Exercise 21(A)

Prove the following identities:

1. sec A – 1/ sec A + 1 = 1 – cos A/ 1 + cos A

Solution:

$$LHS = \frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$$

$$= \frac{1 - \cos A}{1 + \cos A} = RHS$$

- Hence Proved

2. 1 + sin A/ 1 – sin A = cosec A + 1/ cosec A – 1

Solution:

$$LHS = \frac{1 + \sin A}{1 - \sin A}$$

$$RHS = \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1} = \frac{\frac{1}{\sin A} + 1}{\frac{1}{\sin A} - 1}$$

$$= \frac{1 + \sin A}{1 - \sin A}$$

- Hence Proved

3. 1/ tan A + cot A = cos A sin A

Solution:

Taking L.H.S,

$$\frac{1}{\tan A + \cot A} = \sin A \cos A$$

$$LHS = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{1}{\sin A \cos A}} \left(\because \sin^2 A + \cos^2 A = 1\right)$$

$$= \sin A \cos A = RHS$$

- Hence Proved

4. tan A – cot A = 1 – 2 cos^2 A/ sin A cos A

Solution:

Taking LHS,

$$\tan A - \cot A = \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}$$
$$= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A}$$
$$= \frac{1 - \cos^2 A - \cos^2 A}{\sin A \cos A} \left(\because \sin^2 A = 1 - \cos^2 A\right)$$
$$= \frac{1 - 2\cos^2 A}{\sin A \cos A}$$

- Hence Proved

5. $\sin^4 A – \cos^4 A = 2 \sin^2 A – 1$
Solution:

Taking L.H.S,
$\sin^4 A – \cos^4 A$
$= (\sin^2 A)^2 – (\cos^2 A)^2$
$= (\sin^2 A + \cos^2 A) (\sin^2 A – \cos^2 A)$
$= \sin^2 A – \cos^2 A$
$= \sin^2 A – (1 – \sin^2 A)$ [Since, $\cos^2 A = 1 - \sin^2 A$]
$= 2\sin^2 A – 1$

- Hence Proved

6. $(1 – \tan A)^2 + (1 + \tan A)^2 = 2 \sec^2 A$
Solution:

Taking L.H.S,
$(1 – \tan A)^2 + (1 + \tan A)^2$
$= (1 + \tan^2 A + 2 \tan A) + (1 + \tan^2 A - 2 \tan A)$
$= 2 (1 + \tan^2 A)$
$= 2 \sec^2 A$ [Since, $1 + \tan^2 A = \sec^2 A$]

- Hence Proved

7. $\text{cosec}^4 A – \text{cosec}^2 A = \cot^4 A + \cot^2 A$
Solution:

$\text{cosec}^4 A – \text{cosec}^2 A$
$= \text{cosec}^2 A(\text{cosec}^2 A – 1)$
$= (1 + \cot^2 A) (1 + \cot^2 A - 1)$
$= (1 + \cot^2 A) \cot^2 A$
$= \cot^4 A + \cot^2 A$ = R.H.S

- Hence Proved

8. $\sec A (1 – \sin A) (\sec A + \tan A) = 1$

Solution:

Taking L.H.S,
sec A (1 – sin A) (sec A + tan A)

$$= \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$= \frac{(1-\sin A)}{\cos A}\left(\frac{1+\sin A}{\cos A}\right) = \left(\frac{1-\sin^2 A}{\cos^2 A}\right)$$

$$= \left(\frac{\cos^2 A}{\cos^2 A}\right) = 1 = \text{RHS}$$

- Hence Proved

9. cosec A (1 + cos A) (cosec A – cot A) = 1
Solution:

Taking L.H.S,

$$= \frac{1}{\sin A}(1+\cos A)\left(\frac{1}{\sin A}-\frac{\cos A}{\sin A}\right)$$

$$= \frac{(1+\cos A)}{\sin A}\left(\frac{1-\cos A}{\sin A}\right)$$

$$= \frac{1-\cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1 = \text{RHS}$$

- Hence Proved

10. $\sec^2 A + \text{cosec}^2 A = \sec^2 A \cdot \text{cosec}^2 A$
Solution:

Taking L.H.S,

$$= \frac{1}{\cos^2 A}+\frac{1}{\sin^2 A} = \frac{\sin^2 A+\cos^2 A}{\cos^2 A.\sin^2 A}$$

$$= \frac{1}{\cos^2 A.\sin^2 A} = \sec^2 A\,\text{cosec}^2 A = \text{RHS}$$

- Hence Proved

11. $(1 + \tan^2 A) \cot A / \text{cosec}^2 A = \tan A$
Solution:

Taking L.H.S,

$$\frac{\left(1+\tan^2 A\right)\cot A}{\operatorname{cosec}^2 A}$$
$$= \frac{\sec^2 A \cot A}{\operatorname{cosec}^2 A}\left(\because \sec^2 A = 1+\tan^2 A\right)$$
$$= \frac{\frac{1}{\cos^2 A}\times\frac{\cos A}{\sin A}}{\frac{1}{\sin^2 A}} = \frac{\frac{1}{\cos A \sin A}}{\frac{1}{\sin^2 A}}$$
$$= \frac{\sin A}{\cos A} = \tan A$$
= RHS

- Hence Proved

12. $\tan^2 A - \sin^2 A = \tan^2 A . \sin^2 A$
Solution:

Taking L.H.S,
$\tan^2 A - \sin^2 A$
$$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}$$
$$= \frac{\sin^2 A}{\cos^2 A}.\sin^2 A = \tan^2 A.\sin^2 A = RHS$$

- Hence Proved

13. $\cot^2 A - \cos^2 A = \cos^2 A . \cot^2 A$
Solution:

