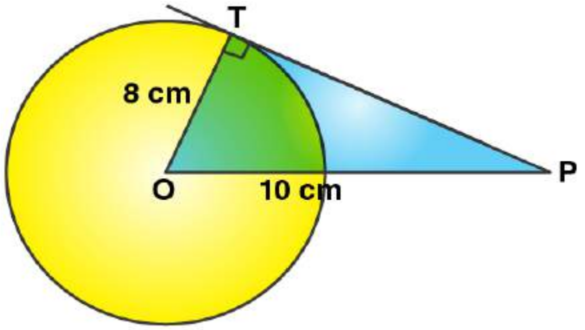
Exercise 18(A)   Page No: 274

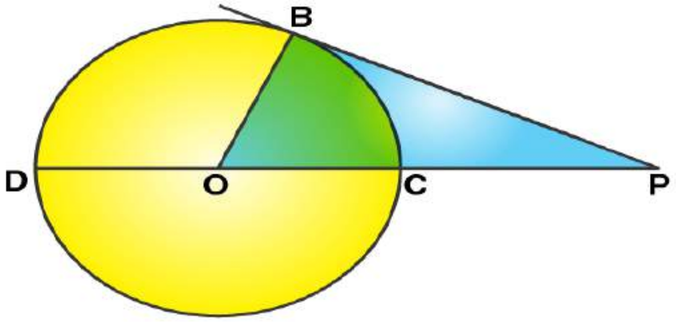
1. The radius of a circle is 8cm. Calculate the length of a tangent drawn to this circle from a point at a distance of 10cm from its centre.

Solution:



Given, a circle with centre O and radius 8 cm.
An external point P from where a tangent is drawn to meet the circle at T.
OP = 10 cm; radius OT = 8 cm
As OT ⊥ PT
In right ΔOTP, we have
$OP^2 = OT^2 + PT^2$ [By Pythagoras Theorem]
$10^2 = 8^2 + PT^2$
$PT^2 = 100 – 64 = 36$
So, PT = 6
Therefore, length of tangent = 6 cm.

2. In the given figure, O is the centre of the circle and AB is a tangent to the circle at B. If AB = 15 cm and AC = 7.5 cm, calculate the radius of the circle.

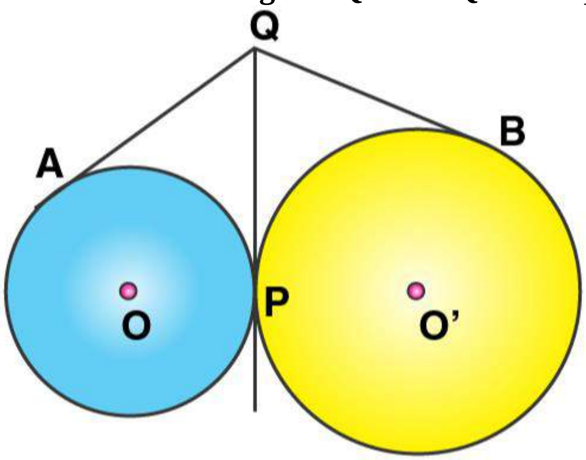


Solution:

Given,
AB = 15 cm, AC = 7.5 cm
Let's assume the radius of the circle to be 'r'.
So, AO = AC + OC = 7.5 + r
In right ΔAOB, we have
$AO^2 = AB^2 + OB^2$ [By Pythagoras Theorem]
$(7.5 + r)^2 = 15^2 + r^2$
$56.25 + r^2 + 15r = 225 + r^2$
$15r = 225 – 56.25$

r = 168.75/ 15
Thus,
r = 11.25 cm

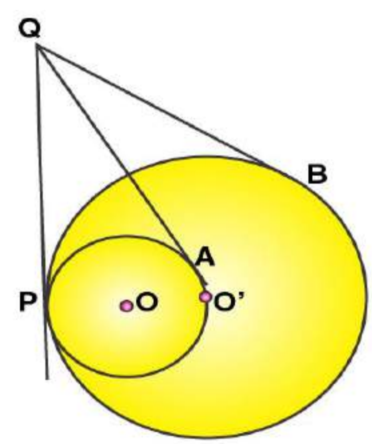
3. Two circles touch each other externally at point P. Q is a point on the common tangent through P. Prove that the tangents QA and QB are equal.



Solution:

Let Q be the point from which, QA and QP are two tangents to the circle with centre O
So, QA = QP .....(a)
Similarly, from point Q, QB and QP are two tangents to the circle with centre O'
So, QB = QP ......(b)
From (a) and (b), we have
QA = QB
Therefore, tangents QA and QB are equal.
- Hence Proved

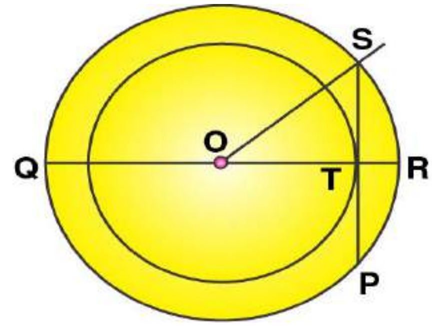
4. Two circles touch each other internally. Show that the tangents drawn to the two circles from any point on the common tangent are equal in length.
Solution:



Let Q be the point on the common tangent from which, two tangents QA and QP are drawn to the circle with centre O.
So, QA = QP .... (1)
Similarly, from point Q, QB and QP are two tangents to the circle with centre O'
So, QB = QP .... (2)
From (1) and (2), we have
QA = QB
Therefore, tangents QA and QB are equal.
- Hence Proved

5. Two circles of radii 5 cm and 3 cm are concentric. Calculate the length of a chord of the outer circle which touches the inner.

Solution:



Given,

OS = 5 cm and OT = 3 cm

In right triangle OST, we have

$ST^2 = OS^2 - OT^2$

$= 25 - 9$

$= 16$

So, ST = 4 cm

As we know, OT is perpendicular to SP and OT bisects chord SP

Hence, SP = 2 x ST = 8 cm

6. Three circles touch each other externally. A triangle is formed when the centers of these circles are joined together. Find the radii of the circles, if the sides of the triangle formed are 6 cm, 8 cm and 9 cm.

Solution:

Let ABC be the triangle formed when centres of 3 circles are joined.

Given,

AB = 6 cm, AC = 8 cm and BC = 9 cm

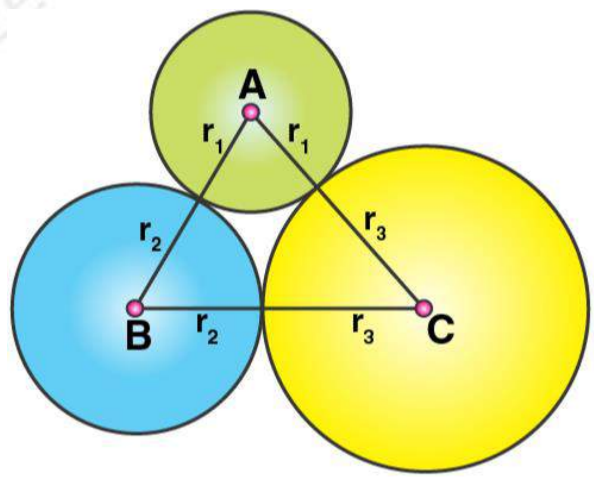
And let the radii of the circles having centres A, B and C be r_1, r_2 and r_3 respectively.

