DAY — **08** SEAT NUMBER | | | | | | |

2019	III	02	1100	J - 60	(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. **(7 Pages)** **Max. Marks : 80**

Note : *(1) All questions are compulsory.*

(2) Figures to the right indicate full marks.

*(3) The question paper consists of **30** questions divided into **FOUR** sections **A, B, C, D**.*

- ***Section A** contains **6** questions of **1 mark** each.*
- ***Section B** contains **8** questions of **2 marks** each. (One of them has internal option)*
- ***Section C** contains **6** questions of **3 marks** each. (Two of them have internal options)*
- ***Section D** contains **10** questions of **4 marks** each. (Three of them have internal options)*

*(4) For each **MCQ**, correct answer must be written along with its **alphabet**,*

e.g. (a).......... / (b).......... / (c).......... /(d)..........etc.

*In case of **MCQ** (Q. No. **1**. to **6**.) evaluation would be done for the first attempt only.*

*(5) Use of logarithmic table is allowed. Use of calculator is **not** allowed.*

*(6) In L. P. P. only rough sketch of graph is expected. Graph paper is **not** necessary.*

(7) Start each section on new page only.

SECTION – A

Select and write the most appropriate answer from the given alternatives for each question : [6]

Q. 1. The principal solutions of $\cot x = -\sqrt{3}$ are ______ . (1)

(a) $\frac{\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{5\pi}{6}, \frac{7\pi}{6}$

(c) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (d) $\frac{\pi}{6}, \frac{11\pi}{6}$

Q. 2. The acute angle between the two planes $x + y + 2z = 3$ and $3x - 2y + 2z = 7$ is ______ . (1)

(a) $\sin^{-1}\left(\frac{5}{\sqrt{102}}\right)$ (b) $\cos^{-1}\left(\frac{5}{\sqrt{102}}\right)$

(c) $\sin^{-1}\left(\frac{15}{\sqrt{102}}\right)$ (d) $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$

Q. 3. The direction ratios of the line which is perpendicular to the lines with direction ratios $-1, 2, 2$ and $0, 2, 1$ are ______. (1)

(a) $-2, -1, -2$ (b) $2, 1, 2$

(c) $2, -1, -2$ (d) $-2, 1, -2$

Q. 4. If $f(x) = (1+2x)^{\frac{1}{x}}$, for $x \neq 0$ is continuous at $x = 0$, then $f(0) =$ ______. (1)

(a) e (b) e^2

(c) 0 (d) 2

Q. 5. $\int \frac{dx}{9x^2+1} =$ ______ . (1)

(a) $\frac{1}{3}\tan^{-1}(2x)+c$ (b) $\frac{1}{3}\tan^{-1}x+c$

(c) $\frac{1}{3}\tan^{-1}(3x)+c$ (d) $\frac{1}{3}\tan^{-1}(6x)+c$

Q. 6. If $y = ae^{5x} + be^{-5x}$, then the differential equation is ______. (1)

(a) $\frac{d^2y}{dx^2} = 25y$ (b) $\frac{d^2y}{dx^2} = -25y$

(c) $\frac{d^2y}{dx^2} = -5y$ (d) $\frac{d^2y}{dx^2} = 5y$

SECTION – B

Q. 7. Write the truth values of the following statements : **[16]**

(i) 2 is a rational number and $\sqrt{2}$ is an irrational number.

(ii) $2+3=5$ or $\sqrt{2}+\sqrt{3}=\sqrt{5}$

(2)

Q. 8. Find the volume of the parallelopiped, if the coterminus edges are given by the vectors $2\hat{i}+5\hat{j}-4\hat{k}$, $5\hat{i}+7\hat{j}+5\hat{k}$, $4\hat{i}+5\hat{j}-2\hat{k}$.

(2)

OR

Find the value of p, if the vectors $\hat{i}-2\hat{j}+\hat{k}$, $2\hat{i}-5\hat{j}+p\hat{k}$ and $5\hat{i}-9\hat{j}+4\hat{k}$ are coplanar.

Q. 9. Show that the points $A(-7, 4, -2)$, $B(-2, 1, 0)$ and $C(3, -2, 2)$ are collinear.

(2)

Q. 10. Write the equation of the plane $3x+4y-2z=5$ in the vector form.

(2)

0	0	6	0

(2)

Q. 11. If $y = x^x$, find $\frac{dy}{dx}$.

Q. 12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at $(-1, -2)$. (2)

Q. 13. Evaluate : $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ (2)

Q. 14. Evaluate : $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ (2)

SECTION – C

[18]

Q. 15. In ΔABC, prove that

$$\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right) \cos\left(\frac{A}{2}\right)$$ (3)

OR

Show that $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

Q. 16. If $A(\bar{a})$ and $B(\bar{b})$ are any two points in the space and $R(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio m : n, then prove that

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$$ (3)

Q. 17. The equation of a line is $2x - 2 = 3y + 1 = 6z - 2$, find its direction ratios and also find the vector equation of the line. (3)

Q. 18. Discuss the continuity of the function

$$f(x)=\frac{\log(2+x)-\log(2-x)}{\tan x}, \text{ for } x\neq 0$$

$$=1 \qquad \text{for } x=0$$

at the point $x=0$ (3)

Q. 19. The probability distribution of a random variable X, the number of defects per 10 meters of a fabric is given by

x	0	1	2	3	4
$P(X=x)$	0·45	0·35	0·15	0·03	0·02

Find the variance of X. (3)

