In set theory, Sets, relations and functions are three different concepts but equally important for JEE Main preparation. The questions from the previous years of JEE Main from this topic are present in this page, along with a detailed solution for each question. These questions include all the important concepts and formulae. Students can expect 2-3 questions from this chapter in the JEE examination.

Question 1: If A = [(x, y) : $x^2 + y^2 = 25$] and B = [(x, y) : $x^2 + 9y^2 = 144$], then A ∩ B contains _______ points.

Solution:

A = Set of all values (x, y) : $x^2 + y^2 = 25 = 5^2$

B = $[x^2 / 144] + [y^2 / 16] = 1$

i.e., $[x^2 / (12)^2] + [y^2 / (4)^2] = 1$.

Clearly, A ∩ B consists of four points.

Question 2: In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is ________.

Solution:

Let the number of newspapers be x.

If every student reads one newspaper, the number of students would be 60x

Since every student reads 5 newspapers, the number of students = [x * 60] / [5] = 300

x = 25

Question 3: Let R be the relation on the set R of all real numbers defined by a R b if and only if |a – b| ≤ 1. Then R is __________.

Solution:

$|a - a| = 0 < 1$

Therefore, a R a ∀ a ∈ R

Therefore, R is reflexive.

Again a R b, $|a - b| \le 1 \Rightarrow |b - a| \le 1 \Rightarrow$ b R a

Therefore, R is symmetric.

Again 1 R [½] and [½] R1 but [½] $\neq$ 1

Therefore, R is not anti-symmetric.

Further, 1 R 2 and 2 R 3 but [1 / R3], [Because, |1 − 3| = 2 > 1]

Hence, R is not transitive.

Question 4: Let a relation R be defined by R = {(4, 5); (1, 4); (4, 6); (7, 6); (3, 7)} then R^{-1} o R is ________.

Solution:

First find R^{-1},

R^{-1} = {(5, 4) ; (4, 1) ; (6, 4) ; (6, 7) ; (7, 3)}.

Obtain the elements of R^{-1} o R.

Pick the element of R and then of R^{-1}.

Since (4, 5) ∈ R and (5, 4) ∈ R^{-1}, we have (4, 4) ∈ R^{-1} o R

Similarly, (1, 4) ∈ R, (4, 1) ∈ R^{-1} ⇒ (1, 1) ∈ R^{-1} o R

(4, 6) ∈ R, (6, 4) ∈ R^{-1} ⇒ (4, 4) ∈ R^{-1} o R,

(4, 6) ∈ R, (6, 7) ∈ R^{-1} ⇒ (4, 7) ∈ R^{-1} o R

(7, 6) ∈ R, (6, 4) ∈ R^{-1} ⇒ (7, 4) ∈ R^{-1} o R,

(7, 6) ∈ R, (6, 7) ∈ R^{-1} ⇒ (7, 7) ∈ R^{-1} o R

(3, 7) ∈ R, (7, 3) ∈ R^{-1} ⇒ (3, 3) ∈ R^{-1} o R,

Hence, R^{-1} o R = {(1, 1); (4, 4); (4, 7); (7, 4), (7, 7); (3, 3)}.

Question 5: If f (x) = [x − 3] / [x + 1], then f [f { f (x) }] equals ________.

Solution:

f [f (x)] = f (x) − 3 f (x) + 1

= {([x − 3] / [x + 1]) − 3} / {([x − 3] / [x + 1]) + 1}

= [x − 3 − 3x − 3] / [x − 3 + x + 1]

= [3 + x] / [1 − x]

f (x) = (2, 4] − {3}.

Now f [f { f (x) }] = f ([3 + x] / [1 − x])

Question 6: If f (x) = cos (log x), then find the value of f (x) * f (4) − [1 / 2] * [f (x / 4) + f (4x)].

Solution:

f (x) = cos (log x)

Now let y = f (x) * f (4) − [1 / 2] * [f (x / 4) + f (4x)]

y = cos (log x) * cos (log 4) − [1 / 2] * [cos log (x / 4) + cos (log 4x)]

y = cos (log x) cos (log 4) − [1 / 2] * [cos (log x −log 4) + cos (log x + log 4)]

y = cos (log x) cos (log 4) − [1 / 2] * [2 cos (log x) cos (log 4)]

y = 0

Question 7: Let f : R → R be defined by f (x) = 2x + |x|, then f (2x) + f (−x) − f (x) = ________.

