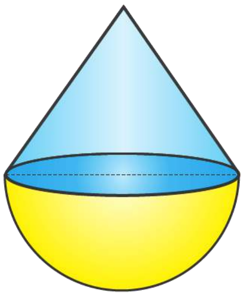
Exercise 20(E) Page No: 311

1. A cone of height 15 cm and diameter 7 cm is mounted on a hemisphere of same diameter. Determine the volume of the solid thus formed.

Solution:



Given,
Height of the cone = 15 cm
Diameter of the cone = 7 cm
So, its radius = 3.5 cm
Radius of the hemisphere = 3.5 cm
Now,
Volume of the solid = Volume of the cone + Volume of the hemisphere
$= 1/3\ \pi r^2 h + 2/3\ \pi r^3$
$= 1/3 \times 22/7 \times (3.5)^2 \times 15 + 2/3 \times 22/7 \times (3.5)^3$
$= 1/3 \times 22/7 \times (3.5)^2 (15 + 2 \times 3.5)$
$= 847/3$
$= 282.33\ cm^3$

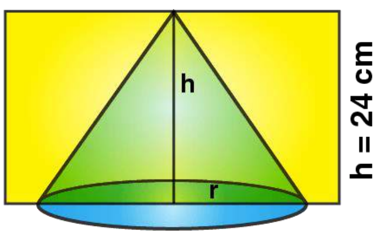
2. A buoy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of the hemisphere. The radius of the base of the cone is 3.5 m and its volume is two-third of the hemisphere. Calculate the height of the cone and the surface area of the buoy, correct to two decimal places.

Solution:

Given,
Radius of the hemisphere part (r) = 3.5 m = 7/2 m
Volume of hemisphere = $2/3\ \pi r^3$
$= 2/3 \times 22/7 \times 7/2 \times 7/2 \times 7/2$
$= 539/6\ m^3$
Volume of conical part = $2/3 \times 539/6\ m^3$ [2/3 rd of the hemisphere]
Let height of the cone = h
Then,
$1/3\ \pi r^2 h = (2 \times 539)/ (3 \times 6)$
$1/3 \times 22/7 \times 7/2 \times 7/2 \times h = (2 \times 539)/ (3 \times 6)$
$h = (2 \times 539 \times 2 \times 2 \times 7 \times 3)/ (3 \times 6 \times 22 \times 7 \times 7)$
h = 14/3 m = 4.67 m
Height of the cone = 4.67 m
Surface area of buoy = $2\ \pi r^2 + \pi rl$

But, we know that
$l = (r^2 + h^2)^{1/2}$
$l = ((7/2)^2 + (14/3)^2)^{1/2}$
$l = (1225/36)^{1/2} = 35/6$ m
Hence,
Surface area = $2\pi r^2 + \pi rl$
= (2 x 22/7 x 7/2 x 7/2) + (22/7 x 7/2 x 35/6)
= 77 + 385/6 = 847/6
= 141.17 m^2 [Corrected to 2 decimal places]

3. From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a conical cavity of diameter 14 cm and depth 24 cm is drilled out. Find:
(i) the surface area of the remaining solid
(ii)the volume of remaining solid
(iii) the weight of the material drilled out if it weighs 7 gm per cm^3.
Solution:



Given,
Dimensions of rectangular solid l = 42 cm, b = 30 cm and h = 20 cm
Conical cavity's diameter = 14 cm
So, its radius = 7 cm
Depth (height) = 24 cm

(i) Total surface area of cuboid = 2 (lb + bh + lh)
= 2 (42 x 30 + 30 x 20 + 20 x 42)
= 2 (1260 + 600 + 840)
= 2 (2700)
= 5400 cm^2
Diameter of the cone = 14 cm
So, its radius = 14/2 = 7 cm
Area of circular base = πr^2 = 22/7 x 7 x 7 = 154 cm^2
We know that,
$l = (7^2 + 24^2)^{1/2} = (49 + 576)^{1/2} = (625)^{1/2} = 25$
Area of curved surface area of cone = πrl = 22/7 x 7 x 25 = 22 x 25 = 550 cm^2
Surface area of remaining part = 5400 + 550 – 154 = 5796 cm^2

(ii) Volume of the rectangular solid = (42 x 30 x 20) cm^3 = 25200 cm^3
Radius of conical cavity (r) = 7 cm
Height (h) = 24 cm

Volume of cone = $1/3\ \pi r^2 h$
= 1/3 x 22/7 x 7 x 7 x 24
= 1232 cm^3

(iii) Weight of material drilled out = 1232 x 7g = 8624g = 8.624 kg

4. The cubical block of side 7 cm is surmounted by a hemisphere of the largest size. Find the surface area of the resulting solid.
Solution:

It's known that, the diameter of the largest hemisphere that can be placed on a face of a cube of side 7 cm will be 7 cm.
So, it's radius = r = 7/2 cm
It's curved surface area = $2\ \pi r^2$ = 2 x 22/7 x 7/2 x 7/2 = 77 cm^2 …… (i)
Now,
Surface area of the top of the resulting solid = Surface area of the top face of the cube - Area of the base of the hemisphere

$= (7 \times 7) - (\frac{22}{7} \times \frac{49}{4})$

$= 49 - \frac{77}{2}$

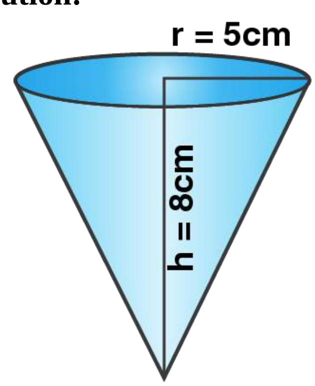
$= \frac{98-77}{2}$

$= \frac{21}{2}$

$= 10.5\ cm^2$…….(ii)

Surface area of the cube = 5 x $(\text{side})^2$ = 5 x 49 = 245 cm^2 …… (iii)
Hence,
Total area of resulting solid = (i) + (ii) + (iii) = 245 + 10.5 + 77 = 332.5 cm^2

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of the top which is open is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.
Solution:



Given,
Height of cone = 8 cm
Radius = 5 cm
Volume = $1/3\ \pi r^2 h$
$= 1/3 \times 22/7 \times 5 \times 5 \times 8\ cm^3$
$= 4400/21\ cm^3$
Hence, the volume of water that flowed out = $¼ \times 4400/21\ cm^3$
$= 1100/21\ cm^3$

Also, given
Radius of each ball = 0.5 cm = ½ cm
Volume of a ball = $4/3\ \pi r^3$
$= 4/3 \times 22/7 \times ½ \times ½ \times ½\ cm^3$
$= 11/21\ cm^3$
Thus, the no. of balls = 1100/21 = 11/21 = 100