So, we have

$r_1 + r_3 = 8$

$r_3 + r_2 = 9$

$r_2 + r_1 = 6$



Adding all the above equations, we get

$r_1 + r_3 + r_3 + r_2 + r_2 + r_1 = 8 + 9 + 6$

$2(r_1 + r_2 + r_3) = 23$

So,

$r_1 + r_2 + r_3 = 11.5$ cm

Now,

$r_1 + 9 = 11.5$ (As $r_2 + r_3 = 9$)

$r_1 = 2.5$ cm

And,

$r_2 + 6 = 11.5$ (As $r_1 + r_3 = 6$)

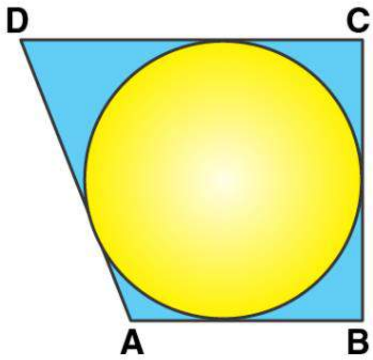
$r_2 = 5.5$ cm

Lastly, $r_3 + 8 = 11.5$ (As $r_2 + r_1 = 8$)

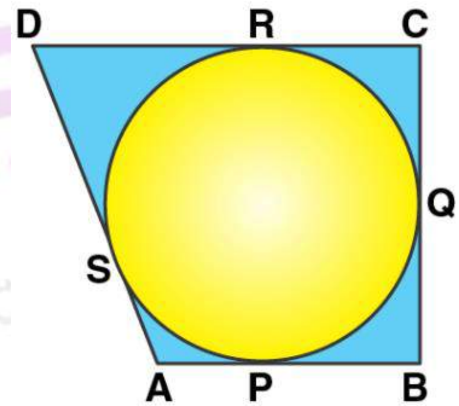
$r_3 = 3.5$ cm

Therefore, the radii of the circles are $r_1 = 2.5$ cm, $r_2 = 5.5$ cm and $r_3 = 3.5$ cm.

7. If the sides of a quadrilateral ABCD touch a circle, prove that AB + CD = BC + AD.

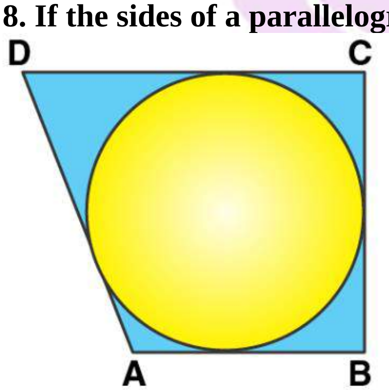


Solution:



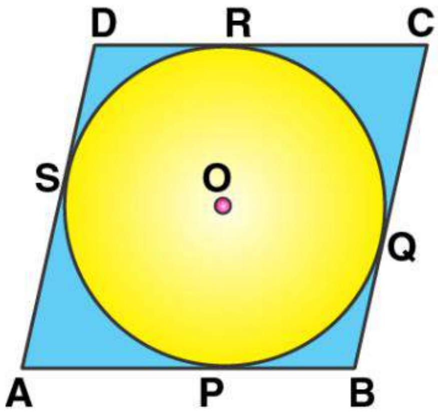
Let a circle touch the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively.
As, AP and AS are tangents to the circle from an external point A, we have
AP = AS .... (1)
Similarly, we also get
BP = BQ .... (2)
CR = CQ .... (3)
DR = DS .... (4)
Adding (1), (2), (3) and (4), we get
AP + BP + CR + DR = AS + DS + BQ + CQ
AB + CD = AD + BC
Therefore,
AB + CD = AD + BC
- Hence Proved

8. If the sides of a parallelogram touch a circle, prove that the parallelogram is a rhombus.



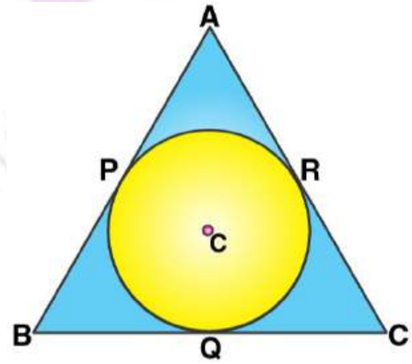
Solution:

Let a circle touch the sides AB, BC, CD and DA of parallelogram ABCD at P, Q, R and S respectively.
Now, from point A, AP and AS are tangents to the circle.
So, AP = AS....... (1)
Similarly, we also have
BP = BQ .... (2)



CR = CQ .... (3)
DR = DS .... (4)
Adding (1), (2), (3) and (4), we get
AP + BP + CR + DR = AS + DS + BQ + CQ
AB + CD = AD + BC
Therefore,
AB + CD = AD + BC
But AB = CD and BC = AD....... (5) [Opposite sides of a parallelogram]
Hence,
AB + AB = BC + BC
2AB = 2 BC
AB = BC .... (6)
From (5) and (6), we conclude that
AB = BC = CD = DA
Thus, ABCD is a rhombus.

9. From the given figure prove that:
AP + BQ + CR = BP + CQ + AR.



Also, show that AP + BQ + CR = ½ x perimeter of triangle ABC.
Solution:

As from point B, BQ and BP are the tangents to the circle
We have, BQ = BP ………..(1)
Similarly, we also get
AP = AR ………….. (2)
And, CR = CQ …… (3)
Adding (1), (2) and (3) we get,
AP + BQ + CR = BP + CQ + AR ……… (4)
Now, adding AP + BQ + CR to both sides in (4), we get
2(AP + BQ + CR) = AP + PQ + CQ + QB + AR + CR
2(AP + BQ + CR) = AB + BC + CA
Therefore, we get
AP + BQ + CR = ½ x (AB + BC + CA)
i.e.
AP + BQ + CR = ½ x perimeter of triangle ABC

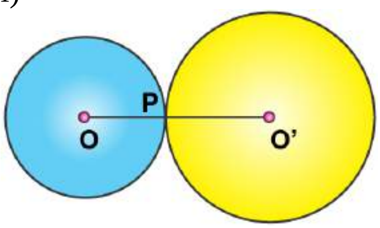
10. In the figure, if AB = AC then prove that BQ = CQ.
Solution:

As, from point A
AP and AR are the tangents to the circle
So, we have AP = AR
Similarly, we also have
BP = BQ and CR = CQ [From points B and C]

Now adding the above equations, we get
AP + BP + CQ = AR + BQ + CR
(AP + BP) + CQ = (AR + CR) + BQ
AB + CQ = AC + BQ ….. (i)
But, as AB = AC [Given]
Therefore, from (i)
CQ = BQ or BQ = CQ

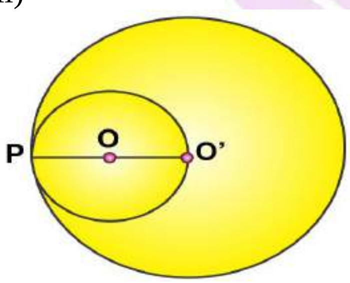
11. Radii of two circles are 6.3 cm and 3.6 cm. State the distance between their centers if -
i) they touch each other externally.
ii) they touch each other internally.
Solution:

Given,
Radius of bigger circle = 6.3 cm and of smaller circle = 3.6 cm
i)



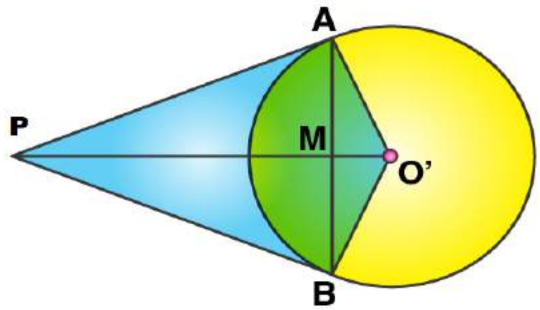
When the two circles touch each other at P externally. O and O' are the centers of the circles. Join OP and O'P.
So, OP = 6.3 cm, O'P = 3.6 cm
Hence, the distance between their centres (OO') is given by
OO' = OP + O'P = 6.3 + 3.6 = 9.9 cm

ii)



