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**Question 1: If (1 + i) (1 + 2i) (1 + 3i) ..... (1 + ni) = a + ib, then what is 2 * 5 * 10....$(1 + n^2)$ is equal to?**

**Solution:**

We have (1 + i) (1 + 2i) (1 + 3i) ..... (1 + ni) = a + ib .....(i)

(1 − i) (1 − 2i) (1 − 3i) ..... (1 − ni) = a − ib .....(ii)

Multiplying (i) and (ii),

we get 2 * 5 * 10 ..... $(1 + n^2) = a^2 + b^2$

**Question 2: If z is a complex number, then the minimum value of |z| + |z − 1| is ______.**

**Solution:**

First, note that |−z|=|z| and |z1 + z2| ≤ |z1| + |z2|

Now |z| + |z − 1| = |z| + |1 − z| ≥ |z + (1 − z)|

= |1|

= 1

Hence, minimum value of |z| + |z − 1| is 1.

**Question 3: For any two complex numbers z1 and z2 and any real numbers a and b; $|(az1 - bz2)|^2 + |(bz1 + az2)|^2$ = ___________.**

**Solution:**

$|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2$

$= a_2 |z_1|^2 + b_2|z_2|^2 - 2 \text{ Re } (ab)|z_1\overline{z2}| + b2 |z_1|^2 + a_2|z_2|^2 + 2 \text{ Re } (ab)|\overline{z1}z2|$

$= (a_2 + b_2) (|z_1|^2 + |z_2|^2)$

**Question 4:** Find the complex number z satisfying the equations

$\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$

**Solution:**

We have $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$

Let z = x + iy, then $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}$

⇒ 3|z – 12| = 5 |z – 8i|

3 |(x – 12) + iy| = 5 |x + (y – 8) i|

9 $(x - 12)^2 + 9y^2 = 25x^2 + 25 (y - 8)^2$ ....(i) and

$\left|\frac{z-4}{z-8}\right| = 1$

⇒ |z – 4| = |z – 8|

|x – 4 + iy| = |x – 8 + iy|

$(x - 4)^2 + y^2 = (x - 8)^2 + y^2$

⇒ x = 6

Putting x = 6 in (i), we get $y^2 - 25y + 136 = 0$

y = 17, 8

Hence, z = 6 + 17i or z = 6 + 8i

**Question 5: If $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ and z is a complex number such that amp $\frac{z-z_1}{z-z_2} = \frac{\pi}{4}$, then the value of |z – 7 – 9i| is equal to __________.**

**Solution:**

Given numbers are $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ and z = x + iy

amp $\frac{z-z_1}{z-z_2} = \frac{\pi}{4}$

amp [(x – 10) + i (y – 6) (x – 4) + i (y – 6)] = π / 4

$\frac{(x-4)(y-6)-(y-6)(x-10)}{(x-4)(x-10)+(y-6)^2} = 1$

$12y - y^2 - 72 + 6y = x^2 - 14x + 40$ .....(i)

Now $|z - 7 - 9i| = |(x - 7) + i(y - 9)|$

From (i), $(x^2 - 14x + 49) + (y^2 - 18y + 81) = 18$

$(x - 7)^2 + (y - 9)^2 = 18$ or

$[(x - 7)^2 + (y - 9)^2]^{½} = [18]^{½} = 3\sqrt{2}$

$|(x - 7) + i(y - 9)| = 3\sqrt{2}$ or

$|z - 7 - 9i| = 3\sqrt{2}$.

**Question 6: Suppose $z_1, z_2, z_3$ are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find the values of $z_3$ and $z_2$.**

**Solution:**

One of the numbers must be a conjugate of $z_1 = 1 + i\sqrt{3}$ i.e. $z_2 = 1 - i\sqrt{3}$ or $z_3 = z_1 e^{i2\pi/3}$ and

$z_2 = z_1 e^{-i2\pi/3}$, $z_3 = (1 + i\sqrt{3})[\cos(2\pi/3) + i\sin(2\pi/3)] = -2$

**Question 7: If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then what is the value of $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$?**

**Solution:**

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

Let $a = \cos\alpha + i\sin\alpha$; $b = \cos\beta + i\sin\beta$ and $c = \cos\gamma + i\sin\gamma$.