Solution:

f (2x) = 2 (2x) + |2x| = 4x + 2 |x|,

y = x^2 + 1,

f (x) = 2x + |x|

f (2x) + f (−x) − f (x) = 4x + 2 |x| + |x| −2x − 2x − |x|

= 2|x|

Question 8: If f (x) = cos [π^2] x + cos[$-\pi^2$] x, then find the function of the angle.

Solution:

f (x) = cos [π^2] x + cos[$-\pi^2$] x

f (x) = cos (9x) + cos (−10x)

= cos (9x) + cos (10x)

= 2 cos (19x / 2) cos (x / 2)

f (π / 2) = 2 cos (19π / 4) cos (π / 4);

f (π / 2) = 2 * −1 / √2 * 1/ √2

= −1

Question 9: If f (x) = $[x^2 - 1] / [x^2 + 1]$, for every real number. Then what is the minimum value of f?

Solution:

Let f (x) = $[x^2 - 1] / [x^2 + 1]$

= $[x^2 + 1 - 2] / [x^2 + 1]$

= $1 - (2 / [x^2 + 1])$ [Because $[x^2 + 1] > 1$ also $(2 / [x^2 + 1]) \leq 2$]

So $1 - [2 / [x^2 + 1]] \geq 1 - 2$;

$-1 \leq f(x) < 1$

Thus, f (x) has the minimum value equal to 1.

Question 10: The function f : R → R defined by f (x) = e^x is ________.

Solution:

Function f: R → R is defined by f (x) = e^x.

Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$ or $e^{x1} = e^{x2}$ or $x_1 = x_2$.

Therefore, f is one-one.

Let f (x) = e^x = y.

Taking log on both sides, we get x = logy.

We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number.

Therefore, function f is into.

Question 11: If f: R → S defined by f (x) = sinx − √3 cosx + 1 is onto, then what is the interval of S?

Solution:

$-\sqrt{[1 + (\sqrt{-3})^2]} \leq (\sin x - \sqrt{3}\cos x) \leq \sqrt{[1 + (\sqrt{-3})^2]}$

$-2 \leq (\sin x - \sqrt{3}\cos x) \leq 2$

$-2 + 1 \leq (\sin x - \sqrt{3}\cos x + 1) \leq 2 + 1$

$-1 \leq (\sin x - \sqrt{3}\cos x + 1) \leq 3$ i.e.,

range = [−1, 3]

For f to be onto S = [−1, 3].

Question 12: What is the domain of the function

$$f(x) = \frac{\sin^{-1}(3 - x)}{\log(|x| - 2)}$$

Solution:

Let $g(x) = \sin^{-1}(3 - x)$

$-1 \le 3 - x \le 1$

Domain of g(x) is [2, 4] and let $h(x) = \log[|x| - 2]$

$|x| - 2 > 0$

$|x| > 2$

$x < -2$ or $x > 2$

$(-\infty, -2) \cup (2, \infty)$

We know that, $(f / g)(x) = f(x) / g(x)\ \forall\ x \in D1 \cap D2 - \{x \in R : g(x) = 0\}$

Domain of $f(x) = (2, 4] - \{3\} = (2, 3) \cup (3, 4]$.

Question 13: If f (x) = a cos (bx + c) + d, then what is the range of f (x)?

Solution:

$f(x) = a\cos(bx + c) + d$ ..(i)

For minimum $\cos(bx + c) = -1$

From (i), $f(x) = -a + d = (d - a)$

For maximum $\cos(bx + c) = 1$

From (i), $f(x) = a + d = (d + a)$

Range of $f(x) = [d - a, d + a]$

Question 14: The function f: R → R is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$, then what is f (R)?

Solution:

$f(x) = \cos^2 x + \sin^4 x$

$y = f(x) = \cos^2 x + \sin^2 x(1 - \cos^2 x)$

$y = \cos^2 x + \sin^2 x - \sin^2 x \cos^2 x$

$y = 1 - \sin^2 x \cos^2 x$

$y = 1 - [1 / 4] * [\sin^2 2x]$

$3 / 4 \leq f(x) \leq 1$, (Because $0 \leq \sin^2 2x \leq 1$)

$f(R) \in [3/4, 1]$

Question 15: If f (x) = 3x − 5, then $f^{-1}(x)$ is _____________.

Solution:

Let $f(x) = y \Rightarrow x = f^{-1}(y)$.

Hence, $f(x) = y = 3x - 5$

$\Rightarrow x = [y + 5] / [3]$

$\Rightarrow f^{-1}(y) = x = [y + 5] / [3]$

$f^{-1}(x) = [x + 5] / [3]$

Also, f is one-one and onto, so f^{-1} exists and is given by $f^{-1}(x) = [x + 5] / [3]$.