When the two circles touch each other at P internally. O and O' are the centers of the circles. Join OP and O'P
So, OP = 6.3 cm, O'P = 3.6 cm
Hence, the distance between their centres (OO') is given by
OO' = OP - O'P = 6.3 - 3.6 = 2.7 cm

12. From a point P outside the circle, with centre O, tangents PA and PB are drawn. Prove that:
i) ∠AOP = ∠BOP
ii) OP is the ⊥ bisector of chord AB.
Solution:



i) In ΔAOP and ΔBOP, we have

$AP = BP$ [Tangents from P to the circle]

$OP = OP$ [Common]

$OA = OB$ [Radii of the same circle]

Hence, by SAS criterion of congruence

$\Delta AOP \cong \Delta BOP$

So, by C.P.C.T we have

$\angle AOP = \angle BOP$

ii) In ΔOAM and ΔOBM, we have

$OA = OB$ [Radii of the same circle]

$\angle AOM = \angle BOM$ [Proved $\angle AOP = \angle BOP$]

$OM = OM$ [Common]

Hence, by SAS criterion of congruence

$\Delta OAM \cong \Delta OBM$

So, by C.P.C.T we have

$AM = MB$

And $\angle OMA = \angle OMB$

But,

$\angle OMA + \angle OMB = 180^o$

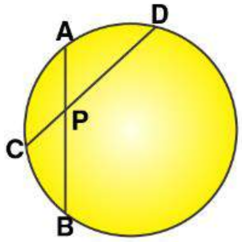
Thus, $\angle OMA = \angle OMB = 90^o$

Therefore, OM or OP is the perpendicular bisector of chord AB.

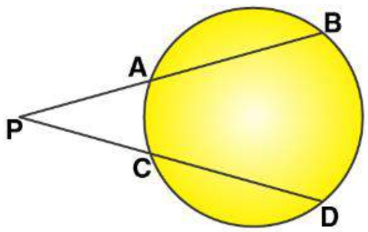
- Hence Proved

Exercise 18(B) Page No: 283

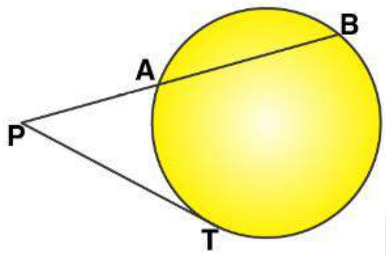
1. (i) In the given figure, 3 x CP = PD = 9 cm and AP = 4.5 cm. Find BP.



(ii) In the given figure, 5 x PA = 3 x AB = 30 cm and PC = 4cm. Find CD.



(iii) In the given figure, tangent PT = 12.5 cm and PA = 10 cm; find AB.



Solution:

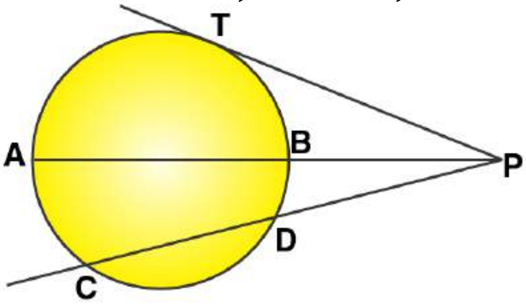
(i) As the two chords AB and CD intersect each other at P, we have
AP x PB = CP x PD
4.5 x PB = 3 x 9 [3CP = 9 cm so, CP = 3 cm]
PB = (3 x 9)/ 4.5 = 6 cm

(ii) As the two chords AB and CD intersect each other at P, we have
AP x PB = CP x PD
But, 5 x PA = 3 x AB = 30 cm
So, PA = 30/5 = 6 cm and AB = 30/3 = 10 cm
And, BP = PA + AB = 6 + 10 = 16 cm
Now, as
AP x PB = CP x PD
6 x 16 = 4 x PD
PD = (6 x 16)/ 4 = 24 cm
CD = PD – PC = 24 – 4 = 20 cm

(iii) As PAB is the secant and PT is the tangent, we have
PT^2 = PA x PB

$12.5^2 = 10 \times PB$
PB = (12.5 x 12.5)/ 10 = 15.625 cm
AB = PB – PA = 15.625 – 10 = 5.625 cm

2. In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm.

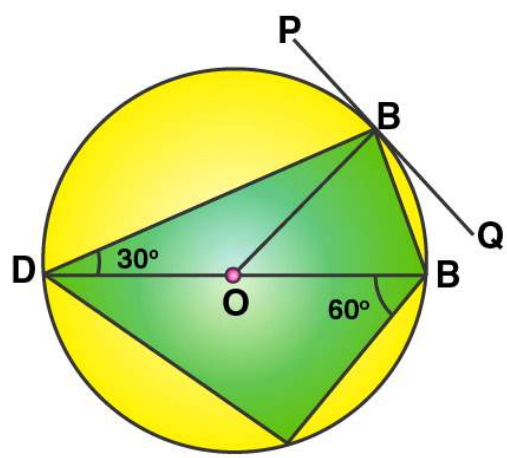


Find
(i) AB.
(ii) the length of tangent PT.
Solution:

(i) PA = AB + BP = (AB + 4) cm
PC = PD + CD = 5 + 7.8 = 12.8 cm
As PA x PB = PC x PD
(AB + 4) x 4 = 12.8 x 5
AB + 4 = (12.8 x 5)/ 4
AB + 4 = 16
Hence, AB = 12 cm

(ii) As we know,
$PT^2 = PC \times PD$
$PT^2 = 12.8 \times 5 = 64$
Thus, PT = 8 cm

3. In the following figure, PQ is the tangent to the circle at A, DB is a diameter and O is the centre of the circle. If ∠ADB = 30º and ∠CBD = 60º; calculate:
i) ∠QAB
ii) ∠PAD
iii) ∠CDB
Solution:

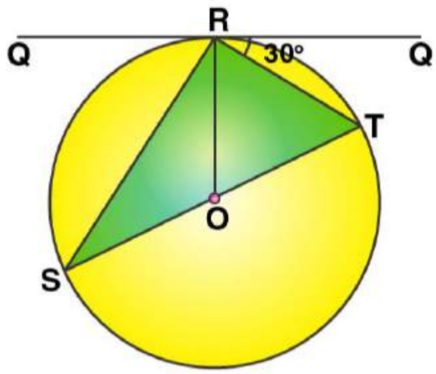


(i) Given, PAQ is a tangent and AB is the chord
∠QAB = ∠ADB = 30º [Angles in the alternate segment]

(ii) OA = OD [radii of the same circle]
So, ∠OAD = ∠ODA = 30º
But, as OA ⊥ PQ
∠PAD = ∠OAP - ∠OAD = 90º – 30º = 60º

(iii) As BD is the diameter, we have
$\angle BCD = 90^o$ [Angle in a semi-circle]
Now in ΔBCD,
$\angle CDB + \angle CBD + \angle BCD = 180^o$
$\angle CDB + 60^o + 90^o = 180^o$
Thus, $\angle CDB = 180^o - 150^o = 30^o$

