same circle]
CA = CB [Tangents to the circle]
Hence, ΔOAC ≅ ΔOBC by SSS congruence criterion
Thus, ∠ACO = ∠BCO = 30°

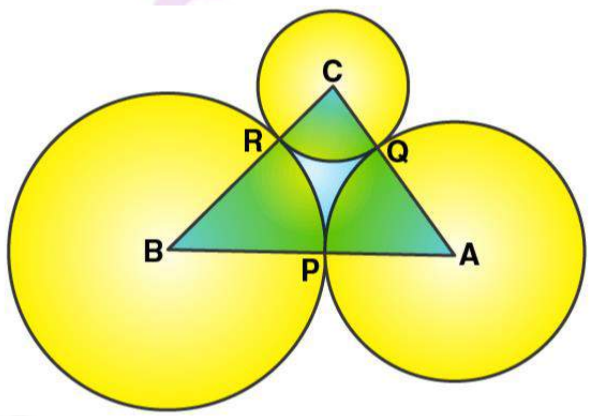
(ii) As ∠ACB = 30° + 30° = 60°

And, $\angle AOB + \angle ACB = 180^o$
$\angle AOB + 60^o = 180^o$
$\angle AOB = 180^o - 60^o$
$\angle AOB = 120^o$

(iii) Arc AB subtends $\angle AOB$ at the center and $\angle APB$ is the remaining part of the circle.
$\angle APB = ½ \angle AOB = ½ \times 120^o = 60^o$

21. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centers. Find the radii of the three circles.

Solution:



Given: ABC is a triangle with AB = 10 cm, BC= 8 cm, AC = 6 cm. Three circles are drawn with centre A, B and C touch each other at P, Q and R respectively.
So, we need to find the radii of the three circles.
Let,
PA = AQ = x
QC = CR = y
RB = BP = z
So, we have
$x + z = 10$ ….. (i)
$z + y = 8$ …… (ii)
$y + x = 6$ ……. (iii)
Adding all the three equations, we have
$2(x + y + z) = 24$
$x + y + z = 24/2 = 12$ ….. (iv)
Subtracting (i), (ii) and (iii) from (iv) we get
$y = 12 – 10 = 2$
$x = 12 – 8 = 4$
$z = 12 – 6 = 6$
Thus, radii of the three circles are 2 cm, 4 cm and 6 cm.