## EXERCISE 16.1 PAGE: 386

**In each of the following Exercises 1 to 7, describe the sample space for the indicated experiment.**

**1. A coin is tossed three times.**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), the possible outcomes may be
So, when 1 coin is tossed once the sample space = 2
Then,
Coin is tossed 3 times the sample space = $2^3 = 8$
Thus, the sample space is S = {HHH, THH, HTH, HHT, TTT, HTT, THT, TTH}

**2. A die is thrown two times.**

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
Then, the total number of sample space = (6 × 6)
= 36
Thus, the sample space is
S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3)(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

**3. A coin is tossed four times.**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
So, when 1 coin is tossed once the sample space = 2
Then,
Coin is tossed 3 times the sample space = $2^4 = 16$
Thus, the sample space is S = {HHHH, THHH, HTHH, HHTH, HHHT, TTTT, HTTT, THTT, TTHT, TTTH, TTHH, HHTT, THTH, HTHT, THHT, HTTH}

**4. A coin is tossed and a die is thrown.**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers comes when the die is thrown.
Then, total number of space = (2 × 6) = 12
Thus, the sample space is,

S={(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}

**5. A coin is tossed and then a die is rolled only in case a head is shown on the coin.**
**Solution:-**
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers comes when the die is thrown.
When head in encountered,
Then, number of space = (1 × 6) = 6
Sample Space $S_H$= {H1, H2, H3, H4, H5, H6}
Now, tail is encountered, Sample space $S_T$= {T}
Therefore the total sample space S = {H1, H2, H3, H4, H5, H6, T}

**6. 2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.**
**Solution:-**
From the question it is given that,
2 boys and 2 girls are in Room X
1 boy and 3 girls in Room Y
Let us assume b1, b2 and g1, g2 be 2 boys and 2 girls are in Room X.
And also assume b3 and g3, g4, g5 be 1 boy and 3 girls in Room Y.
The problem is solved by dividing into two cases
Case 1: Room X is selected
Sample Space $S_x$ = {(X,b1),(X,b2),(X,g1),(X,g2)}
Case 2: Room Y is selected
Sample Space $S_y$ ={(Y,b3),(Y,g3),(Y,g4),(Y,g5)}
The overall sample space
S={(X,b1),(X,b2),(X,g1),(X,g2),(Y,b3),(Y,g3),(Y,g4),(Y,g5)}

**7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.**
**Solution:-**
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers comes when the die is thrown.
And also assume die of red colour be 'R', die of white colour be 'W', die of blue colour be 'B'.

So, the total number of sample space = (6 × 3) = 18
The sample space of the event is
S={(R,1),(R,2),(R,3),(R,4),(R,5),(R,6),(W,1),(W,2),(W,3),(W,4),(W,5),(W,6)
(B,1),(B,2),(B,3),(B,4),(B,5),(B,6)}

**8. An experiment consists of recording boy–girl composition of families with 2 children.**
**(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?**
**(ii) What is the sample space if we are interested in the number of girls in the family?**
**Solution:-**
Let us assume boy be 'B' and girl be 'G'
(i) The sample space if we are interested in knowing whether it is a boy or girl in the order of their births. S = {GG, BB, GB, BG}
(ii) The sample space if we are interested in the number of girls in the family when there are two child in the family then,
Sample space S = {2, 1, 0}

**9. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.**
**Solution:-**
From the question it is given that, a box contains 1 red and 3 identical white balls.
Let us assume 'R' be the event of red ball is drawn and 'W' be the event of white ball is drawn.
Given in the question that white balls are identical, therefore the event of drawing any one of the three white ball is same.
Then, total number of sample space = $(2^2 - 1) = 3$
∴Sample space S = {WW, WR, RW}

**10. An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.**
**Solution:-**
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us take,
Case 1: Head is encountered
Sample space $S_1$ = {HT, HH}

Case 2: Tail is encountered
Sample Space $S_2$ = {(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}
Then,
The Overall Sample space
S = {(HT), (HH), (T1), (T2), (T3), (T4), (T5), (T6)}

**11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non – defective (N). Write the sample space of this experiment.**

**Solution:-**

From the question,
'D' denotes the event of bulb is defective and 'N' denotes event of non-defective bulbs.
Then,
Total number of Sample space = 2 × 2 × 2 = 8
Thus, Sample space S = {DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