4. If PQ is a tangent to the circle at R; calculate:
i) $\angle PRS$
ii) $\angle ROT$



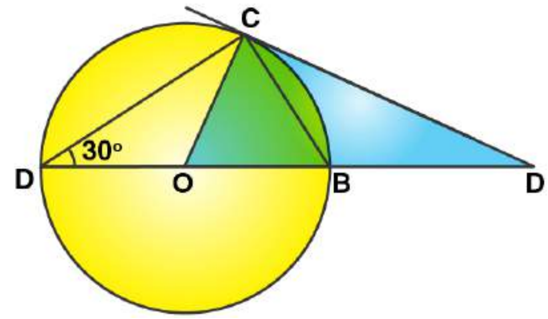
Given: O is the centre of the circle and $\angle TRQ = 30^o$
Solution:

(i) As PQ is the tangent and OR is the radius.
So, $OR \perp PQ$
$\angle ORT = 90^o$
$\angle TRQ = 90^o – 30^o = 60^o$
But in ΔOTR, we have
OT = OR [Radii of same circle]
$\angle OTR = 60^o$ or $\angle STR = 60^o$
But,
$\angle PRS = \angle STR = 60^o$ [Angles in the alternate segment]

(ii) In ΔOTR,
$\angle ORT = 60^o$
$\angle OTR = 60^o$
Thus,
$\angle ROT = 180^o – (60^o + 60^o) = 180^o – 120^o = 60^o$

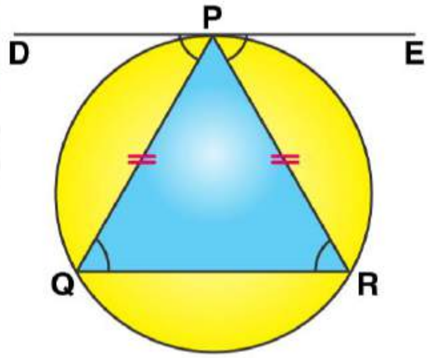
5. AB is diameter and AC is a chord of a circle with centre O such that angle BAC=30^o. The tangent to the circle at C intersects AB produced in D. Show that BC = BD.
Solution:

Join OC.



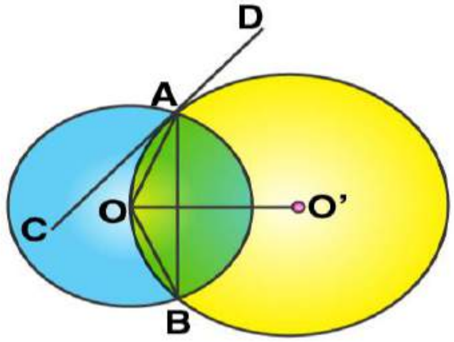
∠BCD = ∠BAC = 30° [Angles in the alternate segment]
It's seen that, arc BC subtends ∠DOC at the center of the circle and ∠BAC at the remaining part of the circle.
So, ∠BOC = 2∠BAC = 2 x 30° = 60°
Now, in ΔOCD
∠BOC or ∠DOC = 60°
∠OCD = 90° [OC ⊥ CD]
∠DOC + ∠ODC = 90°
∠ODC = 90° – 60° = 30°
Now, in ΔBCD
As ∠ODC or ∠BDC = ∠BCD = 30°
Therefore, BC = BD

6. Tangent at P to the circumcircle of triangle PQR is drawn. If this tangent is parallel to side QR, show that triangle PQR is isosceles.
Solution:



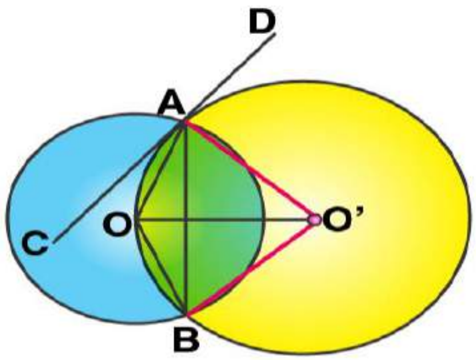
Let DE be the tangent to the circle at P.
And, DE || QR [Given]
∠EPR = ∠PRQ [Alternate angles are equal]
∠DPQ = ∠PQR [Alternate angles are equal] ….. (i)
Let ∠DPQ = x and ∠EPR = y
As the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment, we have
∠DPQ = ∠PRQ …… (ii) [DE is tangent and PQ is chord]
So, from (i) and (ii),
∠PQR = ∠PRQ
PQ = PR
Therefore, triangle PQR is an isosceles triangle.

7. Two circles with centres O and O' are drawn to intersect each other at points A and B. Centre O of one circle lies on the circumference of the other circle and CD is drawn tangent to the circle with centre O' at A. Prove that OA bisects angle BAC.



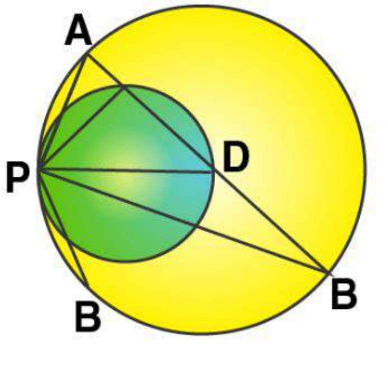
Solution:

Join OA, OB, O'A, O'B and O'O.



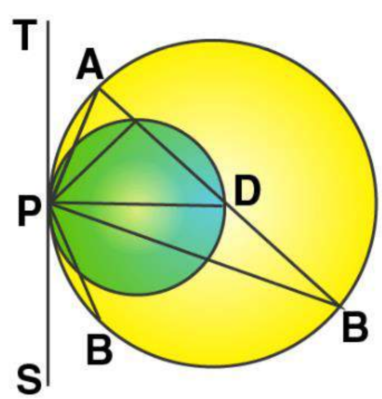
CD is the tangent and AO is the chord.
∠OAC = ∠OBA … (i) [Angles in alternate segment]
In ΔOAB,
OA = OB [Radii of the same circle]
∠OAB = ∠OBA ….. (ii)
From (i) and (ii), we have
∠OAC = ∠OAB
Thus, OA is the bisector of ∠BAC.

8. Two circles touch each other internally at a point P. A chord AB of the bigger circle intersects the other circle in C and D. Prove that: ∠CPA = ∠DPB



Solution:

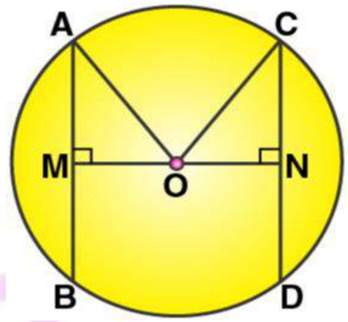
Let's draw a tangent TS at P to the circles given.
As TPS is the tangent and PD is the chord, we have
∠PAB = ∠BPS …. (i) [Angles in alternate segment]
Similarly,
∠PCD = ∠DPS …. (ii)
Now, subtracting (i) from (ii) we have
∠PCD - ∠PAB = ∠DPS - ∠BPS
But in ΔPAC,
Ext. ∠PCD = ∠PAB + ∠CPA
∠PAB + ∠CPA - ∠PAB = ∠DPS - ∠BPS
Thus,
∠CPA = ∠DPB



Exercise 18(C) Page No: 285

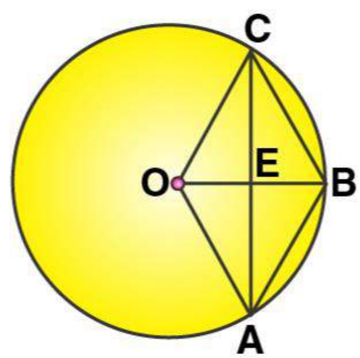
1. Prove that, of any two chords of a circle, the greater chord is nearer to the center.
Solution:

Given: A circle with center O and radius r. AB and CD are two chords such that AB > CD. Also, OM ⊥ AB and ON ⊥ CD.