Taking L.H.S,
$\cot^2 A - \cos^2 A$
$$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A(1-\sin^2 A)}{\sin^2 A}$$
$$= \cos^2 A\frac{\cos^2 A}{\sin^2 A} = \cos^2 A.\cot^2 A = RHS$$

- Hence Proved

14. (cosec A + sin A) (cosec A – sin A) = $\cot^2 A + \cos^2 A$
Solution:

Taking L.H.S,
(cosec A + sin A) (cosec A – sin A)
= $\operatorname{cosec}^2 A - \sin^2 A$
= $(1 + \cot^2 A) - (1 - \cos^2 A)$
= $\cot^2 A + \cos^2 A$ = R.H.S

- Hence Proved

15. (sec A – cos A)(sec A + cos A) = $\sin^2$ A + $\tan^2$ A
Solution:

Taking L.H.S,
(sec A – cos A)(sec A + cos A)
= ($\sec^2$ A – $\cos^2$ A)
= (1 + $\tan^2$ A) – (1 – $\sin^2$ A)
= $\sin^2$ A + $\tan^2$ A = RHS
- Hence Proved

16. (cos A + sin A)^2 + (cosA – sin A)^2 = 2
Solution:

Taking L.H.S,
(cos A + sin A)^2 + (cosA – sin A)^2
= cos2 A + sin2 A + 2cos A sin A + cos2 A – 2cosA.sinA
= 2 ($\cos^2$ A + $\sin^2$ A) = 2 = R.H.S
- Hence Proved

17. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1
Solution:

Taking LHS,
(cosec A – sin A)(sec A – cos A)(tan A + cot A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\tan A + \frac{1}{\tan A}\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A . \cos A}\right)$$

= 1
= RHS
- Hence Proved

18. 1/ sec A + tan A = sec A – tan A
Solution:

Taking LHS,

$$\frac{1}{\sec A + \tan A}$$
$$= \frac{1}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$
$$= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A}$$
$$= \sec A - \tan A$$
= RHS

- Hence Proved

19. cosec A + cot A = 1/ cosec A – cot A
Solution:

Taking LHS,
cosec A + cot A
$$= \frac{\operatorname{cosec} A + \cot A}{1} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A}$$
$$= \frac{\operatorname{cosec}^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A} = \frac{1 + \cot^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A}$$
$$= \frac{1}{\operatorname{cosec} A - \cot A}$$
= RHS

- Hence Proved

20. sec A – tan A/ sec A + tan A = 1 – 2 secA tanA + 2 tan^2 A
Solution:
Taking LHS,
$$\frac{\sec A - \tan A}{\sec A + \tan A}$$
$$= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$
$$= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A}$$
$$= \frac{\sec^2 A + \tan^2 A - 2\sec A \tan A}{1}$$
= 1 + tan^2 A + tan^2 A – 2 sec A tan A
= 1 – 2 sec A tan A + 2 tan^2 A = RHS

- Hence Proved

21. (sin A + cosec A)^2 + (cos A + sec A)^2 = 7 + tan^2 A + cot^2 A
Solution:

Taking LHS,

$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$
$= \sin^2 A + \text{cosec}^2 A + 2 \sin A \text{ cosec } A + \cos^2 A + \sec^2 A + 2\cos A \sec A$
$= (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + \sec^2 A + 2 + 2$
$= 1 + \text{cosec}^2 A + \sec^2 A + 4$
$= 5 + (1 + \cot^2 A) + (1 + \tan^2 A)$
$= 7 + \tan^2 A + \cot^2 A$ = RHS

- Hence Proved

22. $\sec^2 A. \text{cosec}^2 A = \tan^2 A + \cot^2 A + 2$
Solution:

Taking,
RHS = $\tan^2 A + \cot^2 A + 2 = \tan^2 A + \cot^2 A + 2 \tan A. \cot A$
$= (\tan A + \cot A)^2 = (\sin A/\cos A + \cos A/ \sin A)^2$
$= (\sin 2 A + \cos 2 A/ \sin A.\cos A)^2 = 1/ \cos^2 A. \sin^2 A$
$= \sec^2 A. \text{cosec}^2 A$ = LHS

- Hence Proved

23. 1/ 1 + cos A + 1/ 1 – cos A = 2 cosec^2 A
Solution:

Taking LHS,
$$\frac{1}{1+\cos A} + \frac{1}{1-\cos A}$$
$$= \frac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)}$$
$$= \frac{2}{1-\cos^2 A}$$
$$= \frac{2}{\sin^2 A}$$
$$= 2\text{cosec}^2 A$$
= RHS

- Hence Proved

24. 1/ 1 – sin A + 1/ 1 + sin A = 2 sec^2 A
Solution:

Taking LHS,

$\frac{1}{1-\sin A}+\frac{1}{1+\sin A}$

$=\frac{1+\sin A+1-\sin A}{(1-\sin A)(1+\sin A)}$

$=\frac{2}{1-\sin^2 A}$

$=\frac{2}{\cos^2 A}$

$=2\sec^2 A$

= RHS

- Hence Proved

Exercise 21(B) Page No: 237

1. Prove that:

(i) $\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$

(ii) $\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}=2$

(iii) $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=\sec A\operatorname{cosec} A+1$

(iv) $\left(\tan A+\frac{1}{\cos A}\right)^2+\left(\tan A-\frac{1}{\cos A}\right)^2=2\left(\frac{1+\sin^2 A}{1-\sin^2 A}\right)$

