Exercise 20(A)   Page No: 297

1. The height of a circular cylinder is 20 cm and the radius of its base is 7 cm. Find:
(i) the volume
(ii) the total surface area.
Solution:

Given, a circular cylinder whose
Height, h = 20 cm and base radius, r = 7 cm
(i) Volume of cylinder = $\pi r^2 h$ = 22/7 x 7^2 x 20 cm^3
        = 3080 cm^3
(ii) Total surface area of a cylinder = $2\pi r$ (h + r)
        = 2 x 22/7 x 7(20 + 7) cm^2
        = 2 x 22 x 27 cm^3
        = 1188 cm^3

2. The inner radius of a pipe is 2.1 cm. How much water can 12 m of this pipe hold?
Solution:

Given,
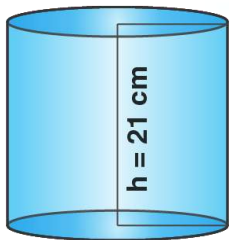The inner radius of the pipe = 2.1 cm
Length of the pipe = 12 m = 1200 cm
Volume of the pipe = $\pi r^2 h$ = 22/7 x 2.1^2 x 1200
        = 16632 cm^3

3. A cylinder of circumference 8 cm and length 21 cm rolls without sliding for 4½ seconds at the
rate of 9 complete rounds per second. Find:
(i) distance travelled by the cylinder in 4½ seconds, and
(ii) the area covered by the cylinder in 4½ seconds
Solution:

Given,
Base circumference of cylinder (c) = 8 cm
So, the radius = c/2π = (8 x 7)/ (2 x 22) = 14/11 cm
Length of the cylinder (h) = 21 cm

(i) If distance covered in one revolution is 8 cm, then distance covered in 9 revolutions = 9 x 8 = 72 cm
or distance covered in 1 second = 72 cm.
Thus, distance covered in 4½ seconds = 72 x (9/2) = 324 cm

(ii) Curved surface area = 2πrh

         = 2 x 22/7 x 14/11 x 21

         = 168 cm^2

So, the area covered in one revolution = 168 cm^2

Then,

The area covered in 9 revolutions = 168 x 9 = 1512 cm^2

Which is also the area covered in 1 second = 1512 cm^2

Therefore, the area covered in 4½ seconds = 1512 x 9/2 = 6804 cm^2

4. How many cubic meters of earth must be dug out to make a well 28 m deep and 2.8 m in diameter? Also, find the cost of plastering its inner surface at Rs 4.50 per sq meter.
Solution:

Given,

Radius of the well = 2.8/2 = 1.4 m

Depth of the well = 28 m

Hence, the volume of earth dug out = πr^{2}h

              = (22/7) x 1.4 x 1.4 x 28

              = 17248/100 = 172.48 m^3

Area of curved surface = 2πrh

          = 2 x 22/7 x 1.4 x 28

          = 246.40 m^2

Now, the cost of plastering at the rate of Rs 4.50 per sq m

= Rs 246.40 x 4.50

= Rs 1108.80

5. What length of solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of external diameter 20 cm, 0.25 cm thick and 15 cm long?
Solution:

Given,

External diameter of hollow cylinder = 20 cm

So, it's radius = 10 cm = R

Thickness = 0.25 cm

Hence, the internal radius = (10 - 0.25) = 9.75 cm = r

Length of cylinder (h) = 15 cm

Now,

Volume = πh (R^2 - r^2) = π x 15(10^2 – 9.75^2) = 15 π (100 – 95.0625)

     = 15 π x 4.9375 cm^3

Now,

Diameter of the solid cylinder = 2 cm

so, radius (r) = 1 cm

Let h be the length of the solid cylinder,

Volume = πr^{2}h = π1^{2}h = πh cm^3

Then, according to given condition in the question

πh = 15 π x 4.9375
h = 15 x 4.9375
h = 74.0625
Thus, the length of solid cylinder = 74.0625 cm

6. A cylinder has a diameter of 20 cm. The area of curved surface is 100 sq cm. Find:
(i) the height of the cylinder correct to one decimal place.
(ii) the volume of the cylinder correct to one decimal place.
Solution:

Given,
The diameter of the cylinder = 20 cm
So, the radius (r) = 10 cm
And the curved surface area = 100 cm^2
Height = h cm
(i) Curved surface area = 2πrh
So,
2πrh = 100 cm^2
2 x 22/7 x 10 x h = 100
h = (100 x 7)/ (22 x 10 x 2) = 35/22
h = 1.6 cm

(ii) Volume of the cylinder = πr^{2}h
    = 22/7 x 10 x 10 x 1.6
    = 502.9 cm^3

7. A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the
weight, in kilogram, of 2 metres of the pipe if 1 cm^3 of the metal weights 7.7 g.
Solution:

Given,
Inner radius of the pipe = r = 5/2 = 2.5 cm
External radius of the pipe = R = Inner radius of the pipe + Thickness of the pipe
                = 2.5 cm + 0.5 cm = 3 cm
Length of the pipe = h = 2m = 200 cm
Volume of the pipe = External Volume – Internal Volume
        = πR^{2}h - πr^{2}h
        = π(R^2 - r^2)h
        = π (R + r)(R - r) h
        = 22/7 (3 – 2.5)(3 + 2.5) x 200
        = 22/7 (0.5) (5.5) x 200 = 1728.6 cm^3
Since, 1 cm^3 of the metal weight 7.7g,
Hence, weight of the pipe = (1728.6 x 7.7)g = (1728.6 x 7.7)/ 1000 kg = 13.31 kg

8. A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a
rectangular solid of iron with dimensions 22 cm x 14 cm x 10.5 cm. Find the rise in level of the

water when the solid is submerged.
Solution:

Given,
Diameter of cylindrical container = 42 cm
So, it's radius (r) = 21 cm
Dimensions of rectangular solid = 22cm x 14cm x 10.5cm
Hence,
The volume of solid = 22 x 14 x 10.5 cm^3 ….. (i)
Let the height of water = h
So, the volume of water in the container will be = $\pi r^2 h$
$= 22/7$ x 21 x 21 x h cm^3 …… (ii)
According to the question, from (i) and (ii) we have
22/7 x 21 x 21 x h = 22 x 14 x 10.5
22 x 3 x 21 x h = 22 x 14 x 10.5
h = (22 x 14 x 10.5)/ (22 x 3 x 21)
= 7/3 = 2.33 cm
Therefore, the water level will be raised to a level of 2.33 cm when the solid is submerged.