Required to prove: OM < ON
Proof:
Join OA and OC.
Then in right ΔAOM, we have
$AO^2 = AM^2 + OM^2$
$r^2 = (½AB)^2 + OM^2$
$r^2 = ¼\ AB^2 + OM^2$ ….. (i)
Again, in right ΔONC, we have
$OC^2 = NC^2 + ON^2$
$r^2 = (½CD)^2 + ON^2$
$r^2 = ¼\ CD^2 + ON^2$ ….. (ii)
On equating (i) and (ii), we get
$¼\ AB^2 + OM^2 = ¼\ CD^2 + ON^2$
But, AB > CD [Given]
So, ON will be greater than OM to be equal on both sides.
Thus,
OM < ON
Hence, AB is nearer to the centre than CD.

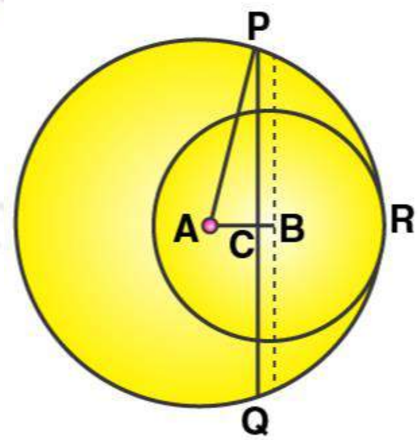
2. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O.
i) If the radius of the circle is 10 cm, find the area of the rhombus.
ii) If the area of the rhombus is 32√3 cm², find the radius of the circle.
Solution:



(i) Given, radius = 10 cm
In rhombus OABC,
OC = 10 cm
So,
OE = ½ x OB = ½ x 10 = 5 cm
Now, in right ΔOCE
$OC^2 = OE^2 + EC^2$
$10^2 = 5^2 + EC^2$
$EC^2 = 100 – 25 = 75$
EC = √75 = 5√3
Hence, AC = 2 x EC = 2 x 5√3 = 10√3
We know that,
Area of rhombus = ½ x OB x AC
= ½ x 10 x 10√3
= 50√3 cm² ≈ 86.6 cm²

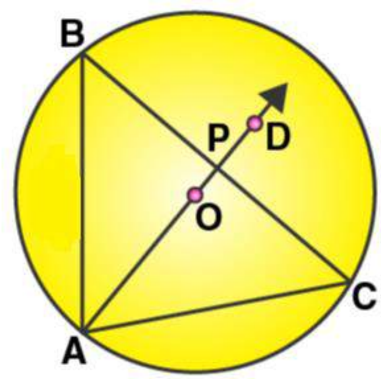
(ii) We have the area of rhombus = $32\sqrt{3}$ cm^2
But area of rhombus OABC = 2 x area of ΔOAB
Area of rhombus OABC = 2 x $(\sqrt{3}/4)$ r^2
Where r is the side of the equilateral triangle OAB.
$2 \times (\sqrt{3}/4) r^2 = 32\sqrt{3}$
$\sqrt{3}/2 \; r^2 = 32\sqrt{3}$
$r^2 = 64$
r = 8
Therefore, the radius of the circle is 8 cm.

3. Two circles with centers A and B, and radii 5 cm and 3 cm, touch each other internally. If the perpendicular bisector of the segment AB meets the bigger circle in P and Q; find the length of PQ.
Solution:



We know that,
If two circles touch internally, then distance between their centres is equal to the difference of their radii. So, AB = (5 - 3) cm = 2 cm.
Also, the common chord PQ is the perpendicular bisector of AB.
Thus, AC = CB = ½ AB = 1 cm
In right ΔACP, we have
$AP^2 = AC^2 + CP^2$ [Pythagoras Theorem]
$5^2 = 1^2 + CP^2$
$CP^2 = 25 - 1 = 24$
CP = $\sqrt{24}$ cm = $2\sqrt{6}$ cm
Now,
PQ = 2 CP
= 2 x $2\sqrt{6}$ cm
= $4\sqrt{6}$ cm
Therefore, the length of PQ is $4\sqrt{6}$ cm.

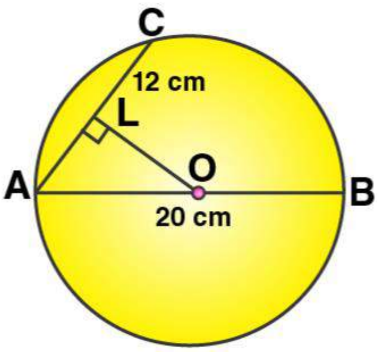
4. Two chords AB and AC of a circle are equal. Prove that the center of the circle, lies on the bisector of the angle BAC.
Solution:



Given: AB and AC are two equal chords of C (O, r).
Required to prove: Centre, O lies on the bisector of $\angle BAC$.
Construction: Join BC. Let the bisector of $\angle BAC$ intersects BC in P.
Proof:
In ΔAPB and ΔAPC,
AB = AC [Given]
$\angle BAP = \angle CAP$ [Given]
AP = AP [Common]
Hence, $\Delta APB \cong \Delta APC$ by SAA congruence criterion
So, by CPCT we have
BP = CP and $\angle APB = \angle APC$

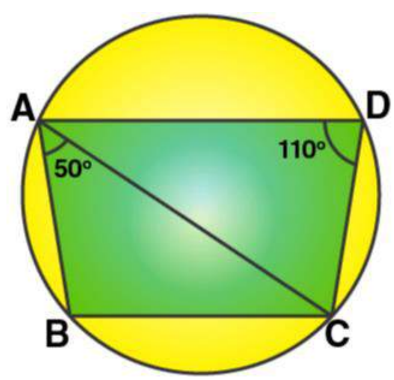
And,
$\angle APB + \angle APC = 180^\circ$ [Linear pair]
$2\angle APB = 180^\circ$ [$\angle APB = \angle APC$]
$\angle APB = 90^\circ$
Now, BP = CP and $\angle APB = 90^\circ$
Therefore, AP is the perpendicular bisector of chord BC.
Hence, AP passes through the centre, O of the circle.

5. The diameter and a chord of circle have a common end-point. If the length of the diameter is 20 cm and the length of the chord is 12 cm, how far is the chord from the center of the circle?
Solution:



We have, AB as the diameter and AC as the chord.
Now, draw OL ⊥ AC
Since OL ⊥ AC and hence it bisects AC, O is the centre of the circle.
Therefore, OA = 10 cm and AL = 6 cm
Now, in right ΔOLA
$AO^2 = AL^2 + OL^2$ [By Pythagoras Theorem]
$10^2 = 6^2 + OL^2$
$OL^2 = 100 – 36 = 64$
OL = 8 cm
Therefore, the chord is at a distance of 8 cm from the centre of the circle.