(v) $2\sin^2 A+\cos^4 A=1+\sin^4 A$

(vi) $\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A+\sin B}=0$

(vii) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

(viii) $(1+\tan A\tan B)^2+(\tan A-\tan B)^2=\sec^2 A\sec^2 B$

(ix) $\frac{1}{\cos A+\sin A-1}+\frac{1}{\cos A+\sin A+1}=\operatorname{cosec} A+\sec A$

Solution:

(i)

$$\text{LHS}=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$

$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}=\frac{\cos^2 A-\sin^2 A}{(\cos A-\sin A)}$$

$$=\sin A+\cos A=\text{RHS}$$

- Hence Proved

(ii) Taking LHS,

$$\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$$

$$=\frac{(\cos^3 A+\sin^3 A)(\cos A-\sin A)+(\cos^3 A-\sin^3 A)(\cos A+\sin A)}{\cos^2 A-\sin^2 A}$$

$$=\frac{\cos^4 A-\cos^3 A\sin A+\sin^3 A\cos A-\sin^4 A+\cos^4 A+\cos^3 A\sin A-\sin^3 A\cos A-\sin^4 A}{\cos^2 A-\sin^2 A}$$

$$=\frac{2(\cos^4 A-\sin^4 A)}{\cos^2 A-\sin^2 A}$$

$$=\frac{2(\cos^2 A+\sin^2 A)(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)}$$

$$=2(\cos^2 A+\sin^2 A)$$

$$=2\ (\because \cos^2 A+\sin^2 A=1)$$

- Hence Proved

(iii) $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$$=\frac{\tan A}{1-\frac{1}{\tan A}}+\frac{\frac{1}{\tan A}}{1-\tan A}$$

$$=\frac{\tan^2 A}{\tan A-1}+\frac{1}{\tan A(1-\tan A)}$$

$$=\frac{\tan^3 A-1}{\tan A(\tan A-1)}$$

$$=\frac{(\tan A-1)(\tan^2 A+1+\tan A)}{\tan A(\tan A-1)}$$

$$=\frac{\sec^2 A+\tan A}{\tan A}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{\sin A}{\cos A}}+1$$

$$=\frac{1}{\sin A\cos A}+1$$

$$=\sec A\operatorname{cosec} A+1$$

- Hence Proved

(iv) $\left(\tan A + \frac{1}{\cos A}\right)^2 + \left(\tan A - \frac{1}{\cos A}\right)^2$

$= \left(\frac{\sin A + 1}{\cos A}\right)^2 + \left(\frac{\sin A - 1}{\cos A}\right)^2$

$= \frac{\sin^2 A + 1 + 2\sin A + \sin^2 A + 1 - 2\sin A}{\cos^2 A}$

$= \frac{2 + 2\sin^2 A}{\cos^2 A}$

$= 2\left(\frac{1 + \sin^2 A}{1 - \sin^2 A}\right)$

- Hence Proved

(v) Taking LHS,
$2 \sin^2 A + \cos^2 A$
$= 2 \sin^2 A + (1 – \sin^2 A)^2$
$= 2 \sin^2 A + 1 + \sin^4 A – 2 \sin^2 A$
$= 1 + \sin^4 A$ = RHS

- Hence Proved

(vi) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$

$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)}$

$= 0$

- Hence Proved

(vii) LHS

$= (\text{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$

$= \sin A \cos A$

$$RHS = \frac{1}{\tan A + \cot A}$$
$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$
$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$
$$= \sin A \cos A$$
$$LHS = RHS$$

- Hence Proved

(viii) $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$
$$= 1 + \tan^2 A \tan^2 B + 2\tan A \tan B + \tan^2 A + \tan^2 B - 2\tan A \tan B$$
$$= 1 + \tan^2 A + \tan^2 B + \tan^2 A \tan^2 B$$
$$= \sec^2 A + \tan^2 B(1 + \tan^2 A)$$
$$= \sec^2 A + \tan^2 B \sec^2 A$$
$$= \sec^2 A(1 + \tan^2 B)$$
$$= \sec^2 A \sec^2 B$$

- Hence Proved

(ix) $\frac{1}{(\cos A + \sin A) - 1} + \frac{1}{(\cos A + \sin A) + 1}$
$$= \frac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A)^2 - 1}$$
$$= \frac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2\cos A \sin A - 1}$$
$$= \frac{2(\cos A + \sin A)}{1 + 2\cos A \sin A - 1} = \frac{\cos A + \sin A}{\cos A \sin A}$$
$$= \frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A}$$
$$= \frac{1}{\sin A} + \frac{1}{\cos A}$$
$$= \operatorname{cosec} A + \sec A$$

- Hence Proved

2. If x cos A + y sin A = m and x sin A – y cos A = n, then prove that:
$x^2 + y^2 = m^2 + n^2$
Solution:

Taking RHS,
$m^2 + n^2$
$= (x \cos A + y \sin A)^2 + (x \sin A - y \cos A)^2$

$= x^2 \cos^2 A + y^2 \sin^2 A + 2xy \cos A \sin A + x^2 \sin^2 A + y^2 \cos^2 A - 2xy \sin A \cos A$
$= x^2 (\cos^2 A + \sin^2 A) + y^2 (\sin^2 A + \cos^2 A)$
$= x^2 + y^2$ [Since, $\cos^2 A + \sin^2 A = 1$]
= RHS

3. If m = a sec A + b tan A and n = a tan A + b sec A, prove that $m^2 - n^2 = a^2 - b^2$
Solution:

Taking LHS,
$m^2 - n^2$
$= (a \sec A + b \tan A)^2 - (a \tan A + b \sec A)^2$
$= a^2 \sec^2 A + b^2 \tan^2 A + 2ab \sec A \tan A - a^2 \tan^2 A - b^2 \sec^2 A - 2ab \tan A \sec A$
$= a^2 (\sec^2 A - \tan^2 A) + b^2 (\tan^2 A - \sec^2 A)$
$= a^2 (1) + b^2 (-1)$ [Since, $\sec^2 A - \tan^2 A = 1$]
$= a^2 - b^2$
= RHS