**12. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
The problem can be solved by dividing it into 3 cases
Case 1: The outcome is Head and the corresponding number on the die shows Odd number
Total number of sample space = (1 × 3) = 3
Sample space $S_{HO}$ = {(H,1), (H,3), (H,5)}
Case 2: The outcome is Head and the corresponding number on the die shows Even number
Total number of sample space = (1 × 3 × 6) = 18
$S_{HE}$={(H,2,1),(H,2,2),(H,2,3),(H,2,4),(H,2,5),(H,2,6),(H,4,1),(H,4,2),(H,4,3),(H,2,4),(H,4,5),(H,4,6), (H,6,1),(H,6,2),(H,6,3),(H,6,4),(H,6,5),(H,6,6)}
Case 3: The outcome is Tail
Total number of sample space=1
Sample space $S_T$ = {(T)}
The overall sample spaces
S={(H,1),(H,3),(H,5),
(H,2,1),(H,2,2),(H,2,3),(H,2,4),(H,2,5),(H,2,6),(H,4,1),(H,4,2),(H,4,3),(H,2,4),(H,4,5),

(H,4,6), (H,6,1),(H,6,2),(H,6,3),(H,6,4),(H,6,5),(H,6,6),(T)}

**13. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment**

**Solution:-**

From the question it is given that, 1, 2, 3, 4 are the numbers written on the four slip.
When two slips are drawn without replacement the first event has 4 possible outcomes and the second event has 3 possible outcomes because 1 slip is already picked.
Therefore, the total number of possible outcomes = (4 × 3) = 12
Thus sample space,
S={(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)}

**14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
The following problem can be divided in two cases
(i) The number on the die is even.
The sample space $S_E$={(2,H),(4,H),(6,H),(2,T),(4,T),(6,T)}
(ii) The number on the die is odd and the coin is tossed twice.
The sample space
$S_o$ = {(1,H,H), (3,H,H), (5,H,H), (1,H,T), (3,H,T), (5,H,T), (1,T,H), (3,T,H), (5,T,H), (1,T,T), (3,T,T), (5,T,T)}
Hence, the overall sample space for the problem= $S_E + S_o$
S = {(2,H), (4,H), (6,H), (2,T), (4,T), (6,T), (1,H,H), (3,H,H), (5,H,H), (1,H,T), (3,H,T), (5,H,T), (1,T,H), (3,T,H), (5,T,H), (1,T,T), (3,T,T), (5,T,T)}

**15. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume $R_1$, $R_2$ denote the event the red balls are drawn and $B_1$, $B_2$, $B_3$ denote the events black ball are drawn.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
(i) Coin shows Tail.

So, the sample space $S_T$ = {($TR_1$), ($TR_2$), ($TB_1$), ($TB_2$), ($TB_3$)}

(ii) Coin shows head.

So, the sample space $S_H$ = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)}

Hence, the overall sample space for the problem= $S_T + S_H$

S = {( $T,R_1$), ($T,R_2$), ($T,B_1$), ($T,B_2$), ($T,B_3$), (H,1), (H,2), (H,3), (H,4), (H,5), (H,6)}

**16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?**

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
As per the condition given in the question, a die is thrown repeatedly until a six comes up.
If six may come up for the first throw or six may come up on second throw this process will goes continuously until the six comes.
The sample space when 6 comes on very first throw $S_1$ = {6}
The sample space when 6 comes on second throw $S_2$ = {(1,6), (2,6), (3,6), (4,6), (5,6)}
This event can go on for infinite times.
So, the sample space is infinitely defined

S = {(6), (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6)......}

## EXERCISE 16.2 PAGE: 393

**1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?**

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
So, S = (1, 2, 3, 4, 5, 6)
As per the conditions given the question
E be the event "die shows 4"
E = (4)
F be the event "die shows even number"
F = (2, 4, 6)
$E \cap F = (4) \cap (2, 4, 6)$
$= 4$
$4 \neq \phi$ … [because there is a common element in E and F]
Therefore E and F are not mutually exclusive event.