9. A cylindrical container with internal radius of its base 10 cm, contains water up to a height of 7 cm. Find the area of wetted surface of the cylinder.
Solution:

Given,
Internal radius of the cylindrical container = 10 cm = r
Height of water = 7 cm = h
So, the surface area of the wetted surface = $2\pi rh + \pi r^2 = \pi r(2h + r) = 22/7$ x 10 x (2 x 7 + 10)
$= 220/7$ x 24
$= 754.29$ cm^2

10. Find the total surface area of an open pipe of length 50 cm, external diameter 20 cm and internal diameter 6 cm.
Solution:

Given,
Length of the open pipe = 50 cm
Its external diameter = 20 cm
So, it's external radius (R) = 10 cm
And, its internal diameter = 6 cm
So, it's internal radius (r) = 3 cm
Then.
Surface area of pipe open from both sides = $2\pi Rh + 2\pi rh = 2\pi h(R + r)$
$= 2$ x 22/7 x 50 x (10 + 3)
$= 4085.71$ cm^2
Area of upper and lower part = $2\pi(R^2 – r^2)$
$= 2$ x 22/7 x (10^2 - 3^2)

= 2 x 22/7 x 91

= 572 cm²

Hence, the total surface area = 4085.71 + 572 = 4657.71 cm³

11. The height and the radius of the base of a cylinder are in the ratio 3:1. If its volume is 1029π cm³; find its total surface area.

Solution:

Given,

The ratio between height and radius of a cylinder = 3:1

Volume = 1029π cm³ …….(i)

Let the radius of the base = r

Then, it's height will be = 3r

Hence,

Volume = πr²h = π x r² x 3r = 3πr³ …… (ii)

Equating (i) and (ii), we get

3πr³ = 1029π

r³ = 1029π/ 3π

r³ = 343

r = 7

Thus, radius = 7 cm and height = 3 x 7 = 21 cm

Now,

Total surface area = 2πr(h + r) = 2 x 22/7 x 7 x (21 + 7)

= 2 x 22/7 x 7 x 28

= 44 x 28

= 1232 cm²

12. The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume.

Solution:

Let the radius of a solid right cylinder (r) = 100 cm

And, let the height of a solid right circular cylinder (h) = 100 cm

So, the volume (original) of a solid right circular cylinder = πr²h

= π x (100)² x 100

= 10000 π cm³

Now, the new radius = r' = 120 cm

New height = h' = 80 cm

So, the volume (new) of a solid right circular cylinder = πr'²h' = π x (120)² x 80

= 1152000 π cm³

Thus, the increase in volume = New volume – Original volume

= 1152000 π cm³ – 1000000 π cm³

= 152000 π cm³

Therefore, percentage change in volume = Increase in volume/Original volume x 100%

= 152000 π cm³/ 1000000 π cm³ x 100%

= 15.2 %

13. The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%.
Find the percentage change in its:
(i) volume (ii) curved surface area
Solution:

Let the original dimensions of the solid right cylinder be radius (r) and height (h) in cm.
Then its volume = $\pi r^2 h$ cm^3
And, curved surface area = $2\pi rh$
Now, after the changes the new dimensions are:
New radius (r') = r – 0.2r = 0.8r and
New height (h') = h + 0.1h = 1.1h
So,
The new volume = $\pi r'^2 h'$ cm^3
                 = $\pi(0.8r)^2(1.1h)$ cm^3
                 = 0.704 $\pi r^2 h$ cm^3
And, the new curved surface area = $2\pi r'h' = 2\pi(0.8r)(1.1h)$
                                 = (0.88) $2\pi rh$
(i) Percentage change in its volume = Decrease in volume/ original volume x 100 %
                 = (Original volume – new volume)/ original volume x 100 %
                 = ($\pi r^2 h$ - 0.704 $\pi r^2 h$)/ $\pi r^2 h$ x 100
                 = 0.296 x 100 = 29.6 %

(ii) Percentage change in its curved surface area = Decrease in CSA/ original CSA x 100 %
    = (Original CSA – new CSA)/ original CSA x 100 %
    = ($2\pi rh$ - (0.88) $2\pi rh$)/ $2\pi rh$ x 100
    = 0.12 x 100 = 12 %

14. Find the minimum length in cm and correct to nearest whole number of the thin metal sheet
required to make a hollow and closed cylindrical box of diameter 20 cm and height 35 cm. Given
that the width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of Rs. 56 per m.
Find the area of metal sheet required, if 10% of it is wasted in cutting, overlapping, etc.
Solution:

Given,
Height of the cylinder box = h = 35 cm
Base radius of the cylinder box = r = 10 cm
Width of metal sheet = 1m = 100 cm
Area of metal sheet required = total surface area of the box
Length x width = $2\pi r(r + h)$
Length x 100 = 2 x 22/7 x 10(10 + 35)
Length x 100 = 22 x 22/7 x 10 x 45
Length = (2 x 22 x 10 x 45)/ (100 x 7) = 28.28 cm = 28 cm (correcting to the nearest whole number)
Thus,
Area of metal sheet = length x width = 28 x 100 = 2800 cm^2 = 0.28 m^2
So, the cost of the sheet at the rate of Rs 56 per m^2 = Rs (56 x 0.28) = Rs 15.68
Let the total sheet required be x.

Then, x – 10% of x = 2800 cm^2

x - 10/100 × x = 2800 cm^2

9x = 2800

x = 3111 cm^2

Therefore, a metal sheet of area 3111 cm^2 is required.

Exercise 20(B)   Page No: 303

1. Find the volume of a cone whose slant height is 17 cm and radius of base is 8 cm.
Solution:

Given,
Slant height of the cone (l) = 17 cm
Base radius (r) = 8 cm
We know that,
$l^2 = r^2 + h^2$
$h^2 = l^2 - r^2 = 17^2 - 8^2 = 289 - 64 = 225$
h = 15 cm
Now, the volume of cone = $1/3$ $\pi r^2 h$ = $1/3$ x 22/7 x 8 x 8 x 15 cm^3
                                             = 7040/7 cm^3
                                             = 1005.71 cm^3

2. The curved surface area of a cone is 12320 cm^2. If the radius of its base is 56 cm, find its height.
Solution:

Given,
Curved surface area of the cone = 12320 cm^2
Radius of the base = 56 cm
Let the slant height be 'l'
Then,
Curved surface area = $\pi r l$ = 12320 cm^2
22/7 x 56 x l = 12320 cm^2
l = (12320 x 7)/ (22 x 56)
l = 70 cm
We know that,
$l^2 = h^2 + r^2$
$70^2 = h^2 + 56^2$
$h^2 = 4900 - 3136 = 1764$
Thus, h = 42 cm

3. The circumference of the base of a 12 m high conical tent is 66 m. Find the volume of the air contained in it.
Solution:

Given,
Circumference of the base (c) = 66 m
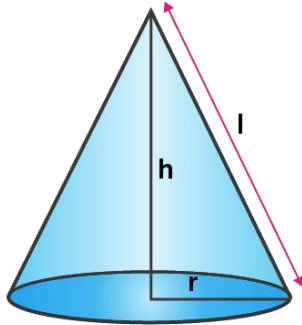Height of the conical tent (h) = 12 m
Radius = c/2π = 66/ 2π = (33 x 7)/22 = 21/2 = 10.5 m
Thus, the volume of the cone tent = $1/3$ $\pi r^2 h$
                                             = $1/3$ x 22/7 x (21/2)^2 x 12
                                             = 1386 m^3
Therefore, the volume of air contained is 1386 m^3.