6. ABCD is a cyclic quadrilateral in which BC is parallel to AD, angle ADC = 110° and angle BAC = 50°. Find angle DAC and angle DCA.
Solution:

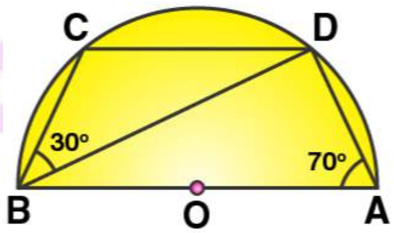


Given, ABCD is a cyclic quadrilateral in which AD || BC
And, $\angle ADC = 110^\circ$, $\angle BAC = 50^\circ$
We know that,
$\angle B + \angle D = 180^\circ$ [Sum of opposite angles of a quadrilateral]
$\angle B + 110^\circ = 180^\circ$
So, $\angle B = 70^\circ$
Now in ΔADC, we have
$\angle BAC + \angle ABC + \angle ACB = 180^\circ$
$50^\circ + 70^\circ + \angle ACB = 180^\circ$
$\angle ACB = 180^\circ – 120^\circ = 60^\circ$

And, as AD || BC we have
$\angle DAC = \angle ACB = 60^o$ [Alternate angles]
Now in ΔADC,
$\angle DAC + \angle ADC + \angle DCA = 180^o$
$60^o + 110^o + \angle DCA = 180^o$
Thus,
$\angle DCA = 180^o - 170^o = 10^o$

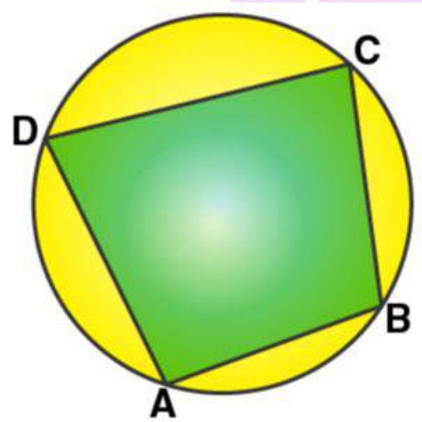
7. In the given figure, C and D are points on the semi-circle described on AB as diameter. Given angle BAD = 70º and angle DBC = 30º, calculate angle BDC.

Solution:



As ABCD is a cyclic quadrilateral, we have
$\angle BCD + \angle BAD = 180^o$ [Opposite angles of a cyclic quadrilateral are supplementary]
$\angle BCD + 70^o = 180^o$
$\angle BCD = 180^o - 70^o = 110^o$
And, by angle sum property of ΔBCD we have
$\angle CBD + \angle BCD + \angle BDC = 180^o$
$30^o + 110^o + \angle BDC = 180^o$
$\angle BDC = 180^o - 140^o$
Thus,
$\angle BDC = 40^o$

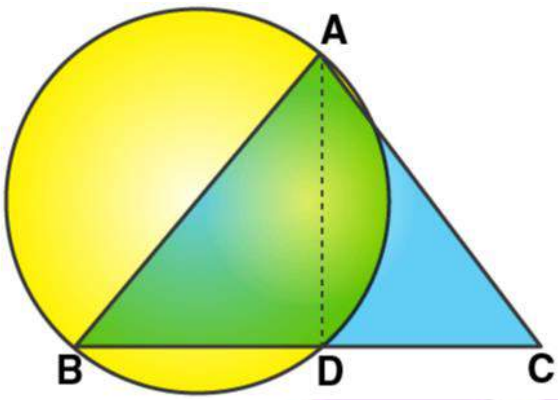
8. In cyclic quadrilateral ABCD, ∠A = 3 ∠C and ∠D = 5 ∠B. Find the measure of each angle of the quadrilateral.
Solution:



Given, cyclic quadrilateral ABCD
So, $\angle A + \angle C = 180^o$ [Opposite angles in a cyclic quadrilateral is supplementary]
$3\angle C + \angle C = 180^o$ [As $\angle A = 3 \angle C$]
$\angle C = 45^o$
Now,
$\angle A = 3 \angle C = 3 \times 45^o$

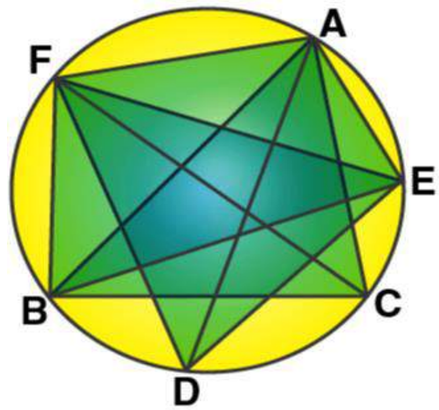
$\angle A = 135^o$
Similarly,
$\angle B + \angle D = 180^o$ [As $\angle D = 5\angle B$]
$\angle B + 5\angle B = 180^o$
$6\angle B = 180^o$
$\angle B = 30^o$
Now,
$\angle D = 5\angle B = 5 \times 30^o$
$\angle D = 150^o$
Therefore,
$\angle A = 135^o$, $\angle B = 30^o$, $\angle C = 45^o$, $\angle D = 150^o$

9. Show that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.
Solution:



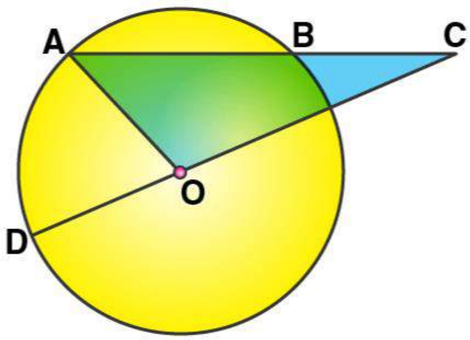
Let's join AD.
And. AB is the diameter.
We have $\angle ADB = 90^o$ [Angle in a semi-circle]
But,
$\angle ADB + \angle ADC = 180^o$ [Linear pair]
So, $\angle ADC = 90^o$
Now, in ΔABD and ΔACD we have
$\angle ADB = \angle ADC$ [each 90^o]
AB = AC [Given]
AD = AD [Common]
Hence, $\Delta ABD \cong \Delta ACD$ by RHS congruence criterion
So, by C.P.C.T
BD = DC
Therefore, the circle bisects base BC at D.

10. Bisectors of vertex angles A, B and C of a triangle ABC intersect its circumcircle at points D, E and F respectively. Prove that angle EDF = 90^o – ½ $\angle A$
Solution:

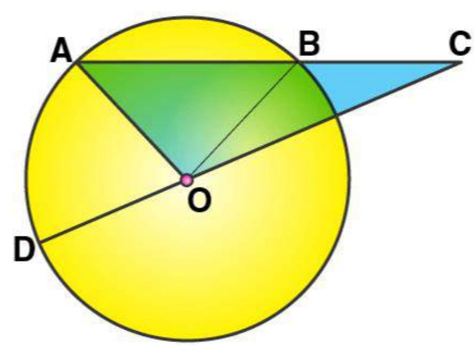


Join ED, EF and DF. Also join BF, FA, AE and EC.
∠EBF = ∠ECF = ∠EDF ….. (i) [Angle in the same segment]
In cyclic quadrilateral AFBE,
∠EBF + ∠EAF = 180° …… (ii)
[Sum of opposite angles in a cyclic quadrilateral is supplementary]
Similarly in cyclic quadrilateral CEAF,
∠EAF + ∠ECF = 180° ……. (iii)
Adding (ii) and (iii) we get,
∠EBF + ∠ECF + 2∠EAF = 360°
∠EDF + ∠EDF + 2∠EAF = 360° [From (i)]
∠EDF + ∠EAF = 180°
∠EDF + ∠1 + ∠BAC + ∠2 = 180°
But, ∠1 = ∠3 and ∠2 and ∠4 [Angles in the same segment]
∠EDF + ∠3 + ∠BAC + ∠4 = 180°
But, ∠4 = ½ ∠C, ∠3 = ½ ∠B
Thus, ∠EDF + ½ ∠B + ∠BAC + ½ ∠C = 180°
∠EDF + ½ ∠B + 2 x ½ ∠A + ½ ∠C = 180°
∠EDF + ½ (∠A + ∠B + ∠C) + ½ ∠A = 180°
∠EDF + ½ (180°) + ½ ∠A = 180°
∠EDF + 90° + ½ ∠A = 180°
∠EDF = 180° – (90° + ½ ∠A)
∠EDF = 90° – ½ ∠A

11. In the figure, AB is the chord of a circle with centre O and DOC is a line segment such that BC = DO. If ∠C = 20°, find angle AOD.