Exercise 21(C)

1. Show that:
(i) tan 10º tan 15º tan 75º tan 80º = 1
Solution:

Taking, tan 10º tan 15º tan 75º tan 80º
= tan (90º - 80º) tan (90º - 75º) tan 75º tan 80º
= cot 80º cot 75º tan 75º tan 80º
= 1 [Since, tan θ x cot θ = 1]

(ii) sin 42º sec 48º + cos 42º cosec 48º = 2
Solution:

Taking, sin 42º sec 48º + cos 42º cosec 48º
= sin 42º sec (90º – 42º) + cos 42º cosec (90º – 42º)
= sin 42º cosec 42º + cos 42º sec 42º
= 1 + 1 [Since, sin θ x cosec θ = 1 and cos θ x sec θ = 1]
= 2

(iii) sin 26º/ sec 64º + cos 26º/ cosec 64º = 1
Solution:

Taking,

$$\frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ}$$

$$= \frac{\sin 26^\circ}{\sec(90^\circ - 26^\circ)} + \frac{\cos 26^\circ}{\operatorname{cosec}(90^\circ - 26^\circ)}$$

$$= \frac{\sin 26^\circ}{\operatorname{cosec} 26^\circ} + \frac{\cos 26^\circ}{\sec 26^\circ}$$

$$= \sin^2 26^\circ + \cos^2 26^\circ$$

$$= 1$$

2. Express each of the following in terms of angles between 0°and 45°:
(i) sin 59°+ tan 63°
(ii) cosec 68°+ cot 72°
(iii) cos 74°+ sec 67°
Solution:

(i) sin 59°+ tan 63°
= sin (90 - 31)°+ tan (90 - 27)°
= cos 31°+ cot 27°

(ii) cosec 68°+ cot 72°

= cosec (90 - 22)°+ cot (90 - 18)°
= sec 22°+ tan 18°

(iii) cos 74°+ sec 67°
= cos (90 - 16)°+ sec (90 - 23)°
= sin 16°+ cosec 23°

3. Show that:

(i) $\frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)} = \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos(90^\circ - A)\cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A)\sin A}{\operatorname{cosec}(90^\circ - A)} = 0$

Solution:

(i) $\frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)}$

$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$

$= \frac{1}{\cos A \sin A}$

$= \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos(90^\circ - A)\cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A)\sin A}{\operatorname{cosec}(90^\circ - A)}$

$= \sin A \cos A - \frac{\sin A \sin A \cos A}{\operatorname{cosec} A} - \frac{\cos A \cos A \sin A}{\sec A}$

= sin A cos A – $\sin^3$ A cos A – $\cos^3$ A sin A
= sin A cos A – sin A cos A ($\sin^2$ A + $\cos^2$ A)
= sin A cos A – sin A cos A (1) [Since, $\sin^2$ A + $\cos^2$ A = 1]
= 0

4. For triangle ABC, show that:
(i) sin (A + B)/ 2 = cos C/2
(ii) tan (B + C)/ 2 = cot A/2
Solution:

We know that, in triangle ABC
∠A + ∠B + ∠C = 180° [Angle sum property of a triangle]

(i) Now,
(∠A + ∠B)/ 2 = 90° - ∠C/ 2
So,
sin ((A + B)/ 2) = sin (90° - C/ 2)
= cos C/ 2

(ii) And,
$(\angle C + \angle B)/2 = 90^\circ - \angle A/2$
So,
$\tan((B + C)/2) = \tan(90^\circ - A/2)$
$= \cot A/2$

5. Evaluate:

(i) $3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(ii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$

(v) $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$

(vi) $2\frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

(vii) $\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$

(viii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

(ix) $14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$

Solution:

(i)
$$3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$
$$= 3\frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$
$$= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2$$

(ii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$
$= 3\cos(90 - 10)^\circ \operatorname{cosec} 10^\circ + 2\cos(90 - 31)^\circ \operatorname{cosec} 31^\circ$
$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\sin 31^\circ \operatorname{cosec} 31^\circ$
$= 3 + 2 = 5$

(iii) $\sin 80^\circ/\cos 10^\circ + \sin 59^\circ \sec 31^\circ$
$= \sin(90 - 10)^\circ/\cos 10^\circ + \sin(90 - 31)^\circ \sec 31^\circ$
$= \cos 10^\circ/\cos 10^\circ + \cos 31^\circ \sec 31^\circ$
$= 1 + 1 = 2$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$
$= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A)$

= cot (35° + A)] – cot (35° + A)
= 0

(v) cosec (65° + A) – sec (25° – A)
= cosec [90° – (25° – A)] – sec (25° – A)
= sec (25° – A) – sec (25° – A)
= 0

(vi)
$$2\frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$$
$$= 2\frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$
$$= 2\frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1$$
$$= 2 - 1 - 1$$
$$= 0$$

(vii)
$$\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$$
$$= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2\frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ}$$
$$= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\frac{\cos^2 15^\circ}{\cos^2 15^\circ}$$

= 1 – 2 = -1

(viii)
$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$$
$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$$
$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2$$
$$= 1 + 1 - 2 = 0$$

(ix) 14 sin 30° + 6 cos 60° – 5 tan 45°
= 14 (1/2) + 6 (1/2) – 5(1)
= 7 + 3 – 5
= 5