4. The radius and height of a right circular cone are in the ratio 5:12 and its volume is 2512 cubic cm. Find the radius and slant height of the cone. (Take π = 3.14)
Solution:

Given,
The ratio between radius and height = 5: 12
Volume of the right circular cone = 2512 cm^3
Let its radius (r) = 5x, its height (h) = 12x and slant height = l
We know that,
$l^2 = r^2 + h^2 = (5x)^2 + (12x)^2 = 25x^2 + 144x^2$
$l^2 = 169x^2$
l = 13x
Now, the volume = 1/3 πr^2 h
1/3 πr^2 h = 2512
1/3 x (3.14) x (5x)^2 x (12x) = 2512
1/3 x (3.14) x (300x^3) = 2512
x^3 = (2512 x 3)/ (3.14 x 300) = 8
x = 2
Thus,
Radius = 5x = 5 x 2 = 10 cm
Height = 12x = 12 x 2 = 24 cm
Slant height = 13x = 13 x 2 = 26 cm

5. Two right circular cones x and y are made, x having three times the radius of y and y having half the volume of x. Calculate the ratio between the heights of x and y.
Solution:
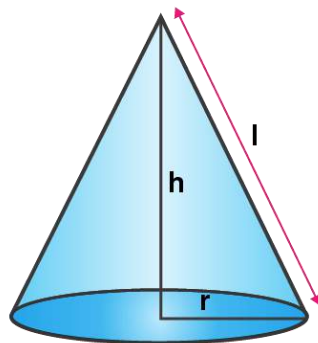
From the question,
Let radius of cone y = r
So, radius of cone x = 3r
Let volume of cone y = V
Then, volume of cone x = 2V
Let h_1 be the height of x and h_2 be the height of y.
Now,
Volume of cone x = $1/3$ $\pi(3r)^2h_1$ = $1/3$ $\pi9r^2h_1$ = $3\pi r^2h_1$
Volume of cone y = $1/3$ πr^2h_2
So,
$2V/V = 3\pi r^2h_1/ (1/3$ $\pi r^2h_2)$
$2 = 3h_1$ x $3/ h_2 = 9h_1/ h_2$
$h_1/h_2 = 2/9$
Hence, $h_1: h_2 = 2: 9$

6. The diameters of two cones are equal. If their slant heights are in the ratio 5:4, find the ratio of their curved surface areas.
Solution:

Let radius of each cone = r
Given that, ratio between their slant heights = 5: 4
Let slant height of the first cone = 5x
And slant height of second cone = 4x
So, curved surface area of the first cone = $\pi rl = \pi r$ x (5x) = $5\pi rx$
And, the curved surface area of the second cone = πr x (4x) = $4\pi rx$
Therefore,
The ratio between them = $5\pi rx: 4\pi rx = 5: 4$

7. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.
Solution:

Let slant height of the first cone = l
So, the slant height of the second cone = 2l
Radius of the first cone = r_1
And, the radius of the second cone = r_2
Now,
Curved surface area of first cone = πr_1l
Curved surface area of second cone = $\pi r_2(2l)$ = $2\pi r_2l$
According to given condition, we have
$\pi r_1l = 2(2\pi r_2l)$
$r_1 = 4r_2$
$r_1/ r_2 = 4/1$
Thus, $r_1: r_2 = 4: 1$

Exercise 20(C)   Page No: 305

1. The surface area of a sphere is 2464 cm², find its volume.
Solution:

Given,
Surface area of the sphere = 2464 cm²
Let the radius of the sphere be r.
Then, surface area of the sphere = $4\pi r^2$
$4\pi r^2$ = 2464
4 x 22/7 x r² = 2464
r² = (2464 x 7)/ (4 x 22) = 196
r = 14 cm
So, volume = 4/3 πr^3
     = 4/3 x 22/7 x 14 x 14 x 14
     = 11498.67 cm³

2. The volume of a sphere is 38808 cm³; find its diameter and the surface area.
Solution:

Given,
Volume of the sphere = 38808 cm³
Let the radius of the sphere = r
4/3 πr^3 = 38808
4/3 x (22/7) x r³ = 38808
r³ = (38808 x 7 x 3)/ (4 x 22) = 9261
r = 21 cm
So, the diameter = 2r = 21 x 2 = 42 cm
And,
Surface area = $4\pi r^2$ = 4 x 22/7 x 21 x 21 cm² = 5544 cm²

3. A spherical ball of lead has been melted and made into identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made?
Solution:

Let the radius of the spherical ball = r
So, the volume = 4/3 πr^3
And, the radius of smaller ball = r/2
Volume of smaller ball = 4/3 $\pi(r/2)^3$ = 4/3 πr^3/8 = πr^3/6
Thus, the number of balls made out of the given ball = (4/3 πr^3)/ (πr^3/6)
                                              = 4/3 x 6 = 8

4. How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm.
Solution:

Given,
Diameter of bigger ball = 8 cm
So, radius of bigger ball = 4 cm
Volume = 4/3 πr^3 = 4/3 π 4^3 = 265π/3 cm^3
Radius of small ball = 1 cm
Then, volume = 4/3 πr^3 = 4/3 π 1^3 = 4π/3 cm^3
Thus,
The number of balls = (265π/3)/ (4π/3)
= 256π/3 x 3/4π
= 64

5. 8 metallic sphere; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.
Solution:

Radius of metallic sphere = 2mm = 1/5 cm
Volume = 4/3 πr^3 = 4/3 x 22/7 x 1/5 x 1/5 x 1/5 = 88/ (21 x 125) cm^3
Volume of 8 spheres = (88 x 8)/ (21 x 125) = 704/ (21 x 125) cm^3 ….. (i)
Let the radius of new sphere = R
Volume = 4/3 πR^3 = 4/3 x 22/7 x R^3 …… (ii)
According the question, equating (i) and (ii) we have
704/ (21 x 125) = 4/3 x 22/7 x R^3
704/ (21 x 125) = 88/21 x R^3
R^3 = (704 x 21)/ (21 x 125 x 88) = 8/125
R = 2/5 = 0.4 cm = 4 mm
Therefore, the radius of the new sphere is 4 mm.

