Get Differential Equations past year questions for JEE Main exams here. These problems on the definition of the differential equation, the order of a differential equation, the degree of a differential equation, general solution, variable separable method, homogeneous differential equation and linear differential equation. The differential equations questions from the previous years of JEE Main are present in this page, along with a detailed solution for each question. These questions include all the important topics and formulae.

Question 1: The equation of the curve which passes through the point (1, 1) and whose slope is given by 2y / x, is ___________.

Solution:

Slope dy / dx = 2y / x

2 ∫dx / x =∫dy / y

2 log x = log y + log c

x^2 = yc

Since it passes through (1, 1), therefore, c = 1.

Hence, $x^2 - y = 0$

$y = x^2$.

Question 2: Equation of curve through the point (1, 0) which satisfies the differential equation $(1 + y^2)$ dx − xy dy = 0, is ________.

Solution:

We have [dx / x] = [y dy] / $[1 + y^2]$

Integrating, we get log |x| = ([1 / 2] * log $[1 + y^2]$) + log c

|x| = c √$(1 + y^2)$−−−−−−−

But it passes through (1, 0), so we get c = 1

Therefore, the solution is $x^2 = y^2 + 1$ or $x^2 - y^2 = 1$.

Question 3: A function $y = f(x)$ has the second-order derivatives $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is __________.

Solution:

Given $f''(x) = 6(x - 1)$

$f'(x) = 3(x - 1)^2 + c_1$ ..(i)

But at point (2, 1) the line $y = 3x - 5$ is tangent to the graph $y = f(x)$.

Hence, $dy / dx|_{x=2} = 3$ or

$f'(2) = 3$.

Then from (i) $f'(2) = 3(2 - 1)^2 + c_1$

$3 = 3 + c_1$

$c_1 = 0$ i.e.,

$f'(x) = 3(x - 1)^2$

Given $f(2) = 1$

$f(x) = (x - 1)^3 + c_2$

$f(2) = 1 + c_2$

$1 = 1 + c_2$

$c_2 = 0$

Hence, $f(x) = (x - 1)^3$.

Question 4: An integrating factor for the differential equation $(1 + y^2)\,dx - (\tan^{-1} y - x)\,dy = 0$

Solution:

$(1 + y^2)\,dx - (\tan^{-1} y - x)\,dy = 0$

$dy / dx = (1 + y^2) / (\tan^{-1} y - x)$

$dx / dy = [\tan^{-1} y / 1 + y^2] - [x / 1 + y^2]$

$[dx / dy] + [x / 1 + y^2] = [\tan^{-1} y] / [1 + y^2]$

This is equation of the form $dx / dy + Px = Q$

So, I.F. = $e^{\int P dy}$

$= e^{\int 1 / 1 + y2.dy}$

$= e^{\tan{-1}y}$

Question 5: If x dy = y (dx + y dy), y > 0 and y (1) = 1, then y (−3) is equal to _________.

Solution:

x dy = y (dx + y dy)

[x dy − ydx] / [y^2] = dy

−d (x / y) = dy

Integrating both sides, we get x / y + y = c   [Because y (1) = 1 ⇒ c = 2; Hence xy + y = 2   ]

For x = −3,

$y^2 - 2y - 3 = 0$

⇒ y = −1 or 3

⇒y = 3   (Because y > 0)

Question 6: ($x^2 + y^2$) dy = xy dx. If y (x_0) = e, y (1) = 1, then the value of x_0 = __________.

Solution:

($x^2 + y^2$) dy = xy dx

x (x dy − y dx) = $-y^2$ dy

[x * (y dx − x dy)] / y^2 = dy

[x / y] d (x / y) = dy / y

Integrating, $x^2 / 2y^2$ = loge y + c

Given y (1) = 1

c = 1 / 2

$x^2 / 2y^2$ = loge y + 1 / 2

Now y (x_0) = e

$x_0^2 / 2e^2$ − loge e − 1 / 2 = 0

$x_0^2 = 3e^2$

$x_0 = \pm \sqrt{3}e$

Question 7: Solution of differential equation $2xy\ dy / dx = x^2 + 3y^2$ is __________.

Solution:

It is homogeneous equation $dy / dx = [x^2 + 3y^2] / 2xy$

Put $y = vx$ and $dy / dx = v + x * [dv / dx]$

So, we get $x * [dv / dx] = [1 + v^2] / 2v$

$2v\ dv / 1 + v^2 = dx / x$

On integrating, we get

$x^2 + y^2 = px^3$. (where p is a constant)

Question 8: The general solution of the differential equation $(x + y)\ dx + x\ dy = 0$ is _______.

Solution:

$(x + y)\ dx + x\ dy = 0$

$x\ dy = -(x + y)\ dx$

$dy / dx = -[x + y] / x$

It is homogeneous equation, hence, put $y = vx$ and $dy / dx = v + x [dv / dx]$, we get

$v + x\ dv / dx = -[x + vx] / x = -[1 + v] / 1$

$x\ dv / dx = -1 - 2v$

$\int dv / [1 + 2v] = -\int dx / x$

$[1 / 2] \log (1 + 2v) = -\log x + \log c$

$\log (1 + 2 [y / x]) = 2 \log [c / x]$

$[x + 2y] / x = (c / x)^2$

$x^2 + 2xy = c$.

