Exercise 20(D) Page No: 308

1. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.
Solution:

Given,
Radius of the sphere = R = 15 cm
So, the volume of sphere melted = 4/3 πR^2 = 4/3 x π x 15 x 15 x 15
Radius of each cone recasted = r = 2.5 cm
And, height of each cone recasted = h = 8 cm
So, volume of each cone recasted = 1/3 $\pi r^2 h$ = 1/3 x π x 2.5 x 2.5 x 8
Thus,
Number of cones recasted = Volume of sphere melted/ Volume of each cone formed

$$= \frac{\frac{4}{3} \times \pi \times 15 \times 15 \times 15}{\frac{1}{3} \times \pi \times 2.5 \times 2.5 \times 8}$$

= 270

2. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.
Solution:

Given,
External diameter of the hollow sphere = 8 cm
So, radius (R) = 4 cm
Internal diameter of the hollow sphere = 4 cm
So, radius (r) = 2 cm
Then, the volume of metal used in hollow sphere

$$= \frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3} \times \frac{22}{7} \times (4^3 - 2^3) = \frac{88}{21}(64 - 8) = \frac{88}{21} \times 56 \text{ cm}^3 \ldots\ldots (i)$$

Also given,
Diameter of cone = 8 cm
Therefore, radius = 4 cm
Let height of the cone = h
Volume = 1//3 $\pi r^2 h$ = 1/3 x 22/7 x 4 x 4 x h = 352/21 h …. (ii)
So, according to the question
From (i) and (ii), we have

$$\frac{352}{21} h = \frac{88}{21} \times 56$$

h = (88 x 56 x 21)/ (21 x 352) = 14 cm
Thus, the height of the cone = 14 cm

3. The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cone of height 32 cm. find the

diameter of the base of the cone.
Solution:

Given,
Height of the solid right circular cone = 32 cm
Internal radius metallic spherical shell = 3 cm
External radius spherical shell = 5 cm
Then, the volume of the spherical shell = $4/3\ \pi\ (5^3 - 3^3)$
$= 4/3 \times 22/7 \times (125 - 27)$
$= 4/3 \times 22/7 \times 98$
Volume of solid right circular cone = $1/3\ \pi r^2 h$
$= 1/3 \times 22/7 \times r^2 \times 32$
According to the question,
$1/3 \times 22/7 \times r^2 \times 32 = 4/3 \times 22/7 \times 98$
$r^2 = (4 \times 98)/ 32$
$r = 7/2 = 3.5$ cm
Therefore, diameter = 2r = 7 cm

4. Total volume of three identical cones is the same as that of a bigger cone whose height is 9 cm and diameter 40 cm. Find the radius of the base of each smaller cone, if height of each is 108 cm.
Solution:

Let the radius of the smaller cone be r cm
Given,
Diameter of bigger cone = 40 cm
So, the radius = 20 cm and height = 9 cm
Volume of larger cone = $1/3\ \pi \times (20)^2 \times 9$
And,
The volume of the smaller cone = $1/3\ \pi \times r^2 \times 108$
From the question, we have
Volume of larger cone = 3 x Volume of smaller cone
$1/3\ \pi \times (20)^2 \times 9 = 3 \times (1/3\ \pi \times r^2 \times 108)$
$r^2 = (20)^2 \times 9/ (108 \times 3)$
$r = 20/6 = 10/3$ cm

5. A solid rectangular block of metal 49 cm by 44 cm by 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.
Solution:

Volume of rectangular block = $49 \times 44 \times 18\ cm^3 = 38808\ cm^3$ …… (i)
Let r be the radius of sphere.
So, the volume = $4/3\ \pi r^3 = 4/3 \times 22/7 \times r^3 = 88/21\ r^3$ ….. (ii)
Then, according to the question
$88/21\ r^3 = 38808$
$r^3 = (38808 \times 21)/ 88 = 441 \times 21$
$= 9261$

r = 21 cm
Therefore, the radius of sphere = 21 cm

6. A hemi-spherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into conical shaped small containers each of diameter 3 cm and height 4 cm. How many containers are necessary to empty the bowl?
Solution:

Given,
Radius of hemispherical bowl = 9 cm
Volume = $2/3\ \pi r^3 = 2/3\ \pi 9^3 = 2/3\ \pi \times 729 = 486\ \pi\ cm^2$
Diameter of each cylindrical bottle = 3 cm
So, the radius = 1.5 cm and height = 4 cm
Volume of bottle = $1/3\ \pi r^2 h = 1/3\ \pi(3/2)^2 \times 4 = 3\ \pi$
Thus,
The number of bottles = $486\ \pi/ 3\ \pi$
= 162