**2. A die is thrown. Describe the following events:**
**(i) A: a number less than 7**
**(ii) B: a number greater than 7**
**(iii) C: a multiple of 3**
**(iv) D: a number less than 4**
**(v) E: an even number greater than 4**
**(vi) F: a number not less than 3**
**Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F^{I}$, $F^{I}$**

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
So, S = (1, 2, 3, 4, 5, 6)
As per the conditions given in the question,
(i) A: a number less than 7
All the numbers in the die are less than 7,
A = (1, 2, 3, 4, 5, 6)

(ii) B: a number greater than 7
There is no number greater than 7 on the die
Then,
B= ($\phi$)

(iii) C: a multiple of 3
There are only two numbers which are multiple of 3.
Then,

C= (3, 6)

(iv) D: a number less than 4
D= (1, 2, 3)

(v) E: an even number greater than 4
E = (6)

(vi) F: a number not less than 3
F= (3, 4, 5, 6)

Also we have to find, A U B, A ∩ B, B U C, E ∩ F, D ∩ E, D - E, A - C, E ∩ F', F'
So,
A ∩ B = (1, 2, 3, 4, 5, 6) ∩ (ϕ)
= (ϕ)

B U C = (ϕ) U (3, 6)
= (3, 6)

E ∩ F = (6) ∩ (3, 4, 5, 6)
= (6)

D ∩ E = (1, 2, 3) ∩ (6)
= (ϕ)

D - E = (1, 2, 3) - (6)
= (1, 2, 3)

A - C = (1, 2, 3, 4, 5, 6) - (3, 6)
= (1, 2, 4, 5)

F' = S – F
= (1, 2, 3, 4, 5, 6) – (3, 4, 5, 6)
= (1, 2)

E ∩ F' = (6) ∩ (1, 2)
= (ϕ)

**3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events: A: the sum is greater than 8, B: 2 occurs on either die C: the sum is at least 7 and a multiple of 3. Which pairs of these events are mutually exclusive?**

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
In the question is given that pair of die is thrown, so sample space will be,

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5). (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

A: the sum is greater than 8

$\therefore$ A= $\begin{Bmatrix} (3,6), (4,5), (5,4), (6,3), (4,6), \\ (5,5), (6,4), (5,6), (6,5), (6,6) \end{Bmatrix}$

Possible sum greater than 8 are 9, 10, 11 & 12

B: 2 occurs on either die

B= $\begin{Bmatrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (1,2), (3,2), (4,2), (5,2), (6,2) \end{Bmatrix}$

In this conditions possibilities are there that the number 2 will come on either first die or second die or both the die simultaneously

C: The sum is at least 7 and multiple of 3

C= $\{(3,6), (4,5), (5,4), (6,3), (6,6)\}$

So the sum can be only 9 or 12

Now, we shall find pairs of these events are mutually exclusive or not.

(i) A∩ B = ϕ

Since there is no common element in A and B

Therefore A & B are mutually exclusive

(ii) B ∩ C = ϕ

Since there is no common element between

Therefore B and C are mutually exclusive.

(iii) A ∩ C {(3,6), (4,5), (5,4), (6,3), (6,6)}

⇒ {(3,6), (4,5), (5,4), (6,3), (6,6)} ≠ ϕ

Since A and C has common elements.
Therefore A and C are mutually exclusive.

**4. Three coins are tossed once. Let A denote the event 'three heads show", B denote the event "two heads and one tail show", C denote the event" three tails show and D denote the event 'a head shows on the first coin". Which events are**
**(i) Mutually exclusive? (ii) Simple? (iii) Compound?**
**Solution:-**
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
But, now three coins are tossed once so the possible sample space contains,
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTH}
Now,
A: 'three heads'
A= (HHH)
B: "two heads and one tail"
B= (HHT, THH, HTH)
C: 'three tails'
C= (TTT)
D: a head shows on the first coin
D= (HHH, HHT, HTH, HTT)
(i) Mutually exclusive
A ∩ B = (HHH) ∩ (HHT, THH, HTH)
= ϕ
Therefore, A and C are mutually exclusive.
A ∩ C = (HHH) ∩ (TTT)
= ϕ
There, A and C are mutually exclusive.
A ∩ D = (HHH) ∩ (HHH, HHT, HTH, HTT)
= (HHH)
A ∩ D ≠ ϕ
So they are not mutually exclusive
B ∩ C = (HHT, HTH, THH) ∩ (TTT)
= ϕ
Since there is no common element in B & C, so they are mutually exclusive.
B ∩ D = (HHT, THH, HTH) ∩ (HHH, HHT, HTH, HTT)
= (HHT, HTH)
B ∩ D ≠ ϕ

Since there are common elements in B & D,
So, they not mutually exclusive.
C ∩ D = (TTT) ∩ (HHH, HHT, HTH, HTT)
= ϕ
Since there is no common element in C & D,
So they are not mutually exclusive.
(ii) Simple event
If an event has only one sample point of a sample space, it is called a simple (or elementary) event.
A = (HHH)
C = (TTT)
Both A & C have only one element,
so they are simple events.
(iii) Compound events
If an event has more than one sample point, it is called a Compound event
B= (HHT, HTH, THH)
D= (HHH, HHT, HTH, HTT)
Both B & D have more than one element,
So, they are compound events.