Therefore, $a + b + c = (\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma) = 0 + i0 = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$ or

$(\cos\alpha + i\sin a)^3 + (\cos\beta + i\sin\beta)^3 + (\cos\gamma + i\sin\gamma)^3$

$= 3(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

$\Rightarrow (\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma)$

$= 3[\cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)]$ or $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$

$= 3\cos(\alpha + \beta + \gamma)$.

**Question 8: If the cube roots of unity are 1, ω, $\omega^2$, then find the roots of the equation $(x - 1)^3 + 8 = 0$.**

**Solution:**

$(x - 1)^3 = -8 \Rightarrow x - 1 = (-8)^{1/3}$

$x - 1 = -2, -2\omega, -2\omega^2$

$x = -1, 1 - 2\omega, 1 - 2\omega^2$

**Question 9: If 1, ω, $\omega^2$, $\omega^3$......., $\omega^{n-1}$ are the n, $n^{th}$ roots of unity, then $(1 - \omega)(1 - \omega^2)$ .....**

**$(1 - \omega^{n-1})$ = _____________.**

**Solution:**

Since 1, ω, $\omega^2$, $\omega^3$......., $\omega^{n-1}$ are the n, $n^{th}$ roots of unity, therefore, we have the identity

$= (x - 1)(x - \omega)(x - \omega^2) \ldots\ldots (x - \omega^{n-1}) = x^n - 1$ or

$(x - \omega)(x - \omega^2)\ldots\ldots(x - \omega^{n-1}) = x^n-1 / x-1$

$= x^{n-1} + x^{n-2} + \ldots\ldots + x + 1$

Putting x = 1 on both sides, we get

$(1 - \omega)(1 - \omega^2)\ldots\ldots (1 - \omega^{n-1}) = n$

**Question 10: If a = cos (2π / 7) + i sin (2π / 7), then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$ is ______________.**

**Solution:**

$a = \cos(2\pi / 7) + i \sin(2\pi / 7)$

$a^7 = [\cos(2\pi / 7) + i \sin(2\pi / 7)]^7$

$= \cos 2\pi + i \sin 2\pi = 1$ .....(i)

$S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$

$S = a + a^2 + a^3 + a^4 + a^5 + a^6$

$= \frac{a(1-a^6)}{1-a}$

or $S = \frac{a-1}{1-a}$

$= -1$ .....(ii)

$P = \alpha * \beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$

$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3$ (From eqn (i)]

$= 3+(a + a^2 + a^3 + a^4 + a^5 + a^6)$

$= 3 + S = 3 - 1 = 2$ [From (ii)]

Required equation is, $x^2 - Sx + P = 0$

$x^2 + x + 2 = 0$.

**Question 11: Let $z_1$ and z2 be nth roots of unity, which are ends of a line segment that subtend a right angle at the origin. Then n must be of the form _____________.**

**Solution:**

$1^{1/n} = \cos [2r\pi / n] + i \sin [2r \pi / n]$

Let $z_1 = [\cos 2r_1\pi / n] + i \sin [2r_1\pi / n]$ and $z_2 = [\cos 2r_2\pi / n] + i \sin [2r_2\pi / n]$.

Then $\angle Z_1 O Z_2 = \text{amp}(z_1 / z_2) = \text{amp}(z_1) - \text{amp}(z_2)$

$= [2 (r_1 - r_2)\pi] / [n]$

$= \pi / 2$

(Given) $n = 4 (r_1 - r_2)$

$= 4 \times$ integer, so n is of the form 4k.

**Question 12: $(\cos \theta + i \sin \theta)^4 / (\sin \theta + i \cos \theta)^5$ is equal to _____________.**

**Solution:**

$(\cos \theta + i \sin \theta)^4 / (\sin \theta + i \cos \theta)^5$

$= (\cos \theta + i \sin \theta)^4 / i^5 ([1 / i] \sin \theta + \cos \theta)^5$

$= (\cos\theta + i \sin \theta)^4 / i (\cos \theta - i \sin \theta)^5$

$= (\cos \theta + i \sin \theta)^4 / i (\cos \theta + i \sin \theta)^{-5}$ (By property) $= 1 / i (\cos \theta + i \sin \theta)^9$

$= \sin(9\theta) - i \cos (9\theta)$.

