Exercise 20(A) Page No: 297

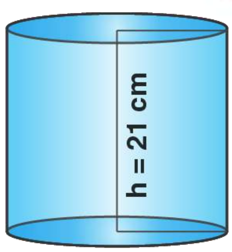
1. The height of a circular cylinder is 20 cm and the radius of its base is 7 cm. Find:
(i) the volume
(ii) the total surface area.
Solution:

Given, a circular cylinder whose
Height, h = 20 cm and base radius, r = 7 cm
(i) Volume of cylinder = $\pi r^2 h$ = 22/7 x 7^2 x 20 cm^3
= 3080 cm^3
(ii) Total surface area of a cylinder = $2\pi r$ (h + r)
= 2 x 22/7 x 7(20 + 7) cm^2
= 2 x 22 x 27 cm^3
= 1188 cm^3

2. The inner radius of a pipe is 2.1 cm. How much water can 12 m of this pipe hold?
Solution:

Given,
The inner radius of the pipe = 2.1 cm
Length of the pipe = 12 m = 1200 cm
Volume of the pipe = $\pi r^2 h$ = 22/7 x 2.1^2 x 1200
= 16632 cm^3

3. A cylinder of circumference 8 cm and length 21 cm rolls without sliding for 4½ seconds at the rate of 9 complete rounds per second. Find:
(i) distance travelled by the cylinder in 4½ seconds, and
(ii) the area covered by the cylinder in 4½ seconds
Solution:



Given,
Base circumference of cylinder (c) = 8 cm
So, the radius = $c/2\pi$ = (8 x 7)/ (2 x 22) = 14/11 cm
Length of the cylinder (h) = 21 cm

(i) If distance covered in one revolution is 8 cm, then distance covered in 9 revolutions = 9 x 8 = 72 cm
or distance covered in 1 second = 72 cm.
Thus, distance covered in 4½ seconds = 72 x (9/2) = 324 cm

(ii) Curved surface area = $2\pi rh$
= 2 x 22/7 x 14/11 x 21
= 168 cm^2
So, the area covered in one revolution = 168 cm^2
Then,
The area covered in 9 revolutions = 168 x 9 = 1512 cm^2
Which is also the area covered in 1 second = 1512 cm^2
Therefore, the area covered in 4½ seconds = 1512 x 9/2 = 6804 cm^2

4. How many cubic meters of earth must be dug out to make a well 28 m deep and 2.8 m in diameter? Also, find the cost of plastering its inner surface at Rs 4.50 per sq meter.
Solution:

Given,
Radius of the well = 2.8/2 = 1.4 m
Depth of the well = 28 m
Hence, the volume of earth dug out = $\pi r^2 h$
= (22/7) x 1.4 x 1.4 x 28
= 17248/100 = 172.48 m^3
Area of curved surface = $2\pi rh$
= 2 x 22/7 x 1.4 x 28
= 246.40 m^2
Now, the cost of plastering at the rate of Rs 4.50 per sq m
= Rs 246.40 x 4.50
= Rs 1108.80

5. What length of solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of external diameter 20 cm, 0.25 cm thick and 15 cm long?
Solution:

Given,
External diameter of hollow cylinder = 20 cm
So, it's radius = 10 cm = R
Thickness = 0.25 cm
Hence, the internal radius = (10 - 0.25) = 9.75 cm = r
Length of cylinder (h) = 15 cm
Now,
Volume = $\pi h (R^2 - r^2) = \pi \times 15(10^2 – 9.75^2) = 15\pi (100 – 95.0625)$
= $15\pi \times 4.9375$ cm^3
Now,
Diameter of the solid cylinder = 2 cm
so, radius (r) = 1 cm
Let h be the length of the solid cylinder,
Volume = $\pi r^2 h = \pi 1^2 h = \pi h$ cm^3
Then, according to given condition in the question

$\pi h = 15\ \pi \times 4.9375$
$h = 15 \times 4.9375$
$h = 74.0625$
Thus, the length of solid cylinder = 74.0625 cm