**5. Three coins are tossed. Describe**
**(i) Two events which are mutually exclusive.**
**(ii) Three events which are mutually exclusive and exhaustive.**
**(iii) Two events, which are not mutually exclusive.**
**(iv) Two events which are mutually exclusive but not exhaustive.**
**(v) Three events which are mutually exclusive but not exhaustive.**
**Solution:-**
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
But, now three coins are tossed once so the possible sample space contains,
S= (HHH, HHT, HTH, HTT, THH,THT, TTH, TTT)
(i) Two events which are mutually exclusive.
Let us assume A be the event of getting only head
A = (HHH)
And also let us assume B be the event of getting only Tail
B = (TTT)
So, A ∩ B = ϕ
Since there is no common element in A& B so these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive

Now,

Let us assume P be the event of getting exactly two tails

P = (HTT, TTH, THT)

Let us assume Q be the event of getting at least two heads

Q = (HHT, HTH, THH, HHH)

Let us assume R be the event of getting only one tail

C= (TTT)

P ∩ Q = (HTT, TTH, THT) ∩ (HHT, HTH, THH, HHH)

= ϕ

Since there is no common element in P and Q,

Therefore, they are mutually exclusive

Q ∩ R = (HHT, HTH, THH, HHH) ∩ (TTT)

= ϕ

Since there is no common element in Q and R

Hence, they are mutually exclusive.

P ∩ R = (HTT, TTH, THT) ∩ (TTT)

= ϕ

Since there is no common element in P and R,

So they are mutually exclusive.

Now, P and Q, Q and R, and P and R are mutually exclusive

∴ P, Q, and R are mutually exclusive.

And also,

P ∪ Q ∪ R = (HTT, TTH, THT, HHT, HTH, THH, HHH, TTT) = S

Hence P, Q and R are exhaustive events.

(iii) Two events, which are not mutually exclusive

Let us assume 'A' be the event of getting at least two heads

A = (HHH, HHT, THH, HTH)

Let us assume 'B' be the event of getting only head

B= (HHH)

Now A ∩ B = (HHH, HHT, THH, HTH) ∩ (HHH)

= (HHH)

A ∩ B ≠ ϕ

Since there is a common element in A and B,

So they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive

Let us assume 'P' be the event of getting only Head

$P = (HHH)$

Let us assume 'Q' be the event of getting only tail

$Q = (TTT)$

$P \cap Q = (HHH) \cap (TTT)$

$= \phi$

Since there is no common element in P and Q,

These are mutually exclusive events.

But,

$P \cup Q = (HHH) \cup (TTT)$

$= \{HHH, TTT\}$

$P \cup Q \neq S$

Since $P \cup Q \neq S$ these are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive

Let us assume 'X' be the event of getting only head

$X = (HHH)$

Let us assume 'Y' be the event of getting only tail

$Y = (TTT)$

Let us assume 'Z' be the event of getting exactly two heads

$Z= (HHT, THH, HTH)$

Now,

$X \cap Y = (HHH) \cap (TTT)$

$= \phi$

$X \cap Z = (HHH) \cap (HHT, THH, HTH)$

$= \phi$

$Y \cap Z = (TTT) \cap (HHT, THH, HTH)$

$= \phi$

Therefore, they are mutually exclusive

Also

$X \cup Y \cup Z = (HHH\ TTT, HHT, THH, HTH)$

$X \cup Y \cup Z \neq S$

So, X, Y and Z are not exhaustive.

Hence it is proved that X, Y and X are mutually exclusive but not exhaustive.