**Question 13: Given $z = (1 + i\sqrt{3})^{100}$, then find the value of Re (z) / Im (z).**

**Solution:**

Let $z = (1 + i\sqrt{3})$

$r = \sqrt{[3 + 1]} = 2$ and $r \cos\theta = 1$, $r \sin\theta = \sqrt{3}$ $\tan\theta = \sqrt{3} = \tan \pi / 3 \Rightarrow \theta = \pi / 3$.

$z = 2 (\cos \pi / 3 + i \sin \pi / 3)$

$z^{100} = [2 (\cos \pi / 3 + i \sin \pi / 3)]^{100}$

$= 2^{100} (\cos 100\pi / 3 + i \sin 100\pi / 3)$

$= 2^{100} (-\cos \pi / 3 - i \sin \pi / 3)$

$= 2^{100}(-1 / 2 - i \sqrt{3} / 2)$

$Re(z) / Im(z) = [-1/2] / [-\sqrt{3} / 2] = 1 / \sqrt{3}$.

**Question 14: If x = a + b, y = aα + bβ and z = aβ + bα, where α and β are complex cube roots of unity, then what is the value of xyz?**

**Solution:**

If $x = a + b$, $y = a\alpha + b\beta$ and $z = a\beta + b\alpha$, then $xyz = (a + b) (a\omega + b\omega^2) (a\omega^2 + b\omega)$,where $\alpha = \omega$ and $\beta = \omega^2$

$= (a + b) (a^2 + ab\omega^2 + ab\omega + b^2)$

$= (a + b) (a^2 - ab + b^2)$

$= a^3 + b^3$

**Question 15: If ω is an imaginary cube root of unity, $(1 + \omega - \omega^2)^7$ equals to ___________.**

**Solution:**

$(1 + \omega - \omega^2)^7 = (1 + \omega + \omega^2 - 2\omega^2)^7$

$= (-2\omega^2)^7$

$= -128\omega^{14}$

$= -128\omega^{12}\omega^2$

$= -128\omega^2$

**Question 16: If α, β, γ are the cube roots of p (p < 0), then for any x, y and z, find the value of [xα + yβ + zγ] / [xβ + yγ + zα].**

**Solution:**

Since $p < 0$.

Let $p = -q$, where q is positive.

Therefore, $p^{1/3} = -q1^{/3}(1)^{1/3}$.

Hence $\alpha = -q^{1/3}$, $\beta = -q^{1/3}\,\omega$ and $\gamma = -q^{1/3}\omega^2$

The given expression $[x + y\omega + z\omega^2] / [x\omega + y\omega^2 + z] = (1 / \omega) * [z\omega + y\omega^2 + z] / [x\omega + y\omega^2 + z]$

$= \omega^2$.

**Question 17: The common roots of the equations $x^{12} - 1 = 0$, $x^4 + x^2 + 1 = 0$ are __________.**

**Solution:**

$x^{12} - 1 = (x^6 + 1)(x^6 - 1)$

$= (x^6 + 1)(x^2 - 1)(x^4 + x^2 + 1)$

Common roots are given by $x^4 + x^2 + 1 = 0$

$x^2 = [-1 \pm i\sqrt{3}] / [2] = \omega, \omega^2$ or $\omega^4, \omega^2$ (Because $\omega^3 = 1$) or

$x = \pm\,\omega^2, \pm\,\omega$

**Question 18: Given that the equation $z^2 + (p + iq)z + r + is = 0$, where p, q, r, s are real and non-zero has a real root, then how are p, q, r and s related?**

**Solution:**

Given that $z^2 + (p + iq)z + r + is = 0$ ......(i)

Let $z = \alpha$ (where α is real) be a root of (i), then

$\alpha^2 + (p + iq)\alpha + r + is = 0$ or

$\alpha^2 + p\alpha + r + i(q\alpha + s) = 0$

Equating real and imaginary parts, we have $\alpha^2 + p\alpha + r = 0$ and $q\alpha + s = 0$

Eliminating α, we get

$(-s / q)^2 + p(-s / q) + r = 0$ or

$s^2 - pqs + q^2r = 0$ or

$pqs = s^2 + q^2r$

**Question 19: The difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and a≠b, then what is the relation between a and b?**

**Solution:**

Let α, β and γ,δ be the roots of the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, respectively therefore,

$\alpha + \beta = -a$, $\alpha\beta = b$ and $\delta + \gamma = -b$, $\gamma\delta = a$.