OR

For the following probability density function (p. d. f.) of X, find : (i) $P(X<1)$, (ii) $P(|X|<1)$

$$\text{if } f(x)=\frac{x^2}{18}, \quad -3<x<3$$

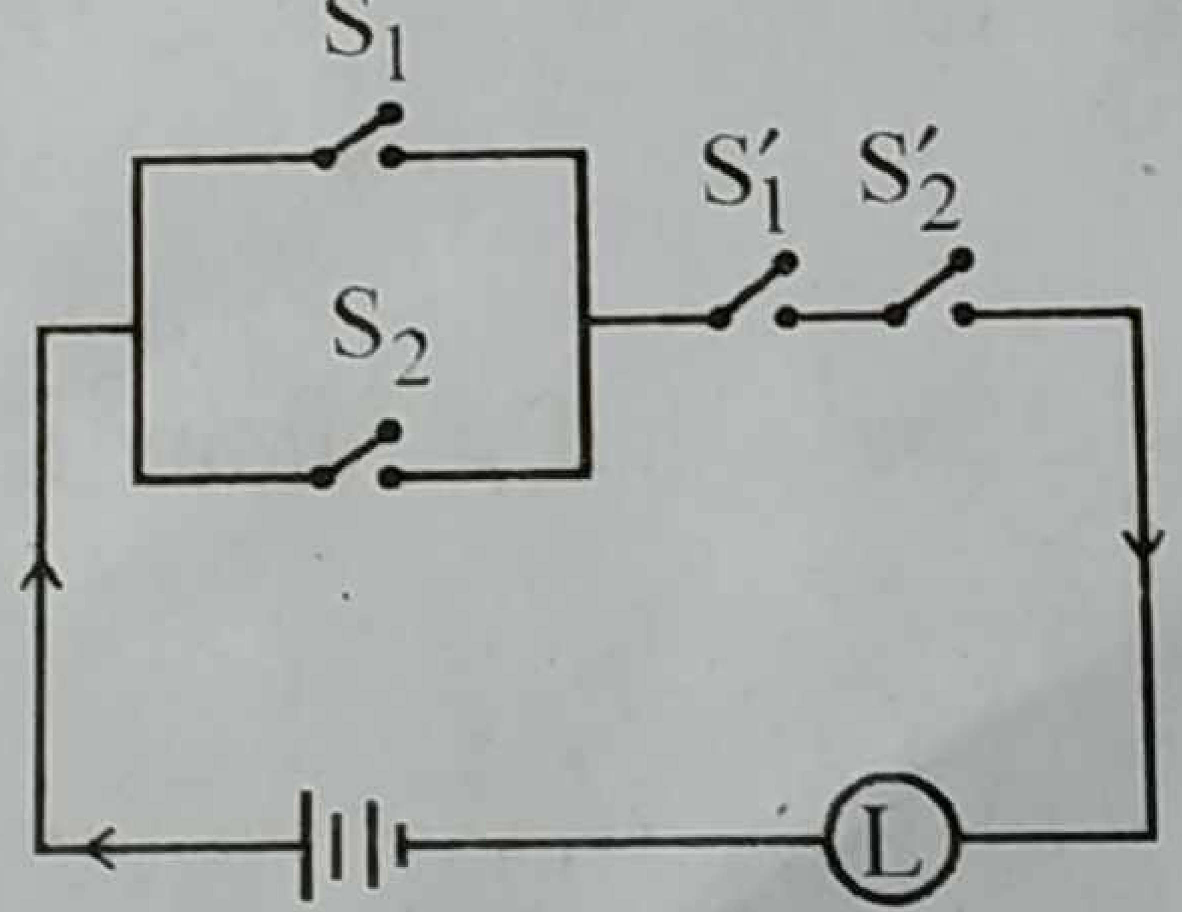
$$=0 \;, \quad \text{otherwise}$$

Q. 20. Given is $X \sim B(n,p)$.

If $E(X)=6$, Var. $(X)=4\cdot 2$, find n and p. (3)

SECTION – D

Q. 21. Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result. **[40]**



(4)

Q. 22. If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices. (4)

Q. 23. In ΔABC, with usual notations prove that

$$b^2 = c^2 + a^2 - 2ca\cos B$$ (4)

OR

In ΔABC, with usual notations prove that

$$(a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right) = c^2.$$

Q. 24. Find 'p' and 'q' if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of perpendicular lines. (4)

Q. 25. Maximize : $z = 3x + 5y$ Subject to

$x + 4y \le 24$, $3x + y \le 21$,

$x + y \le 9$, $x \ge 0, y \ge 0$ (4)

Q. 26. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t, then prove that y is a differentiable function of x and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{ where } \frac{dx}{dt} \ne 0$$

Hence find $\frac{dy}{dx}$ if $x = a\cos^2 t$ and $y = a\sin^2 t$. (4)

Q. 27. $f(x)=(x-1)(x-2)(x-3),\ x\in[0,4]$, find '$c$' if LMVT can be applied. (4)

OR

A rod of 108 meters long is bent to form a rectangle. Find its dimensions if the area is maximum.

Q. 28. Prove that : $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|+c$ (4)

Q. 29. Show that : $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$ (4)

Q. 30. Solve the differential equation : (4)

$$\frac{dy}{dx}+y\sec x=\tan x$$

OR

Solve the differential equation :

$$(x+y)\frac{dy}{dx}=1$$

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