**6. Two dice are thrown. The events A, B and C are as follows:**

**A: getting an even number on the first die.**
**B: getting an odd number on the first die.**
**C: getting the sum of the numbers on the dice ≤ 5.**
**Describe the events**

| | | |
|---|---|---|
| **(i) $A^I$** | **(ii) not B** | **(iii) A or B** |
| **(iv) A and B** | **(v) A but not C** | **(vi) B or C** |
| **(vii) B and C** | **(viii) $A \cap B^I \cap C^I$** | |

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
In the question is given that pair of die is thrown, so sample space will be,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B: getting an odd number on the first die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C: getting the sum of the numbers on the dice ≤ 5

$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$

Then,

$$\text{(i) } A' = \left\{ \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix} \right\} = B$$

(ii) $B' = \left\{\begin{matrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(4,1),(4,2),(4,3),(4,4),(4,5).(4,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\} = A$

(iii) $A \cup B\ (A\ or\ B) = \left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5).(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\} = S$

(iv) A and B (A ∩ B) = ϕ

(v) $A\ but\ not\ C = A - C = \left\{\begin{matrix}(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5).(4,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$

(vi) $B\ or\ C = B \cup C = \left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\(2,1),(2,2),(2,3),\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{matrix}\right\}$

(vii) $B\ and\ C = B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii)

$$C' = \left\{\begin{matrix}(1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2),\\(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \left\{\begin{matrix}(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),\\(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$$

**7. Refer to question 6 above, state true or false: (give reason for your answer)**
**(i) A and B are mutually exclusive**
**(ii) A and B are mutually exclusive and exhaustive**
**(iii) A = B$^{I}$**
**(iv) A and C are mutually exclusive**
**(v) A and B$^{I}$ are mutually exclusive.**

**(vi) $A^{I}$, $B^{I}$, C are mutually exclusive and exhaustive.**
**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
In the question is given that pair of die is thrown, so sample space will be,
By referring the question 6 above,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B: getting an odd number on the first die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$$

(i) A and B are mutually exclusive
So, $(A \cap B) = \phi$
So, A & B are mutually exclusive.
Hence, the given statement is true.

(ii) A and B are mutually exclusive and exhaustive

$$A \cup B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = S$$

$\Rightarrow A \cup B = S$

From statement (i) we have A and B are mutually exclusive.

∴ A and B are mutually exclusive and exhaustive.

Hence, the statement is true.

(iii) A = B

$$B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \end{Bmatrix} = A$$

Therefore, the statement is true.

(iv) A and C are mutually exclusive

We have,

$A \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\}$

$A \cap C \neq \phi$

A and C are not mutually exclusive

Hence, the given statement is false

(v) A and $B^l$ are mutually exclusive.

We have,

$A \cap B^l = A \cap A = A$

$\therefore A \cap B^l \neq \phi$

So, A and $B^l$ not mutually exclusive.

Therefore, the given statement is false.

(vi) $A^l$, $B^l$, C are mutually exclusive and exhaustive.

Here $A^l = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \end{Bmatrix}$

$$B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

And $C$ ={(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)}

$A' \cap B' = \phi$

Hence there is no common element in A' and B'

So they are mutually exclusive.

$B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\}$

$B' \cap C \neq \phi$

They are not mutually exclusive.

Now, since $B'$ and C are not mutually exclusive,

Therefore A', B' and C are not mutually exclusive and exhaustive.

So, the given statement is false.

## EXERCISE 16.3 PAGE: 386

**1. Which of the following cannot be valid assignment of probabilities for outcomes of sample Space S = {$\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$, $\omega_5$, $\omega_6$, $\omega_7$}**

**Assignment**

| Assignment | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ | $\omega_7$ |
|---|---|---|---|---|---|---|---|
| (a) | 0.1 | 0.01 | 0.05 | 0.03 | 0.01 | 0.2 | 0.6 |
| (b) | 1/7 | 1/7 | 1/7 | 1/7 | 1/7 | 1/7 | 1/7 |
| (c) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| (d) | -0.1 | 0.2 | 0.3 | 0.4 | -0.2 | 0.1 | 0.3 |
| (e) | 1/14 | 2/14 | 3/14 | 4/14 | 5/14 | 6/14 | 15/14 |

**Solution:-**

(a) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$

Therefore, the given assignment is valid.

b) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$= (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7)$

$= 7/7$

$= 1$

Therefore, the given assignment is valid.

c) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$

$= 2.8 > 1$

Therefore, the 2nd condition is not satisfied

Which states that $p(w_i) \leq 1$

So, the given assignment is not valid.

d) The conditions of axiomatic approach don't hold true in the given assignment, that is

1) Each of the number $p(w_i)$ is less than zero but also negative

To be true each of the number $p(w_i)$ should be less than zero and positive

So, the assignment is not valid

e) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii):

Sum of probabilities

= (1/14) + (2/14) + (3/14) + (4/14) + (5/14) + (6/14) + (7/14)

= (28/14) ≥ 1

The second condition doesn't hold true so the assignment is not valid.