6. The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their:
(i) radii
(ii) surface areas
Solution:

Given,
The volume of first sphere = 27 x volume of second sphere
Let the radius of the first sphere = r_1
And, radius of second sphere = r_2
(i)   Then, according to the question we have
        4/3 πr_1^3 = 27 (4/3 πr_2^3)
        r_1^3/ r_2^3 = 27
        r_1/ r_2 = 3/ 1
        Thus, r_1: r_2 = 3: 1

(ii)   Surface area of the first sphere = 4 πr_1^2
        And the surface area of second sphere = 4 πr_2^2
        Ratio of their surface areas = 4 πr_1^2/ 4 πr_2^2 = r_1^2/ r_2^2 = 3^2/ 1^2 = 9
        Hence, the ratio = 9: 1

Exercise 20(D)   Page No: 308

1. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.
Solution:

Given,
Radius of the sphere = R = 15 cm
So, the volume of sphere melted = 4/3 πR^2 = 4/3 x π x 15 x 15 x 15
Radius of each cone recasted = r = 2.5 cm
And, height of each cone recasted = h = 8 cm
So, volume of each cone recasted = 1/3 πr^{2}h = 1/3 x π x 2.5 x 2.5 x 8
Thus,
Number of cones recasted = Volume of sphere melted/ Volume of each cone formed

$$= \cfrac{\frac{4}{3} \times \pi \times 15 \times 15 \times 15}{\frac{1}{3} \times \pi \times 2.5 \times 2.5 \times 8}$$

= 270

2. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.
Solution:

Given,
External diameter of the hollow sphere = 8 cm
So, radius (R) = 4 cm
Internal diameter of the hollow sphere = 4 cm
So, radius (r) = 2 cm
Then, the volume of metal used in hollow sphere

$$= \frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3} \times \frac{22}{7} \times (4^3 - 2^3) = \frac{88}{21}(64 - 8) = \frac{88}{21} \times 56 \ cm^3....(i)$$

Also given,
Diameter of cone = 8 cm
Therefore, radius = 4 cm
Let height of the cone = h
Volume = 1//3 πr^{2}h = 1/3 x 22/7 x 4 x 4 x h = 352/21 h …. (ii)
So, according to the question
From (i) and (ii), we have

$$\frac{352}{21}h = \frac{88}{21} \times 56$$

h = (88 x 56 x 21)/ (21 x 352) = 14 cm
Thus, the height of the cone = 14 cm

3. The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cone of height 32 cm. find the

diameter of the base of the cone.
Solution:

Given,
Height of the solid right circular cone = 32 cm
Internal radius metallic spherical shell = 3 cm
External radius spherical shell = 5 cm
Then, the volume of the spherical shell = 4/3 π (5^3 - 3^3)
$\qquad\qquad\qquad\qquad\qquad\qquad$ = 4/3 x 22/7 x (125 - 27)
$\qquad\qquad\qquad\qquad\qquad\qquad$ = 4/3 x 22/7 x 98
Volume of solid right circular cone = 1/3 πr^2h
$\qquad\qquad\qquad\qquad\qquad\qquad$ = 1/3 x 22/7 x r^2 x 32

According to the question,
1/3 x 22/7 x r^2 x 32 = 4/3 x 22/7 x 98
r^2 = (4 x 98)/ 32
r = 7/2 = 3.5 cm
Therefore, diameter = 2r = 7 cm

4. Total volume of three identical cones is the same as that of a bigger cone whose height is 9 cm and diameter 40 cm. Find the radius of the base of each smaller cone, if height of each is 108 cm.
Solution:

Let the radius of the smaller cone be r cm
Given,
Diameter of bigger cone = 40 cm
So, the radius = 20 cm and height = 9 cm
Volume of larger cone = 1/3 π x $(20)^2$ x 9
And,
The volume of the smaller cone = 1/3 π x r^2 x 108
From the question, we have
Volume of larger cone = 3 x Volume of smaller cone
1/3 π x $(20)^2$ x 9 = 3 x (1/3 π x r^2 x 108)
r^2 = $(20)^2$ x 9/ (108 x 3)
r = 20/6 = 10/3 cm

5. A solid rectangular block of metal 49 cm by 44 cm by 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.
Solution:

Volume of rectangular block = 49 x 44 x 18 cm^3 = 38808 cm^3 …… (i)
Let r be the radius of sphere.
So, the volume = 4/3 πr^3 = 4/3 x 22/7 x r^3 = 88/21 r^3 ….. (ii)
Then, according to the question
88/21 r^3 = 38808
r^3 = (38808 x 21)/ 88 = 441 x 21
$\qquad$ = 9261

r = 21 cm
Therefore, the radius of sphere = 21 cm

6. A hemi-spherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into conical shaped small containers each of diameter 3 cm and height 4 cm. How many containers are necessary to empty the bowl?
Solution:

Given,
Radius of hemispherical bowl = 9 cm
Volume = 2/3 πr^3 = 2/3 $\pi 9^3$ = 2/3 π x 729 = 486 π cm^2
Diameter of each cylindrical bottle = 3 cm
So, the radius = 1.5 cm and height = 4 cm
Volume of bottle = 1/3 $\pi r^2 h$ = 1/3 $\pi (3/2)^2$ x 4 = 3 π
Thus,
The number of bottles = 486 π/ 3 π
                     = 162

Exercise 20(E)   Page No: 311

1. A cone of height 15 cm and diameter 7 cm is mounted on a hemisphere of same diameter. Determine the volume of the solid thus formed.
Solution:
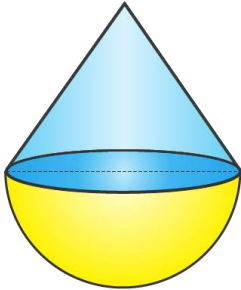
Given,
Height of the cone = 15 cm
Diameter of the cone = 7 cm
So, its radius = 3.5 cm
Radius of the hemisphere = 3.5 cm
Now,
Volume of the solid = Volume of the cone + Volume of the hemisphere
            = 1/3 πr^2 h + 2/3 πr^3
            = 1/3 x 22/7 x $(3.5)^2$ x 15 + 2/3 x 22/7 x $(3.5)^3$
            = 1/3 x 22/7 x $(3.5)^2$ (15 + 2x3.5)
            = 847/3
            = 282.33 cm^3