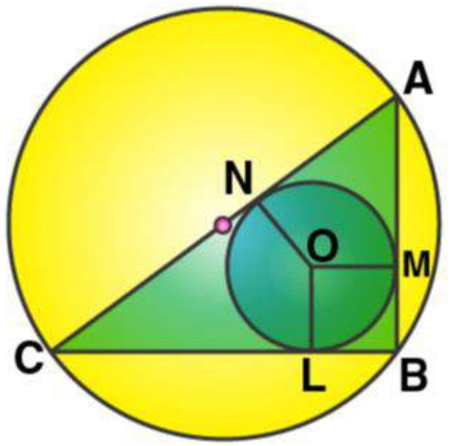
Solution:



Join OB.
In ΔOBC, we have
BC = OD = OB [Radii of the same circle]
∠BOC = ∠BCO = 20°
And ext. ∠ABO = ∠BCO + ∠BOC
Ext. ∠ABO = 20° + 20° = 40° ….. (1)
Now in ΔOAB,
OA = OB [Radii of the same circle]

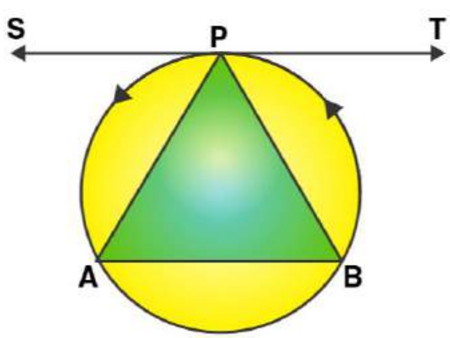
$\angle OAB = \angle OBA = 40^o$ [from (1)]
$\angle AOB = 180^o - 40^o - 40^o = 100^o$
As DOC is a straight line,
$\angle AOD + \angle AOB + \angle BOC = 180^o$
$\angle AOD + 100^o + 20^o = 180^o$
$\angle AOD = 180^o - 120^o$
Thus, $\angle AOD = 60^o$

12. Prove that the perimeter of a right triangle is equal to the sum of the diameter of its incircle and twice the diameter of its circumcircle.
Solution:



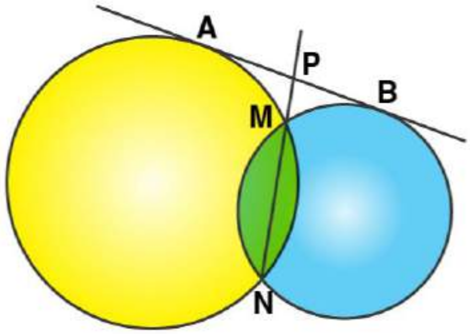
Let's join OL, OM and ON.
And, let D and d be the diameter of the circumcircle and incircle.
Also, let R and r be the radius of the circumcircle and incircle.
Now, in circumcircle of ΔABC,
$\angle B = 90^o$
Thus, AC is the diameter of the circumcircle i.e. AC = D
Let the radius of the incircle be 'r'
OL = OM = ON = r
Now, from B, BL and BM are the tangents to the incircle.
So, BL = BM = r
Similarly,
AM = AN and CL = CN = R
[Tangents from the point outside the circle]
Now,
AB + BC + CA = AM + BM + BL + CL + CA
= AN + r + r + CN + CA
= AN + CN + 2r + CA
= AC + AC + 2r
= 2AC + 2r
= 2D + d
- Hence Proved

13. P is the midpoint of an arc APB of a circle. Prove that the tangent drawn at P will be parallel to the chord AB.
Solution:



First join AP and BP.
As TPS is a tangent and PA is the chord of the circle.
$\angle BPT = \angle PAB$ [Angles in alternate segments]
But,
$\angle PBA = \angle PAB$ [Since PA = PB]
Thus, $\angle BPT = \angle PBA$
But these are alternate angles,
Hence, TPS || AB

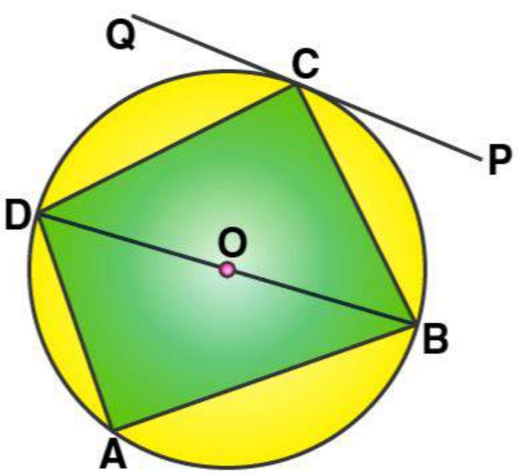
14. In the given figure, MN is the common chord of two intersecting circles and AB is their common tangent.
Prove that the line NM produced bisects AB at P.



Solution:

From P, AP is the tangent and PMN is the secant for first circle.
$AP^2 = PM \times PN$ …. (1)
Again from P, PB is the tangent and PMN is the secant for second circle.
$PB^2 = PM \times PN$ …. (2)
From (i) and (ii), we have
$AP^2 = PB^2$
AP = PB
Thus, P is the midpoint of AB.

15. In the given figure, ABCD is a cyclic quadrilateral, PQ is tangent to the circle at point C and BD is its diameter. If ∠DCQ = 40º and ∠ABD = 60º, find:
i) ∠DBC
ii) ∠BCP
iii) ∠ADB
Solution:

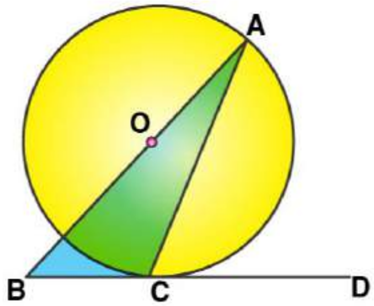


PQ is a tangent and CD is a chord.
$\angle DCQ = \angle DBC$ [Angles in the alternate segment]
$\angle DBC = 40^o$ [As $\angle DCQ = 40^o$]

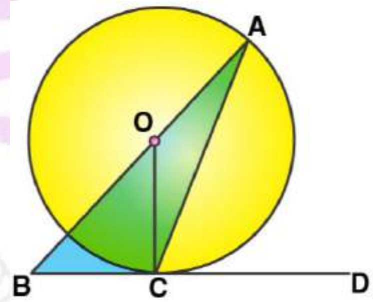
(ii) $\angle DCQ + \angle DCB + \angle BCP = 180^o$
$40^o + 90^o + \angle BCP = 180^o$ [As $\angle DCB = 90^o$]
$\angle BCP = 180^o - 130^o = 50^o$

(iii) In ΔABD,
$\angle BAD = 90^o$ [Angle in a semi-circle], $\angle ABD = 60^o$ [Given]
$\angle ADB = 180^o - (90^o + 60^o)$
$\angle ADB = 180^o - 150^o = 30^o$

16. The given figure shows a circle with centre O and BCD is a tangent to it at C. Show that: ∠ACD + ∠BAC = 90º

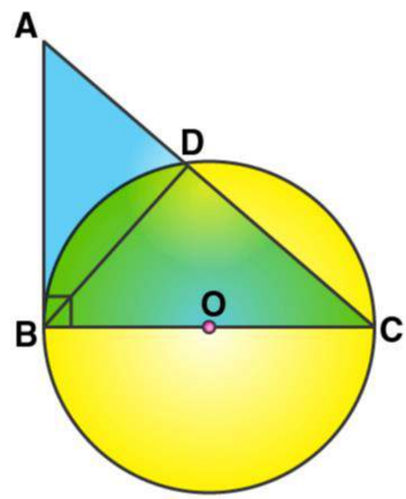


Solution:



Let's join OC.
BCD is the tangent and OC is the radius.
As, OC ⊥ BD
∠OCD = 90º
∠OCD + ∠ACD = 90º …. (i)
But, in ∆OCA
OA = OC [Radii of the same circle]
Thus, ∠OCA = ∠OAC
Substituting in (i), we get
∠OAC + ∠ACD = 90º
Hence, ∠BAC + ∠ACD = 90º

17. ABC is a right triangle with angle B = 90º. A circle with BC as diameter meets by hypotenuse AC at point D. Prove that:
i) AC x AD = AB^2
ii) BD^2 = AD x DC.
Solution:

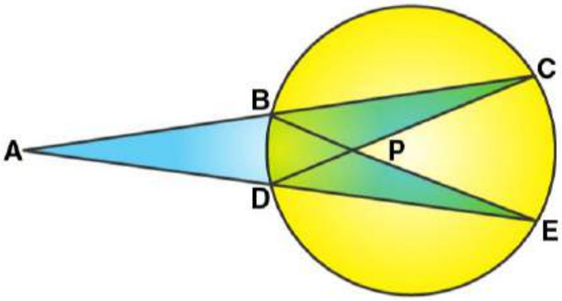


i) In ∆ABC, we have
∠B = 90º and BC is the diameter of the circle.
Hence, AB is the tangent to the circle at B.
Now, as AB is tangent and ADC is the secant we have
AB^2 = AD x AC

ii) In ∆ADB,
∠D = 90º
So, ∠A + ∠ABD = 90º …… (i)
But in ∆ABC, ∠B = 90º
∠A + ∠C = 90º ……. (ii)
From (i) and (ii),
∠C = ∠ABD
Now in ∆ABD and ∆CBD, we have
∠BDA = ∠BDC = 90º

∠ABD = ∠BCD
Hence, ΔABD ~ ΔCBD by AA postulate
So, we have
BD/DC = AD/BD
Therefore,
BD^2 = AD x DC

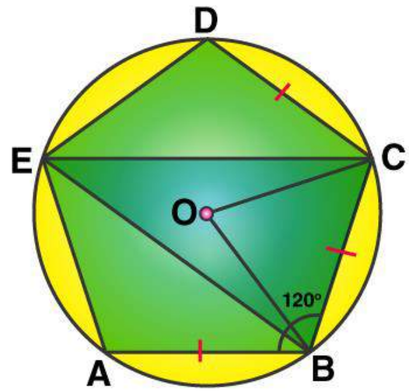
18. In the given figure, AC = AE.



Show that:
i) CP = EP
ii) BP = DP
Solution:

In ΔADC and ΔABE,
∠ACD = ∠AEB [Angles in the same segment]
AC = AE [Given]
∠A = ∠A [Common]
Hence, ΔADC ≅ ΔABE by ASA postulate
So, by C.P.C.T we have
AB = AD
But, AC = AE [Given]
So, AC – AB = AE – AD
BC = DE
In ΔBPC and ΔDPE,
∠C = ∠E [Angles in the same segment]
BC = DE
∠CBP = ∠CDE [Angles in the same segment]
Hence, ΔBPC ≅ ΔDPE by ASA postulate
So, by C.P.C.T we have
BP = DP and CP = PE

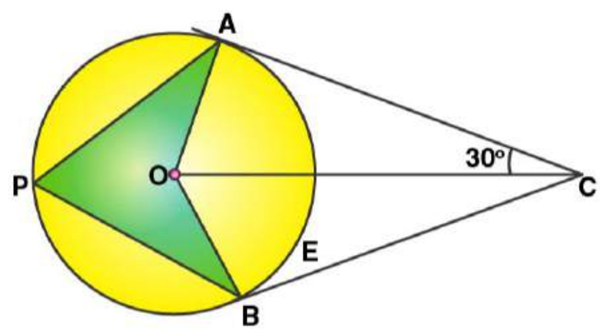
19. ABCDE is a cyclic pentagon with centre of its circumcircle at point O such that AB = BC = CD and angle ABC = 120^o
Calculate:
i) ∠BEC
ii) ∠BED
Solution:



i) Join OC and OB.
AB = BC = CD and ∠ABC = 120° [Given]
So, ∠BCD = ∠ABC = 120°
OB and OC are the bisectors of ∠ABC and ∠BCD and respectively.
So, ∠OBC = ∠BCO = 60°
In ΔBOC,
∠BOC = 180° – (∠OBC + ∠BOC)
∠BOC = 180° – (60° + 60°) = 180° – 120°
∠BOC = 60°
Arc BC subtends ∠BOC at the centre and ∠BEC at the remaining part of the circle.
∠BEC = ½ ∠BOC = ½ x 60° = 30°

ii) In cyclic quadrilateral BCDE, we have
∠BED + ∠BCD = 180°
∠BED + 120° = 180°
Thus, ∠BED = 60°

20. In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If angle ACO = 30°, find:
(i) angle BCO
(ii) angle AOB
(iii) angle APB
Solution:



In the given fig, O is the centre of the circle and, CA and CB are the tangents to the circle from C.
Also, ∠ACO = 30°
P is any point on the circle. P and B are joined.
To find:
(i) ∠BCO
(ii) ∠AOB
(iii) ∠APB
Proof:

(i) In ΔOAC and ΔOBC, we have
OC = OC [Common]
OA = OB [Radii of the same circle]
CA = CB [Tangents to the circle]
Hence, ΔOAC ≅ ΔOBC by SSS congruence criterion
Thus, ∠ACO = ∠BCO = 30°

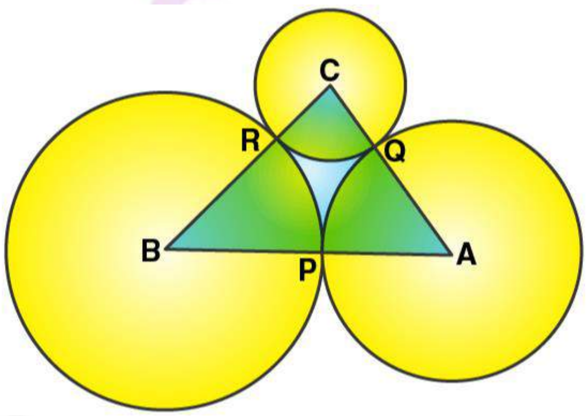
(ii) As ∠ACB = 30° + 30° = 60°

And, ∠AOB + ∠ACB = 180°
∠AOB + 60° = 180°
∠AOB = 180° – 60°
∠AOB = 120°

(iii) Arc AB subtends ∠AOB at the center and ∠APB is the remaining part of the circle.
∠APB = ½ ∠AOB = ½ x 120° = 60°

21. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centers. Find the radii of the three circles.

Solution:



Given: ABC is a triangle with AB = 10 cm, BC= 8 cm, AC = 6 cm. Three circles are drawn with centre A, B and C touch each other at P, Q and R respectively.
So, we need to find the radii of the three circles.
Let,
PA = AQ = x
QC = CR = y
RB = BP = z
So, we have
x + z = 10 ….. (i)
z + y = 8 …… (ii)
y + x = 6 ……. (iii)
Adding all the three equations, we have
2(x + y + z) = 24
x + y + z = 24/2 = 12 ….. (iv)
Subtracting (i), (ii) and (iii) from (iv) we get
y = 12 – 10 = 2
x = 12 – 8 = 4
z = 12 – 6 = 6
Thus, radii of the three circles are 2 cm, 4 cm and 6 cm.