Question 9: If y′= [x − y] / [x + y], then its solution is _____________.

Solution:

Given dy / dx = x −y / x + y.

Put y = vx

dy / dx = v + x * [dv / dx]

v + x * [dv / dx] = [x − vx] / [x + vx]

v + x [dv / dx] = [1 − v] / [1 + v]

$[1 + v] / [2 - (1 + v)^2]$ dv = dx / x

Integrating both sides, $\int[1 + v] / [2 - (1 + v)^2]\, dv = \int dx / x$

Put $(1 + v)^2 = t$

⇒ 2 (1 + v) dv = dt

$[1 / 2] \int dt / [2 - t] = \int dx / x$

[− 1 / 2] log (2 − t) = log xc

$[-1 / 2] \log [2 - (1 + v)^2] = \log xc$

$[-1 / 2] \log [-v^2 - 2v + 1] = \log xc$

$\log 1 / \sqrt{[1 - 2v - v^2]} = \log xc$

$x^2c^2 (1 - 2v - v^2) = 1$

$y^2 + 2xy - x^2 = c_1$.

Question 10: What is the general solution of the differential equation (2x − y + 1) dx + (2y − x + 1) dy = 0?

Solution:

(2x −y + 1) dx + (2y − x + 1) dy = 0

dy / dx = 2x − y + 1x − 2y −1, put x = X + h, y = Y + k

dY / dX = [2X − Y + 2h − k + 1] / [X − 2Y + h −2k − 1]

2h − k + 1 = 0

h − 2k − 1 = 0

On solving h = −1, k = −1;

$dY/dX = [2X - Y] / [X - 2Y]$

Put $Y = vX$;

$dY / dX = v + [X\, dv / dX]$

$v + [X\, dv / dX] = [2X - vX] / [X - 2vX] = [2 - v] / [1 - 2v]$

$X\, dv / dX = [2 - 2v + 2v^2] / [1 - 2v] = 2 (v^2 - v + 1) / [1 - 2v]$

$dX / X = (1 - 2v) / 2 (v^2 - v + 1)\, dv$

Put $v^2 - v + 1 = t$

$(2v - 1)\, dv = dt$

$dX / X = -dt / 2t$

$\log X = \log t^{-1/2} + \log c$

$X = t^{-1/2} c$

$X = (v^2 - v + 1)^{-1/2} * c$

$X^2 (v^2 - v + 1) = \text{constant}$

$(x + 1)^2 (([y + 1)^2 / (x + 1)^2] - [(y+1) / (x + 1)] + 1) = \text{constant}$

$(y + 1)^2 - (y + 1) (x + 1) + (x + 1)^2 = c$

$y^2 + x^2 - xy + x + y = c$

Question 11: The solution of the differential equation $dy / dx = 1 + x + y + xy$ is __________.

Solution:

$dy / dx = 1 + x + y + xy$

$dy / dx = (1 + x) (1 + y)$

$dy / dx + \sin ([x + y] / 2)$

$= \sin ([x - y] / 2)$

On integrating, we get

$\log (1 + y) = x^2 / 2 + x + c$

Question 12: The differential equation for all the straight lines, which are at a unit distance from the origin is ___________.

Solution:

Since the equation of lines whose distance from the origin is unity, is given by

$x \cos\alpha + y \sin\alpha = 1$ .....(i)

Differentiate w.r.t. x, we get $\cos \alpha + [dy / dx] \sin \alpha = 0$ .....(ii)

On eliminating the 'α'with the help of (i) and (ii)

$\sin \alpha (y - x [dy / dx]) = 1$

$(y - x [dy / dx]) = \operatorname{cosec} \alpha$ .....(iii)

$dy / dx = -\cot \alpha$

$(dy / dx)^2 = \cot 2\alpha$ .....(iv)

Therefore, by (iii) and (iv),

$1 + (dy / dx)^2 = (y - x [dy / dx])^2$

Question 13: The differential equation corresponding to primitive $y = e^{cx}$ is, or the elimination of the arbitrary constant m from the equation $y = e^{mx}$ gives the differential equation ________.

Solution:

$y = e^{mx}$

$\log y = mx \Rightarrow m = [\log y] / x$

Now $y = e^{mx}$

$dy / dx = me^{mx}$

$= ([\log y] / x) * y$

$= (y / x) \log y$

Question 14: The differential equation of all parabolas whose axes are parallel to y-axis is

_________________.

Solution:

The equation of a member of the family of parabolas having an axis parallel to y-axis is

$y = Ax^2 + Bx + C$ .....(i) where A, B, C are arbitrary constants.

Differentiating (i) w.r.t. x, we get

$dy / dx = 2Ax + B$ .....(ii) which on differentiating w.r.t. x gives

$d^2y / dx^2 = 2A$ .....(iii)

Differentiating w.r.t. x again, we get

$d^3y / dx^3 = 0$.

Question 15: Order of the differential equation of the family of all concentric circles centred at (h, k) is

_____________.

Solution:

$(x - h)^2 + (y - k)^2 = r^2$

Here r is arbitrary constant.

Order of differential equation = 1.