6. A cylinder has a diameter of 20 cm. The area of curved surface is 100 sq cm. Find:
(i) the height of the cylinder correct to one decimal place.
(ii) the volume of the cylinder correct to one decimal place.
Solution:

Given,
The diameter of the cylinder = 20 cm
So, the radius (r) = 10 cm
And the curved surface area = 100 cm^2
Height = h cm
(i) Curved surface area = $2\pi rh$
So,
$2\pi rh = 100\ cm^2$
$2 \times 22/7 \times 10 \times h = 100$
$h = (100 \times 7)/ (22 \times 10 \times 2) = 35/22$
h = 1.6 cm

(ii) Volume of the cylinder = $\pi r^2 h$
= $22/7 \times 10 \times 10 \times 1.6$
= 502.9 cm^3

7. A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the weight, in kilogram, of 2 metres of the pipe if 1 cm^3 of the metal weights 7.7 g.
Solution:

Given,
Inner radius of the pipe = r = 5/2 = 2.5 cm
External radius of the pipe = R = Inner radius of the pipe + Thickness of the pipe
= 2.5 cm + 0.5 cm = 3 cm
Length of the pipe = h = 2m = 200 cm
Volume of the pipe = External Volume – Internal Volume
= $\pi R^2 h - \pi r^2 h$
= $\pi (R^2 - r^2) h$
= $\pi\ (R + r)(R - r)\ h$
= $22/7\ (3 – 2.5)(3 + 2.5) \times 200$
= $22/7\ (0.5)\ (5.5) \times 200 = 1728.6\ cm^3$
Since, 1 cm^3 of the metal weight 7.7g,
Hence, weight of the pipe = (1728.6 x 7.7)g = (1728.6 x 7.7)/ 1000 kg = 13.31 kg

8. A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm x 14 cm x 10.5 cm. Find the rise in level of the

water when the solid is submerged.
Solution:

Given,
Diameter of cylindrical container = 42 cm
So, it's radius (r) = 21 cm
Dimensions of rectangular solid = 22cm x 14cm x 10.5cm
Hence,
The volume of solid = 22 x 14 x 10.5 cm^3 ….. (i)
Let the height of water = h
So, the volume of water in the container will be = $\pi r^2 h$
= 22/7 x 21 x 21 x h cm^3 …… (ii)
According to the question, from (i) and (ii) we have
22/7 x 21 x 21 x h = 22 x 14 x 10.5
22 x 3 x 21 x h = 22 x 14 x 10.5
h = (22 x 14 x 10.5)/ (22 x 3 x 21)
= 7/3 = 2.33 cm
Therefore, the water level will be raised to a level of 2.33 cm when the solid is submerged.

9. A cylindrical container with internal radius of its base 10 cm, contains water up to a height of 7 cm. Find the area of wetted surface of the cylinder.
Solution:

Given,
Internal radius of the cylindrical container = 10 cm = r
Height of water = 7 cm = h
So, the surface area of the wetted surface = $2\pi rh + \pi r^2 = \pi r(2h + r)$ = 22/7 x 10 x (2 x 7 + 10)
= 220/7 x 24
= 754.29 cm^2

10. Find the total surface area of an open pipe of length 50 cm, external diameter 20 cm and internal diameter 6 cm.
Solution:

Given,
Length of the open pipe = 50 cm
Its external diameter = 20 cm
So, it's external radius (R) = 10 cm
And, its internal diameter = 6 cm
So, it's internal radius (r) = 3 cm
Then.
Surface area of pipe open from both sides = $2\pi Rh + 2\pi rh = 2\pi h(R + r)$
= 2 x 22/7 x 50 x (10 + 3)
= 4085.71 cm^2
Area of upper and lower part = $2\pi(R^2 – r^2)$
= 2 x 22/7 x $(10^2 - 3^2)$

= 2 x 22/7 x 91
= 572 cm^2
Hence, the total surface area = 4085.71 + 572 = 4657.71 cm^3