6. A triangle ABC is right angled at B; find the value of (sec A. cosec C – tan A. cot C)/ sin B
Solution:

As, ABC is a right angled triangle right angled at B
So, A + C = 90°

(sec A. cosec C – tan A. cot C)/ sin B
= (sec (90º - C). cosec C – tan (90º - C). cot C)/ sin 90º
= (cosec C. cosec C – cot C. cot C)/ 1 = $cosec^2$ C – cot^2 C
= 1 [Since, $cosec^2$ C – cot^2 C = 1]

Exercise 21(D)

1. Use tables to find sine of:
(i) 21°
(ii) 34° 42'
(iii) 47° 32'
(iv) 62° 57'
(v) 10° 20' + 20° 45'
Solution:

(i) sin 21° = 0.3584
(ii) sin 34° 42'= 0.5693
(iii) sin 47° 32'= sin (47° 30' + 2') =0.7373 + 0.0004 = 0.7377
(iv) sin 62° 57' = sin (62° 54' + 3') = 0.8902 + 0.0004 = 0.8906
(v) sin (10° 20' + 20° 45') = sin 30°65' = sin 31°5' = 0.5150 + 0.0012 = 0.5162

2. Use tables to find cosine of:
(i) 2° 4’
(ii) 8° 12’
(iii) 26° 32’
(iv) 65° 41’
(v) 9° 23’ + 15° 54’
Solution:

(i) cos 2° 4’ = 0.9994 - 0.0001 = 0.9993
(ii) cos 8° 12’ = cos 0.9898
(iii) cos 26° 32’ = cos (26° 30’ + 2’) = 0.8949 - 0.0003 = 0.8946
(iv) cos 65° 41’ = cos (65° 36’ + 5’) = 0.4131 -0.0013 = 0.4118
(v) cos (9° 23’ + 15° 54’) = cos 24° 77’ = cos 25° 17’ = cos (25° 12’ + 5’) = 0.9048 - 0.0006 = 0.9042

3. Use trigonometrical tables to find tangent of:
(i) 37°
(ii) 42° 18'
(iii) 17° 27'
Solution:

(i) tan 37° = 0.7536
(ii) tan 42° 18' = 0.9099
(iii) tan 17° 27' = tan (17° 24' + 3') = 0.3134 + 0.0010 = 0.3144

Exercise 21(E)

1. Prove the following identities:

(i) $\frac{1}{\cos A+\sin A}+\frac{1}{\cos A-\sin A}=\frac{2\cos A}{2\cos^2 A-1}$

(ii) $\operatorname{cosec} A-\cot A=\frac{\sin A}{1+\cos A}$

(iii) $1-\frac{\sin^2 A}{1+\cos A}=\cos A$

(iv) $\frac{1-\cos A}{\sin A}+\frac{\sin A}{1-\cos A}=2\operatorname{cosec} A$

(v) $\frac{\cot A}{1-\tan A}+\frac{\tan A}{1-\cot A}=1+\tan A+\cot A$

(vi) $\frac{\cos A}{1+\sin A}+\tan A=\sec A$

(vii) $\frac{\sin A}{1-\cos A}-\cot A=\operatorname{cosec} A$

(viii) $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}=\frac{\cos A}{1-\sin A}$

(ix) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\frac{\cos A}{1-\sin A}$

(x) $\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{\sin A}{1+\cos A}$

(xi) $\frac{1+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}=2\tan A$

(xii) $\frac{(\operatorname{cosec} A-\cot A)^2+1}{\sec A(\operatorname{cosec} A-\cot A)}=2\cot A$

(xiii) $\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)=0$

(xiv) $\frac{(1-2\sin^2 A)^2}{\cos^4 A-\sin^4 A}=2\cos^2 A-1$

(xv) $\sec^4 A(1-\sin^4 A)-2\tan^2 A=1$

(xvi) $\operatorname{cosec}^4 A(1-\cos^4 A)-2\cot^2 A=1$

(xvii) $(1+\tan A+\sec A)(1+\cot A-\operatorname{cosec} A)=2$

Solution:

(i) Taking LHS,

1/ (cos A + sin A) + 1/ (cos A – sin A)

$$= \frac{\cos A + \sin A + \cos A - \sin A}{(\cos A + \sin A)(\cos A - \sin A)}$$

$$= \frac{2\cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{2\cos A}{\cos^2 A - (1 - \cos^2 A)}$$

$$= \frac{2\cos A}{2\cos^2 A - 1}$$

= RHS

- Hence Proved

(ii) Taking LHS, cosec A – cot A

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \frac{1 - \cos A}{\sin A}$$

$$= \frac{1 - \cos A}{\sin A} \times \frac{1 + \cos A}{1 + \cos A}$$

$$= \frac{1 - \cos^2 A}{\sin A(1 + \cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1 + \cos A)}$$

$$= \frac{\sin A}{1 + \cos A}$$

= RHS

- Hence Proved

(iii) Taking LHS, 1 – $\sin^2$ A/ (1 + cos A)

$$= \frac{1 + \cos A - \sin^2 A}{1 + \cos A}$$

$$= \frac{\cos A + \cos^2 A}{1 + \cos A}$$

$$= \frac{\cos A(1 + \cos A)}{1 + \cos A}$$

$= \cos A$

= RHS

- Hence Proved

(iv) Taking LHS,
(1 - cos A)/ sin A + sin A/ (1 – cos A)

$= \frac{(1-\cos A)^2 + \sin^2 A}{\sin A(1-\cos A)}$

$= \frac{1+\cos^2 A - 2\cos A + \sin^2 A}{\sin A(1-\cos A)}$

$= \frac{2-2\cos A}{\sin A(1-\cos A)}$

$= \frac{2(1-\cos A)}{\sin A(1-\cos A)}$

$= 2\text{cosec}A$

= RHS

- Hence Proved

(v) Taking LHS, cot A/ (1 – tan A) + tan A/ (1 – cot A)