2. A buoy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of the hemisphere. The radius of the base of the cone is 3.5 m and its volume is two-third of the hemisphere. Calculate the height of the cone and the surface area of the buoy, correct to two decimal places.
Solution:

Given,
Radius of the hemisphere part (r) = 3.5 m = 7/2 m
Volume of hemisphere = 2/3 πr^3
                = 2/3 x 22/7 x 7/2 x 7/2 x 7/2
                = 539/6 m^3
Volume of conical part = 2/3 x 539/6 m^3 [2/3 rd of the hemisphere]
Let height of the cone = h
Then,
1/3 πr^2 h = (2 x 539)/ (3 x 6)
1/3 x 22/7 x 7/2 x 7/2 x h = (2 x 539)/ (3 x 6)
h = (2 x 539 x 2 x 2 x 7 x 3)/ (3 x 6 x 22 x 7 x 7)
h = 14/3 m = 4.67 m
Height of the cone = 4.67 m
Surface area of buoy = 2 πr^2 + πrl

But, we know that
l = $(r^2 + h^2)^{1/2}$
l = $((7/2)^2 + (14/3)^2)^{1/2}$
l = $(1225/36)^{1/2}$ = 35/6 m
Hence,
Surface area = $2\pi r^2 + \pi r l$
       = (2 x 22/7 x 7/2 x 7/2) + (22/7 x 7/2 x 35/6)
       = 77 + 385/6 = 847/6
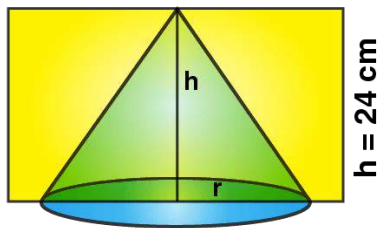   = 141.17 m^2 [Corrected to 2 decimal places]

3. From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a conical cavity of diameter 14 cm and depth 24 cm is drilled out. Find:
(i) the surface area of the remaining solid
(ii) the volume of remaining solid
(iii) the weight of the material drilled out if it weighs 7 gm per cm³.
Solution:

Given,
Dimensions of rectangular solid l = 42 cm, b = 30 cm and h = 20 cm
Conical cavity's diameter = 14 cm
So, its radius = 7 cm
Depth (height) = 24 cm

(i) Total surface area of cuboid = 2 (lb + bh + lh)
             = 2 (42 x 30 + 30 x 20 + 20 x 42)
             = 2 (1260 + 600 + 840)
             = 2 (2700)
             = 5400 cm^2
Diameter of the cone = 14 cm
So, its radius = 14/2 = 7 cm
Area of circular base = πr^2 = 22/7 x 7 x 7 = 154 cm^2
We know that,
l = $(7^2 + 24^2)^{1/2}$ = $(49 + 576)^{1/2}$ = $(625)^{1/2}$ = 25
Area of curved surface area of cone = $\pi r l$ = 22/7 x 7 x 25 = 22 x 25 = 550 cm^2
Surface area of remaining part = 5400 + 550 – 154 = 5796 cm^2

(ii) Volume of the rectangular solid = (42 x 30 x 20) cm^3 = 25200 cm^3
Radius of conical cavity (r) = 7 cm
Height (h) = 24 cm

Volume of cone = 1/3 πr² h
                          = 1/3 x 22/7 x 7 x 7 x 24
                          = 1232 cm³

(iii) Weight of material drilled out = 1232 x 7g = 8624g = 8.624 kg

4. The cubical block of side 7 cm is surmounted by a hemisphere of the largest size. Find the surface area of the resulting solid.
Solution:

It's known that, the diameter of the largest hemisphere that can be placed on a face of a cube of side 7 cm will be 7 cm.
So, it's radius = r = 7/2 cm
It's curved surface area = 2 πr² = 2 x 22/7 x 7/2 x 7/2 = 77 cm² …… (i)
Now,
Surface area of the top of the resulting solid = Surface area of the top face of the cube - Area of the base of the hemisphere

$$= (7 \times 7) - \left(\frac{22}{7} \times \frac{49}{4}\right)$$

$$= 49 - \frac{77}{2}$$

$$= \frac{98 - 77}{2}$$

$$= \frac{21}{2}$$

$$= 10.5 \text{ cm}^2 \dots\dots(ii)$$

Surface area of the cube = 5 x (side)² = 5 x 49 = 245 cm² …… (iii)
Hence,
Total area of resulting solid = (i) + (ii) + (iii) = 245 + 10.5 + 77 = 332.5 cm²

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of the top which is open is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.
Solution:
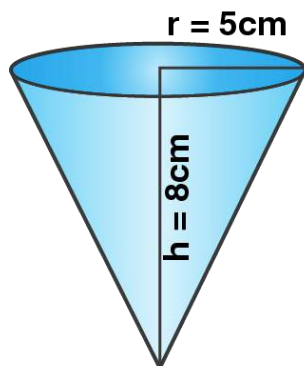
Given,
Height of cone = 8 cm
Radius = 5 cm
Volume = 1/3 πr^2 h

   = 1/3 x 22/7 x 5 x 5 x 8 cm³

   = 4400/21 cm³

Hence, the volume of water that flowed out = ¼ x 4400/21 cm³

= 1100/21 cm³

Also, given
Radius of each ball = 0.5 cm = ½ cm
Volume of a ball = 4/3 πr^3

   = 4/3 x 22/7 x ½ x ½ x ½ cm³

   = 11/21 cm³

Thus, the no. of balls = 1100/21 = 11/21 = 100

Exercise 20(F)   Page No: 315

1. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base are removed. Find the volume of the remaining solid.
Solution:

Given,
Height of the cylinder (h) = 10 cm
And radius of the base (r) = 6 cm
Volume of the cylinder = $\pi r^2 h$
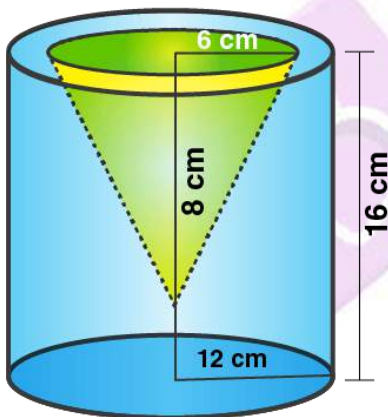Height of the cone = 10 cm
Radius of the base of cone = 6 cm
Volume of the cone = 1/3 $\pi r^2 h$
Volume of the remaining part = $\pi r^2 h$ – 1/3 $\pi r^2 h$
$\qquad\qquad\qquad\qquad\qquad$ = 2/3 $\pi r^2 h$
$\qquad\qquad\qquad\qquad\qquad$ = 2/3 x 22/7 x 6 x 6 x 10
$\qquad\qquad\qquad\qquad\qquad$ = 5280/7 cm^3
$\qquad\qquad\qquad\qquad\qquad$ = 754 (2/7) cm^3