11. The height and the radius of the base of a cylinder are in the ratio 3:1. If its volume is 1029π cm^3; find its total surface area.
Solution:
Given,
The ratio between height and radius of a cylinder = 3:1
Volume = 1029π cm^3 …….(i)
Let the radius of the base = r
Then, it's height will be = 3r
Hence,
Volume = $\pi r^2 h = \pi \times r^2 \times 3r = 3\pi r^3$ …… (ii)
Equating (i) and (ii), we get
$3\pi r^3 = 1029\pi$
$r^3 = 1029\pi / 3\pi$
$r^3 = 343$
r = 7
Thus, radius = 7 cm and height = 3 x 7 = 21 cm
Now,
Total surface area = 2πr(h + r) = 2 x 22/7 x 7 x (21 + 7)
= 2 x 22/7 x 7 x 28
= 44 x 28
= 1232 cm^2

12. The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume.
Solution:

Let the radius of a solid right cylinder (r) = 100 cm
And, let the height of a solid right circular cylinder (h) = 100 cm
So, the volume (original) of a solid right circular cylinder = $\pi r^2 h$
= $\pi \times (100)^2 \times 100$
= 10000 π cm^3
Now, the new radius = r' = 120 cm
New height = h' = 80 cm
So, the volume (new) of a solid right circular cylinder = $\pi r'^2 h' = \pi \times (120)^2 \times 80$
= 1152000 π cm^3
Thus, the increase in volume = New volume – Original volume
= 1152000 π cm^3 – 1000000 π cm^3
= 152000 π cm^3
Therefore, percentage change in volume = Increase in volume/Original volume x 100%
= 152000 π cm^3/ 1000000 π cm^3 x 100%
= 15.2 %

13. The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%. Find the percentage change in its:
(i) volume (ii) curved surface area
Solution:

Let the original dimensions of the solid right cylinder be radius (r) and height (h) in cm.
Then its volume = $\pi r^2 h$ cm^3
And, curved surface area = $2\pi rh$
Now, after the changes the new dimensions are:
New radius (r') = r – 0.2r = 0.8r and
New height (h') = h + 0.1h = 1.1h
So,
The new volume = $\pi r'^2 h'$ cm^3
= $\pi(0.8r)^2(1.1h)$ cm^3
= $0.704\ \pi r^2 h$ cm^3
And, the new curved surface area = $2\pi r'h' = 2\pi(0.8r)(1.1h)$
= $(0.88)\ 2\pi rh$
(i) Percentage change in its volume = Decrease in volume/ original volume x 100 %
= (Original volume – new volume)/ original volume x 100 %
= $(\pi r^2 h - 0.704\ \pi r^2 h)/\ \pi r^2 h \times 100$
= 0.296 x 100 = 29.6 %

(ii) Percentage change in its curved surface area = Decrease in CSA/ original CSA x 100 %
= (Original CSA – new CSA)/ original CSA x 100 %
= $(2\pi rh - (0.88)\ 2\pi rh)/\ 2\pi rh \times 100$
= 0.12 x 100 = 12 %

14. Find the minimum length in cm and correct to nearest whole number of the thin metal sheet required to make a hollow and closed cylindrical box of diameter 20 cm and height 35 cm. Given that the width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of Rs. 56 per m. Find the area of metal sheet required, if 10% of it is wasted in cutting, overlapping, etc.
Solution:

Given,
Height of the cylinder box = h = 35 cm
Base radius of the cylinder box = r = 10 cm
Width of metal sheet = 1m = 100 cm
Area of metal sheet required = total surface area of the box
Length x width = $2\pi r(r + h)$
Length x 100 = 2 x 22/7 x 10(10 + 35)
Length x 100 = 22 x 22/7 x 10 x 45
Length = (2 x 22 x 10 x 45)/ (100 x 7) = 28.28 cm = 28 cm (correcting to the nearest whole number)
Thus,
Area of metal sheet = length x width = 28 x 100 = 2800 cm^2 = 0.28 m^2
So, the cost of the sheet at the rate of Rs 56 per m^2 = Rs (56 x 0.28) = Rs 15.68
Let the total sheet required be x.

Then, $x - 10\%$ of $x = 2800\ cm^2$
$x - 10/100 \times x = 2800\ cm^2$
$9x = 2800$
$x = 3111\ cm^2$
Therefore, a metal sheet of area $3111\ cm^2$ is required.