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha + \beta)^2 - \alpha\beta$

$= (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0$ (Because a≠b)

**Question 20: If $b_1 b_2 = 2(c_1 + c_2)$, then at least one of the equations $x^2 + b_1x + c1 = 0$ and $x^2 + b_2x + C_2 = 0$ has _____________ roots.**

**Solution:**

Let D1 and D2 be discriminants of $x^2 + b_1x + c1 = 0$ and $x^2 + b_2x + C_2 = 0$, respectively.

Then,

$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$

$= (b_1^2 + b_2^2) - 4(c_1 + c_2)$

$= b_1^2 + b_2^2 - 2b_1b_2$ [Because $b_1b_2 = 2(c_1 + c_2)$] $= (b_1 - b_2)^2 \geq 0$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$ or $D_1$ and $D_2$ both are positive.

Hence, at least one of the equations has real roots.

**Question 21: If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most 2m then b lies in what interval?**

**Solution:**

Let the roots be $\alpha$, $\beta$

$\alpha + \beta = -2a$ and $\alpha\beta = b$

Given, $|\alpha - \beta| \leq 2m$

or $|\alpha - \beta|^2 \leq (2m)^2$ or

$(\alpha + \beta)^2 - 4ab \leq 4m^2$ or

$4a^2 - 4b \leq 4m^2$

$\Rightarrow a^2 - m^2 \leq b$ and discriminant $D > 0$ or

$4a^2 - 4b > 0$

$\Rightarrow a^2 - m^2 \leq b$ and $b < a^2$.

Hence, $b \in [a^2 - m^2, a^2)$.

**Question 22: If $([1 + i] / [1 - i])^m = 1$, then what is the least integral value of m?**

**Solution:**

$[1 + i] / [1 - i] = ([1 + i] / [1 - i]) \times [1 + i] / [1 + i]$

$= [(1 + i)^2] / [2]$

$= 2i / 2$

$= i$

$([1 + i] / [1 - i])^m = 1$ (as given)

So, the least value of $m = 4$ {Because $i^4 = 1$}

**Question 23: If $(1 - i) x + (1 + i) y = 1 - 3i$, then $(x, y) =$ _______________.**

**Solution:**

$(1 - i) x + (1 + i) y = 1 - 3i$

$\Rightarrow (x + y) + i (-x + y) = 1 - 3i$

Equating real and imaginary parts, we get $x + y = 1$ and $-x + y = -3$;

So, $x = 2$, $y = -1$.

Thus, the point is $(2, -1)$.

**Question 24: $[3 + 2i \sin\theta] / [1 - 2i \sin\theta]$ will be purely imaginary if $\theta$ = ___________.**

**Solution:**

$[3 + 2i \sin\theta] / [1 - 2i \sin\theta]$ will be purely imaginary, if the real part vanishes, i.e.,

$[3 - 4 \sin^2 \theta] / [1 + 4 \sin^2\theta] = 0$

$3 - 4 \sin^2 \theta$ (only if $\theta$ be real)

$\sin\theta = \pm\sqrt{3} / 2$

$= \sin(\pm \pi / 3)$

$\theta = n\pi + (-1)^n (\pm \pi / 3)$

$= n\pi \pm \pi / 3$

**Question 25: The real values of x and y for which the equation is $(x + iy) (2 - 3i) = 4 + i$ is satisfied, are __________.**

**Solution:**

Equation $(x + iy) (2 - 3i) = 4 + i$

$(2x + 3y) + i (-3x + 2y) = 4 + i$

Equating real and imaginary parts, we get

$2x + 3y = 4$ ......(i)

$-3x + 2y = 1$ ......(ii)

From (i) and (ii), we get

$x = 5 / 13, y = 14 / 13$