$= \frac{\frac{1}{\tan A}}{1-\tan A} + \frac{\tan A}{1-\frac{1}{\tan A}}$

$= \frac{1}{\tan A(1-\tan A)} + \frac{\tan^2 A}{\tan A - 1}$

$= \frac{1-\tan^3 A}{\tan A(1-\tan A)}$

$= \frac{(1-\tan A)(1+\tan A+\tan^2 A)}{\tan A(1-\tan A)}$

$= \frac{1+\tan A+\tan^2 A}{\tan A}$

$= \cot A + 1 + \tan A$

= RHS

- Hence Proved

(vi) Taking LHS, cos A/ (1 + sin A) + tan A

$= \frac{\cos A}{1+\sin A} + \frac{\sin A}{\cos A}$

$= \frac{\cos^2 A + \sin A + \sin^2 A}{(1+\sin A)\cos A}$

$= \frac{1+\sin A}{(1+\sin A)\cos A}$

$= \frac{1}{\cos A}$

$= \sec A$

= RHS

- Hence Proved

(viii) Taking LHS, sin A/ (1 – cos A) – cot A

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{\sin A - (\cos A - 1)}{\sin A - (\cos A - 1)}$$

$$= \frac{(\sin A - \cos A + 1)^2}{\sin^2 A - (\cos A - 1)^2}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 - 2\sin A \cos A - 2\cos A + 2\sin A}{\sin^2 A - \cos^2 A - 1 + 2\cos A}$$

$$= \frac{1 + 1 - 2\sin A \cos A - 2\cos A + 2\sin A}{-\cos^2 A - \cos^2 A + 2\cos A}$$

$$= \frac{2(1 - \cos A) + 2\sin A(1 - \cos A)}{2\cos A(1 - \cos A)}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \frac{1 + \sin A}{\cos A} \times \frac{1 - \sin A}{1 - \sin A}$$

$$= \frac{\cos^2 A}{\cos A(1 - \sin A)}$$

$$= \frac{\cos A}{1 - \sin A}$$

= RHS

- Hence Proved

(ix) Taking LHS,

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 - \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{1 - \sin^2 A}{(1 - \sin A)^2}}$$

$$= \sqrt{\frac{\cos^2 A}{(1 - \sin A)^2}}$$

$$= \frac{\cos A}{1 - \sin A}$$

= RHS

- Hence Proved

(x) Taking LHS,

$$\sqrt{\frac{1-\cos A}{1+\cos A}}$$
$$=\sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1+\cos A}{1+\cos A}}$$
$$=\sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$
$$=\sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$
$$=\frac{\sin A}{1+\cos A}$$
= RHS

- Hence Proved

(xi) Taking LHS,

$$\frac{1+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}$$
$$=\frac{(\sec^2 A-\tan^2 A)+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}$$
$$=\frac{(\sec A-\tan A)(\sec A+\tan A)+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}$$
$$=\frac{(\sec A+\tan A)+(\sec A-\tan A)}{\operatorname{cosec} A}$$
$$=\frac{2\sec A}{\operatorname{cosec} A}$$
$$=2\frac{\frac{1}{\cos A}}{\frac{1}{\sin A}}$$
$$=2\tan A$$
= RHS

- Hence Proved

(xii) Taking LHS,

$$\frac{(\operatorname{cosec}A-\cot A)^2+1}{\sec A(\operatorname{cosec}A-\cot A)}$$

$$=\frac{(\operatorname{cosec}A-\cot A)^2+(\operatorname{cosec}^2A-\cot^2A)}{\sec A(\operatorname{cosec}A-\cot A)}$$

$$=\frac{(\operatorname{cosec}A-\cot A)^2+(\operatorname{cosec}A-\cot A)(\operatorname{cosec}A+\cot A)}{\sec A(\operatorname{cosec}A-\cot A)}$$

$$=\frac{(\operatorname{cosec}A-\cot A)+(\operatorname{cosec}A+\cot A)}{\sec A}$$

$$=\frac{2\operatorname{cosec}A}{\sec A}$$

$= 2\cot A$

= RHS

- Hence Proved

(xiii) Taking LHS,

$$\cot^2A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2A\left(\frac{\sin A-1}{1+\sec A}\right)$$

$$=\cot^2A\left(\frac{\sec A-1}{1+\sin A}\times\frac{\sec A+1}{\sec A+1}\right)+\sec^2A\left(\frac{\sin A-1}{1+\sec A}\right)$$

$$=\cot^2A\left[\frac{\sec^2A-1}{(1+\sin A)(\sec A+1)}\right]+\sec^2A\left(\frac{\sin A-1}{1+\sec A}\right)$$

$$=\cot^2A\left[\frac{\tan^2A}{(1+\sin A)(\sec A+1)}\right]+\sec^2A\left(\frac{\sin A-1}{1+\sec A}\right)$$

$$=\frac{1}{(1+\sin A)(\sec A+1)}+\sec^2A\left(\frac{\sin A-1}{1+\sec A}\right)$$

$$=\frac{1+\sec^2A(\sin A-1)(1+\sin A)}{(1+\sin A)(\sec A+1)}$$

$$=\frac{1+\sec^2A(\sin^2A-1)}{(1+\sin A)(\sec A+1)}$$

$$=\frac{1+\sec^2A(-\cos^2A)}{(1+\sin A)(\sec A+1)}$$

$$=\frac{1-1}{(1+\sin A)(\sec A+1)}$$

$= 0$

= RHS

- Hence Proved

(xiv) Taking LHS,

$\frac{(1-2\sin^2 A)^2}{\cos^4 A - \sin^4 A}$

$= \frac{(1-2\sin^2 A)^2}{(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)}$