2. From a solid cylinder whose height is 16 cm and radius is 12 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume and total surface area of the remaining solid.
Solution:

Given,
Radius of solid cylinder (R) = 12 cm
And, Height (H) = 16 cm
Volume = $\pi R^2 h$ = 22/7 x 12 x 12 x 16
$\qquad$ = 50688/7 cm^3
Also given,
Radius of cone (r) = 6 cm, and height (h) = 8 cm.
Volume = 1/3 $\pi r^2 h$
$\qquad\qquad$ = 1/3 x 22/7 x 6 x 6 x 8
$\qquad\qquad$ = 2112/7 cm^3

(i) Volume of remaining solid = 50688/7 – 2112/7

= 48567/7

= 6939.43 cm³

(ii) Slant height of cone l = (h² + r²)$^{1/2}$

= (6² + 8²)$^{1/2}$ = (36 + 64)$^{1/2}$ = (100)$^{1/2}$ = 10 cm

Thus,

Total surface area of remaining solid = curved surface area of cylinder + curved surface area of cone + base area of cylinder + area of circular ring on upper side of cylinder

$$= 2\pi Rh + \pi r l + \pi R^2 + \pi(R^2 - r^2)$$

$$= (2 \times \frac{22}{7} \times 12 \times 16) + (\frac{22}{7} \times 6 \times 10) + (\frac{22}{7} \times 12 \times 12) + (\frac{22}{7}(12^2 - 6^2))$$

$$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{22}{7}(144 - 36)$$

$$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{2376}{7}$$

= 15312/7

= 2187.43 cm²

3. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 80 m, calculate the total area of canvas required. Also, find the total cost of canvas used at Rs 15 per meter if the width is 1.5 m

Solution:
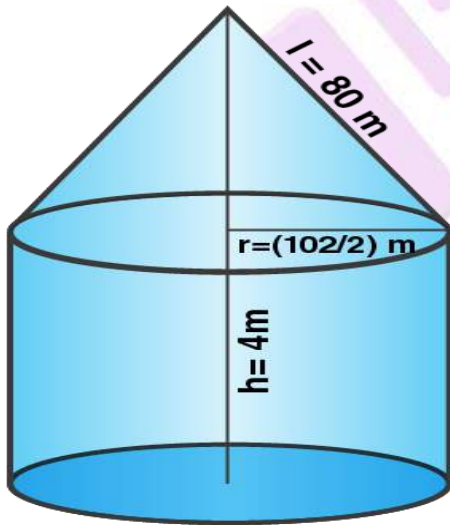
Given,

Radius of the cylindrical part of the tent (r) = 105/2 m

Slant height (l) = 80 m

So, the total curved surface area of the tent = 2πr h + πrl

= (2 x 22/7 x 105/2 x 4) + (22/7 x 105/2 x 80)

= 1320 + 13200

= 14520 m²

Width of canvas used = 1.5 m
Length of canvas = 14520/1.5 = 9680 m
Hence,
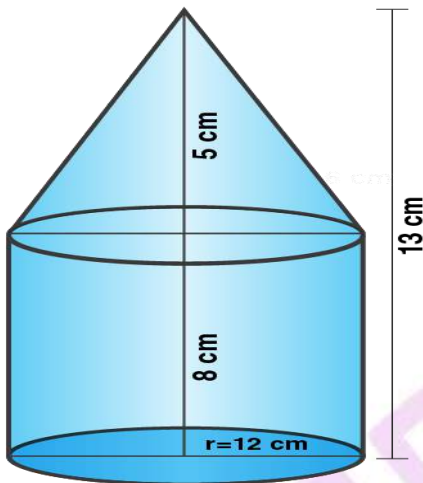Total cost of canvas at the rate of Rs 15 per meter = 9680 x 15 = Rs 145200

4. A circus tent is cylindrical to a height of 8 m surmounted by a conical part. If total height of the tent is 13 m and the diameter of its base is 24 m; calculate:
(i) total surface area of the tent
(ii) area of canvas, required to make this tent allowing 10% of the canvas used for folds and stitching.
Solution:

Given,
Height of the cylindrical part = H = 8 m
Height of the conical part = h = (13 – 8) m = 5 m
Diameter = 24 m
So, its radius = 24/2 = 12 m
Then, slant height (l) = $(r^2 + h^2)^{1/2}$
                        = $(12^2 + 5^2)^{1/2}$ = $(144 + 25)^{1/2}$ = $(169)^{1/2}$ = 13 m

(i) Total surface area of the tent = 2πrh + πrl = πr(2h + l)
                        = 22/7 x 12 x (2 x 8 + 13)
                        = 264/7 (16 + 3)
                        = 264/7 x 29
                        = 7656/7 m²
                        = 1093.71 m²
(ii) Area of canvas used in stitching = total area of canvas
Total area of canvas = 7656/7 + total area of canvas/10
Total area of canvas – Total area of canvas/10 = 7656/10
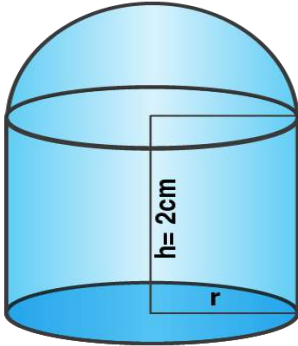Total area of canvas (1 – 1/10) = 7656/7
Total area of canvas x 9/10 = 7656/7
Total area of canvas = 7656/7 x 10/9 = 76560/63
Therefore, the total area of canvas = 1215.23 m²

5. A cylindrical boiler, 2 m high, is 3.5 m in diameter. It has a hemispherical lid. Find the volume of its interior, including the part covered by the lid.
Solution:

Given,
Diameter of cylindrical boiler = 3.5 m
So, the radius (r) = 3.5/2 = 35/20 = 7/4 m
Height (h) = 2m
Radius of hemisphere (R) = 7/4 m
Total volume of the boiler = $\pi r^2 h + 2/3\ \pi r^3$
$\qquad\qquad\qquad\qquad = \pi r^2 (h + 2/3r)$
$\qquad\qquad\qquad\qquad = 22/7\ x\ 7/4\ x\ 7/4\ (2 + 2/3\ x\ 7/4)$
$\qquad\qquad\qquad\qquad = 77/8\ (2 + 7/6)$
$\qquad\qquad\qquad\qquad = 77/8\ x\ 19/6 = 1463/48$
$\qquad\qquad\qquad\qquad = 30.48\ m^3$

6. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylindrical part is 4(2/3) m and the diameter of hemisphere is 3.5 m. Calculate the capacity and the internal surface area of the vessel.
Solution:
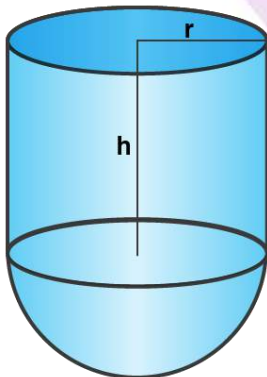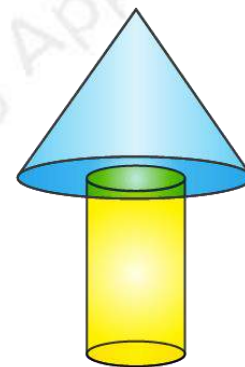

Given,
Diameter of the base = 3.5 m
So, its radius = 3.5/2 m = 1.75 m = 7/4 m
Height of cylindrical part = 4 + 2/3 = 14/3

(i) Capacity (volume) of the vessel = $\pi r^2 h + 2/3 \pi r^2 = \pi r^2 (h + 2/3r)$
$\qquad\qquad\qquad\qquad\qquad = 22/7 \times 7/4 \times 7/4(14/3 + 2/3 \times 7/4)$
$\qquad\qquad\qquad\qquad\qquad = 77/8(14/3 + 7/6)$
$\qquad\qquad\qquad\qquad\qquad = 77/8(28+7/6)$
$\qquad\qquad\qquad\qquad\qquad = 77/8 \times 35/6$
$\qquad\qquad\qquad\qquad\qquad = 2695/48$
$\qquad\qquad\qquad\qquad\qquad = 56.15 \text{ m}^3$

(ii) Internal curved surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
$= 2 \times 22/7 \times 7/4(14/3 + 7/4)$
$= 11(56 + 21)/12$
$= 11 \times 77/12$
$= 847/12$
$= 70.58 \text{ m}^2$

7. A wooden toy is in the shape of a cone mounted on a cylinder as shown alongside.
If the height of the cone is 24 cm, the total height of the toy is
60 cm and the radius of the base of the cone = twice the
radius of the base of the cylinder = 10 cm; find the total
surface area of the toy. [Take π = 3.14]

Solution:

Given,
Height of the cone (h) = 24 cm
Height of the cylinder (H) = 36 cm
Radius of the cone (r) = twice the radius of the cylinder = 10 cm
Radius of the cylinder (R) = 5 cm
Then, we know the slant height of the cone = $(r^2 + h^2)^{1/2} = (10^2 + 24^2)^{1/2} = (100 + 576)^{1/2}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = (676)^{1/2} = 26 \text{ cm}$
Now, the surface area of the toy = curved area of the conical point + curved area of the cylinder
$\qquad\qquad\qquad\qquad\qquad = \pi rl + \pi r^2 + 2\pi RH$
$\qquad\qquad\qquad\qquad\qquad = \pi (rl + r^2 + 2RH)$
$\qquad\qquad\qquad\qquad\qquad = 3.14 (10 \times 26 + 10^2 + 2 \times 5 \times 36)$
$\qquad\qquad\qquad\qquad\qquad = 3.14 (260 + 100 + 360)$
$\qquad\qquad\qquad\qquad\qquad = 3.14 (720)$
$\qquad\qquad\qquad\qquad\qquad = 2260.8 \text{ cm}^2$

Exercise 20(G)   Page No: 316

1. What is the least number of solid metallic spheres, each of 6 cm diameter, that should be melted and recast to form a solid metal cone whose height is 45 cm and diameter is 12 cm?
Solution:

Given,
Diameter of solid metallic sphere = 6 cm
So, its radius = 3 cm
Height of solid metal cone = 45
Diameter of metal cone = 12 cm
So, its radius = 6 cm
Now,
Let the number of solid metallic spheres be 'n'
Volume of 1 sphere = 4/3 π (3)^3
Volume of metallic cone = 1/3 π6^2 x 45
Hence, n = Volume of metal cone/ Volume of 1 sphere
n = (1/3 π6^2 x 45)/ (4/3 π (3)^3)
   = (6 x 6 x 45)/ (4 x 3 x 3 x 3)
n = 15

2. A largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere. (Answer correct to the nearest integer)
Solution:

Radius of the largest sphere that can be formed inside the cylinder will be equal to the radius of the cylinder.
So, radius of the largest sphere = 7 cm
Volume of sphere = 4/3 π 7^3
                 = 4/3 x 22/7 x 7 x 7 x 7
                 = 4312/3
                 = 1437 cm^3

3. A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in identical cones of height 12 cm and diameter 6 cm having a hemi-spherical shape on the top. Find the number of cones required.
Solution:

Given,
Diameter of the cylinder = 12 cm
So, its radius = 6 cm
Height of the cylinder = 15 cm
Diameter of the cone = 6 cm
So, its radius = 3 cm
Height of the cone = 12 cm
Radius of the hemisphere = 3 cm

Now,
Let the number of cones be 'n'.
Volume of the cylinder = $\pi r^2 h$ = $\pi\ 6^2$ x 15
Volume of an ice-cream cone with ice-cream = Volume of cone + Volume of hemisphere
$$= 1/3\ \pi(3)^2\ x\ 12 + 2/3\ \pi(3)^2$$
$$= 36\ \pi + 18\ \pi$$
$$= 54\ \pi\ cm^3$$
Number of cones = Volume of cylinder/ Volume of ice-cream cone
$$= \pi\ 6^2\ x\ 15\ /\ 54\ \pi$$
$$= (36\ x\ 15)/\ 54$$
$$= 10$$
Therefore, the number of cones required = 10

4. A solid is in the form of a cone standing on a hemisphere with both their radii being equal to 8 cm and the height of cone is equal to its radius. Find in terms of π, the volume of the solid.
Solution:

Given,
Radius of both cone and hemisphere = 8 cm
And, height = 8 cm
Hence,
Volume of the solid = Volume of cone + volume of hemisphere
$$= 1/3\ \pi r^2 h + 2/3\ \pi r^3$$
$$= 1/3\ \pi(8)^2(8) + 2/3\ \pi(8)^3$$
$$= \pi(8)^3$$
$$= 512\ \pi\ cm^3$$

5. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of wire.
Solution:

Given,
Diameter of the sphere = 6 cm
So, its radius = 3 cm
And, volume = $4/3\ \pi r^3$ = 4/3 x 22/7 x 3 x 3 x 3 = $792/7\ cm^3$ ….. (i)
Diameter of cylindrical wire = 0.2 cm
So, the radius of the wire = 0.2/2 = 0.1 = 1/10 cm
Now, let length of wire = h
Volume = $\pi r^2\ h$ = 22/7 x 1/10 x 1/10 x h cm^3 = $22h/700\ cm^3$ ….. (ii)
According to the question, equating (i) and (ii) we have
22h/700 = 792/7
h = 792/7 x 700/22
h = 3600 cm = 36 m
Therefore, length of the wire = 36 m

6. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the

cube.
Solution:

Let edge of the cube = a
Then, volume of the cube = a x a x a = a³
The sphere that exactly fits in the cube will have radius 'a/2'
So,
Volume of sphere = 4/3 πr³ = 4/3 x 22/7 x (a/2)³ = 4/3 x 22/7 x a³/8 = 11/21 a³
Now,
Volume of cube: Volume of sphere
= a³ : 11/21 a³
= 1 : 11/21
= 21 : 11

7. An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm³ of iron has 8 gm of mass (approx). (Take π = 355/113)
Solution:

Given,
Radius of the base of poles (r) = 6 cm
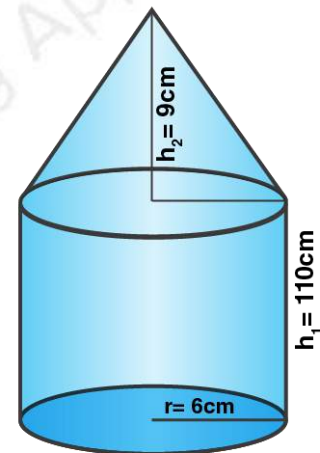Height of the cylindrical part (h₁) = 110 cm
Height of the conical part (h₂) = 9 cm
Now,
Total volume of the iron pole = πr²h₁ + 1/3 πr²h₂ = πr² (h₁ + 1/3h₂)
                 = 355/113 x 6 x 6(110 + 1/3 x 9)
                 = 355/113 x 36 x 113
                 = 12780 cm³
Weight of 1 cm³ = 8 gm (Given)
Thus, the total weight = 12780 x 8 = 102240 gm = 102.24 kg

8. In the following diagram a rectangular platform with a semicircular end on one side is 22 meters long from one end to the other end. If the length of the half circumference is 11 meters, find the cost of constructing the platform, 1.5 meters high at the rate of Rs 4 per cubic meters.
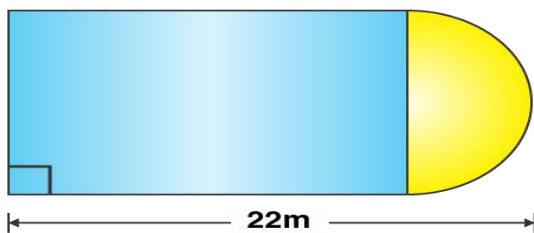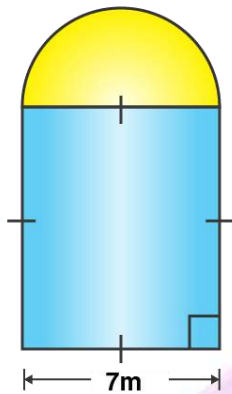
Solution:

Given,
Length of the platform = 22 m

Circumference of semi-circle (c) = 11 m
So, its radius = (c x 2)/ (2 x π) = (11 x 7)/ 22 = 7/2 m
Thus, the breadth of the part = 7/2 x 2 = 7 m
And, length = 22 – 7/2 = 37/2 = 18.5 m
Now, area of platform = l x b + ½ πr^2
                        = 18.5 x 7 + ½ x 22/7 x 7/2 x 7/2 m^2
                        = 129.6 + 77/4 m^2
                        = 148.75 m^2
Also given, height of the platform = 1.5 m
So, the volume = 148.75 x 1.5 = 223.125 m^3
Rate of construction = Rs 4 per m^3
Therefore,
Total expenditure = Rs 4 x 223.125 = Rs 892.50

9. The cross-section of a tunnel is a square of side 7 m surmounted by a semicircle as shown in the following figure.

The tunnel is 80 m long. Calculate:
(i) its volume
(ii) the surface area of the tunnel (excluding the floor) and
(iii) its floor area
Solution:

Given,
Side of square (a) = 7m
And, the radius of semi-circle = 7/2 m
Length of the tunnel = 80 m
Area of cross section of the front part = a^2 + ½ πr^2
                                     = 7 x 7 + ½ x 22/7 x 7/2 x 7/2
                                     = 49 + 77/4 m^2
                                     = (196 + 77)/ 4
                                     = 273/4 m^2
(i) Thus, the volume of the turnel = area of cross section x length of the turnel
                               = 273/4 x 80
                               = 5460 m^3

(ii) Circumference of the front of tunnel (excluding the floor) = 2a + ½ x 2πr
= 2 x 7 + 22/7 x 7/2
= 14 + 11 = 25 cm
Hence, the surface area of the inner part of the tunnel = 25 x 80 = 2000 m²

(iii) Area of floor = l x b = 7 x 80 = 560 m²

10. A cylindrical water tank of diameter 2.8m and height 4.2m is being fed by a pipe of diameter 7 cm through which water flows at the rate of 4m/s. Calculate, in minutes, the time it takes to fill the tank.
Solution:

Given,
Diameter of cylindrical tank = 2.8 m
So, its radius = 1.4 m
Height = 4.2 m
Volume of water filled in it = πr²h
= 22/7 x 1.4 x 1.4 x 4.2 m³
= 181.104/7 m³
= 25.872 m³ …… (i)
Diameter of the pipe = 7 cm
So, the radius (r) = 7/2 cm
Now, let the length of water in the pipe = h_1
Volume = πr²h_1
= 22/7 x 7/2 x 7/2 x h_1
= 77/2h_1 cm³ …… (ii)
According to the question, equating (i) and (ii) we have
77/2h_1 cm³ = 25.872 x 100³ cm³ (Converted from m³ to cm³)

$$h_1 = \frac{25.872 \times 100^3 \times 2}{77} cm$$

$$h_1 = \frac{25.872 \times 100^3 \times 2}{77 \times 100} m$$

$h_1 = 0.672 \times 100^2 m$

h_1 = 6720 m
Thus, the time taken at the speed of 4 m per second = 6720/(4 x 60) min = 28 min