$= \frac{(1-2\sin^2 A)^2}{1 - \sin^2 A - \sin^2 A}$

$= \frac{(1-2\sin^2 A)^2}{1 - 2\sin^2 A}$

$= 1 - 2\sin^2 A$

$= 1 - 2(1 - \cos^2 A)$

$= 2\cos^2 A - 1$

= RHS

- Hence Proved

(xv) Taking LHS,

$\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A$

$= \sec^4 A(1 - \sin^2 A) (1 + \sin^2 A) - 2 \tan^2 A$

$= \sec^4 A(\cos^2 A) (1 + \sin^2 A) - 2 \tan^2 A$

$= \sec^2 A + \sin^2 A/ \cos^2 A - 2 \tan^2 A$

$= \sec^2 A - \tan^2 A$

= 1 = RHS

- Hence Proved

(xvi) $\text{cosec}^4 A(1 - \cos^4 A) - 2 \cot^2 A$

$= \text{cosec}^4 A (1 - \cos^2 A) (1 + \cos^2 A) - 2 \cot^2 A$

$= \text{cosec}^4 A (\sin^2 A) (1 + \cos^2 A) - 2 \cot^2 A$

$= \text{cosec}^2 A (1 + \cos^2 A) - 2 \cot^2 A$

$= \text{cosec}^2 A + \cos^2 A/\sin^2 A - 2 \cot^2 A$

$= \text{cosec}^2 A + \cot^2 A - 2 \cot^2 A$

$= \text{cosec}^2 A - \cot^2 A$

= 1 = RHS

- Hence Proved

(xvii) (1 + tan A + sec A) (1 + cot A – cosec A)

= 1 + cot A – cosec A + tan A + 1 – sec A + sec A + cosec A – cosec A sec A

= 2 + cos A/sin A+ sin A/cos A – 1/(sin A cos A)

$= 2 + (\cos^2 A + \sin^2 A)/ \sin A \cos A - 1/(\sin A \cos A)$

= 2 + 1/(sin A cos A) - 1/(sin A cos A)

= 2 = RHS

- Hence Proved

2. If sin A + cos A = p
and sec A + cosec A = q, then prove that: $q(p^2 - 1) = 2p$

Solution:

Taking the LHS, we have

$q(p^2 - 1) = (\sec A + \text{cosec } A) [(\sin A + \cos A)^2 - 1]$

$= (\sec A + \text{cosec } A) [\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1]$

$= (\sec A + \text{cosec } A) [1 + 2 \sin A \cos A - 1]$

$= (\sec A + \text{cosec } A) [2 \sin A \cos A]$

$= 2\sin A + 2 \cos A$

$= 2p$

3. If x = a cos θ and y = b cot θ, show that:

$a^2/ x^2 - b^2/ y^2 = 1$

Solution:

Taking LHS,

$a^2/ x^2 - b^2/ y^2$

$= \frac{a^2}{a^2 \cos^2 \theta} - \frac{b^2}{b^2 \cot^2 \theta}$

$= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$

$= \frac{1 - \sin^2 \theta}{\cos^2 \theta}$

$= \frac{\cos^2 \theta}{\cos^2 \theta}$

$= 1$

4. If sec A + tan A = p, show that:

$\sin A = (p^2 - 1)/ (p^2 + 1)$

Solution:

Taking RHS, $(p^2 - 1)/ (p^2 + 1)$

$= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1}$

$= \frac{\sec^2 A + \tan^2 A + 2 \tan A \sec A - 1}{\sec^2 A + \tan^2 A + 2 \tan A \sec A + 1}$

$= \frac{\tan^2 A + \tan^2 A + 2 \tan A \sec A}{\sec^2 A + \sec^2 A + 2 \tan A \sec A}$

$= \frac{2 \tan^2 A + 2 \tan A \sec A}{2 \sec^2 A + 2 \tan A \sec A}$

$= \frac{2 \tan A(\tan A + \sec A)}{2 \sec A(\tan A + \sec A)}$

$= \sin A$

5. If tan A = n tan B and sin A = m sin B, prove that:
$\cos^2 A = m^2 - 1/ n^2 - 1$
Solution:

Given,
tan A = n tan B
n = tan A/ tan B
And, sin A = m sin B
m = sin A/ sin B
Now, taking RHS and substitute for m and n
$m^2 - 1/ n^2 - 1$

$$= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1}$$

$$= \frac{\tan^2 B(\sin^2 A - \sin^2 B)}{\sin^2 B(\tan^2 A - \tan^2 B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 B\left(\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}\right)}$$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{\sin^2 A \cos^2 B - (1 - \cos^2 B)\cos^2 A}$$

$$= \frac{\cos^2 A(1 - \cos^2 A - 1 + \cos^2 B)}{\cos^2 B(\sin^2 A + \cos^2 A) - \cos^2 A}$$

$$= \frac{\cos^2 A(\cos^2 B - \cos^2 A)}{\cos^2 B - \cos^2 A}$$

$$= \cos^2 A$$

6. (i) If 2 sin A – 1 = 0, show that:
$\sin 3A = 3 \sin A - 4 \sin^3 A$
(ii) If $4 \cos^2 A - 3 = 0$, show that:
$\cos 3A = 4 \cos^2 A - 3 \cos A$
Solution:

(i) Given, 2 sin A – 1 = 0
So, sin A = ½
We know, sin 30° = 1/2
Hence, A = 30°
Now, taking LHS
sin 3A = sin 3(30°) = sin 30° = 1
RHS = $3 \sin 30° - 4 \sin^3 30° = 3 (1/2) - 4 (1/2)^3 = 3 - 4(1/8) = 3/2 - ½ = 1$

Therefore, LHS = RHS

(ii) Given, $4\cos^2 A - 3 = 0$
$4\cos^2 A = 3$
$\cos^2 A = 3/4$
$\cos A = \sqrt{3}/2$
We know, $\cos 30^\circ = \sqrt{3}/2$
Hence, $A = 30^\circ$
Now, taking
LHS = $\cos 3A = \cos 3(30^\circ) = \cos 90^\circ = 0$
RHS = $4\cos^3 A - 3\cos A = 4\cos^3 30^\circ - 3\cos 30^\circ = 4(\sqrt{3}/2)^3 - 3(\sqrt{3}/2)$
$= 4(3\sqrt{3}/8) - 3\sqrt{3}/2$
$= 3\sqrt{3}/2 - 3\sqrt{3}/2$
$= 0$
Therefore, LHS = RHS

7. Evaluate:

(i) $2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$

(ii) $\sec 26^\circ \sin 64^\circ + \frac{\text{coe}\sec 33^\circ}{\sec 57^\circ}$

(iii) $\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$

(iv) $\cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$

(v) $\sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$

(vi) $\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(vii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(viii) $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$

Solution:

(i)
$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2\left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ}\right)^2 + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ}\right)$$
$= 2(1)^2 + 1^2 - 3$
$= 2 + 1 - 3 = 0$

(ii)

$$\sec 26^\circ \sin 64^\circ + \frac{\text{coe}\sec 33^\circ}{\sec 57^\circ}$$

$$= \sec(90^\circ - 64^\circ)\sin 64^\circ + \frac{\text{coe}\sec(90^\circ - 57^\circ)}{\sec 57^\circ}$$

$$= \operatorname{cosec} 64^\circ \sin 64^\circ + \frac{\sec 57^\circ}{\sec 57^\circ}$$

= 1 + 1 = 2

(iii)

$$\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$$

$$= \frac{5\sin(90^\circ - 24^\circ)}{\cos 24^\circ} - \frac{2\cot(90^\circ - 5^\circ)}{\tan 5^\circ}$$

$$= \frac{5\cos 24^\circ}{\cos 24^\circ} - \frac{2\tan 5^\circ}{\tan 5^\circ}$$

$$= 5 - 2 = 3$$

(iv) cos 40^o cosec 50^o + sin 50^o sec 40^o
= cos (90 – 50)^o cosec 50^o + sin (90 – 50)^o sec 40^o
= sin 50^o cosec 50^o + cos 40^o sec 40^o
= 1 + 1 = 2

(v) sin 27^o sin 63^o – cos 63^o cos 27^o
= sin (90 - 63)^o sin 63^o – cos 63^o cos (90 - 63)^o
= cos 63^o sin 63^o – cos 63^o sin 63^o
= 0

(vi)

$$\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$= \frac{3\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$

$$= \frac{3\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ}$$

$$= 3 - 1 = 2$$

(vii) 3 cos 80^o cosec 10^o + 2 cos 59^o cosec 31^o
= 3 cos (90 - 10)^o cosec 10^o + 2 cos (90 - 31)^o cosec 31^o
= 3 sin 10^o cosec 10^o + 2 sin 31^o cosec 31^o
= 3 + 2 = 5

(viii)

$$\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$
$$= \frac{\cos(90^\circ - 15^\circ)}{\sin 15^\circ} + \frac{\sin(90^\circ - 78^\circ)}{\cos 78^\circ} - \frac{\cos(90^\circ - 72^\circ)}{\sin 72^\circ}$$
$$= \frac{\sin 15^\circ}{\sin 15^\circ} + \frac{\cos 78^\circ}{\cos 78^\circ} - \frac{\sin 72^\circ}{\sin 72^\circ}$$
$$= 1 + 1 - 1 = 1$$

8. Prove that:
(i) tan (55º + x) = cot (35º - x)
(ii) sec (70º - θ) = cosec (20º + θ)
(iii) sin (28º + A) = cos (62º – A)
(iv) 1/ (1 + cos (90º - A)) + 1/(1 – cos (90º - A)) = 2 cosec² (90º - A)
(v) 1/ (1 + sin (90º - A)) + 1/(1 – sin (90º - A)) = 2 sec² (90º - A)
Solution:

(i) tan (55º + x) = tan [90º – (35º - x)] = cot (35º - x)
(ii) sec (70º - **θ**) = sec [90º – (20º + **θ**)] = cosec (20º + **θ**)
(iii) sin (28º + A) = sin [90º – (62º - A)] = cos (62º - A)

(iv) $$\frac{1}{1 + \cos(90^\circ - A)} + \frac{1}{1 - \cos(90^\circ - A)}$$
$$= \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A}$$
$$= \frac{1 - \sin A + 1 + \sin A}{(1 + \sin A)(1 - \sin A)}$$
$$= \frac{2}{1 - \sin^2 A}$$
$$= \frac{2}{\cos^2 A}$$
$$= 2\sec^2 A$$
$$= 2\operatorname{cosec}^2(90^\circ - A)$$

(v) $$\frac{1}{1 + \sin(90^\circ - A)} + \frac{1}{1 - \sin(90^\circ - A)}$$
$$= \frac{1}{1 + \cos A} + \frac{1}{1 - \cos A}$$
$$= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)}$$
$$= \frac{2}{1 - \cos^2 A}$$
$$= 2\operatorname{cosec}^2 A$$
$$= 2\sec^2(90^\circ - A)$$