**2. A coin is tossed twice, what is the probability that at least one tail occurs?**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

Here coin is tossed twice, then sample space is S = (TT, HH, TH, HT)

∴ Number of possible outcomes n (S) = 4

Let A be the event of getting at least one tail

∴ n (A) = 3

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

P(A) = n(A)/n(S)

= ¾

**3. A die is thrown, find the probability of following events:**

**(i) A prime number will appear,**

**(ii) A number greater than or equal to 3 will appear,**

**(iii) A number less than or equal to one will appear,**

**(iv) A number more than 6 will appear,**

**(v) A number less than 6 will appear.**

**Solution:-**

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.

Here S = {1, 2, 3, 4, 5, 6}

∴n(S) = 6

(i) A prime number will appear,

Let us assume 'A' be the event of getting a prime number,

A = {2, 3, 5}

Then, n(A) = 3

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

∴P(A) = n(A)/n(S)

= 3/6

= ½

(ii) A number greater than or equal to 3 will appear,

Let us assume 'B' be the event of getting a number greater than or equal to 3,

B = {3, 4, 5, 6}

Then, n(B) = 4
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(B) = n(B)/n(S)
= 4/6
= 2/3
(iii) A number less than or equal to one will appear,
Let us assume 'C' be the event of getting a number less than or equal to 1,
C = {1}
Then, n (C) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(C) = n(C)/n(S)
= 1/6
(iv) A number more than 6 will appear,
Let us assume 'D' be the event of getting a number more than 6, then
D = {0)}
Then, n (D) = 0
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(D) = n(D)/n(S)
= 0/6
= 0
(v) A number less than 6 will appear.
Let us assume 'E' be the event of getting a number less than 6, then
E= (1, 2, 3, 4, 5)
Then, n (E) = 5
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(E) = n(E)/n(S)
= 5/6

**4. A card is selected from a pack of 52 cards.**
**(a) How many points are there in the sample space?**
**(b) Calculate the probability that the card is an ace of spades.**
**(c) Calculate the probability that the card is (i) an ace (ii) black card**
**Solution:-**
From the question it is given that, there are 52 cards in the deck.
(a) Number of points in the sample space = 52 (given)
∴n(S) = 52
(b) Let us assume 'A' be the event of drawing an ace of spades.

A= 1
Then, n (A) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(A) = n(A)/n(S)
= 1/52
(c) Let us assume 'B' be the event of drawing an ace. There are four aces.
Then, n (B)= 4
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(B) = n(B)/n(S)
= 4/52
= 1/13
(d) Let us assume 'C' be the event of drawing a black card. There are 26 black cards.
Then, n (C) = 26
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(C) = n(C)/n(S)
= 26/52
= ½

**5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12**
**Solution:-**
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
So, the sample space S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}
Then, n(S) = 12
(i) Let us assume 'P' be the event having sum of numbers as 3.
P = {(1, 2)},
Then, n (P) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(P) = n(P)/n(S)
= 1/12
(ii) Let us assume 'Q' be the event having sum of number as 12.
Then Q = {(6, 6)}, n (Q) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(Q) = n(Q)/n(S)
= 1/12

**6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?**

**Solution:-**

From the question it is given that, there are four men and six women on the city council.
Here total members in the council = 4 + 6 = 10,
Hence, the sample space has 10 points
∴ n (S) = 10
Number of women are 6 … [given]
Let us assume 'A' be the event of selecting a woman
Then n (A) = 6
P(Event) = Number of outcomes favourable to event/Total number of possible outcomes
∴P(A) = n(A)/n(S)
= 6/10 … [divide both numerator and denominators by 2]
= 3/5

**7. A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.50 for each tail that turns up.**
**From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
But, now coin is tossed four times so the possible sample space contains,
S = (HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, TTTH, TTHT, THTT, HTTT, TTTT)
As per the condition given the question, a person will win or lose money depending up on the face of the coin so,
(i) For 4 heads = 1 + 1 + 1 + 1 = ₹ 4
So, he wins ₹ 4
(ii) For 3 heads and 1 tail = 1 + 1 + 1 – 1.50
= 3 – 1.50
= ₹ 1.50
So, he will be winning ₹ 1.50
(iii) For 2 heads and 2 tails = 1 + 1 – 1.50 – 1.50
= 2 - 3
= - ₹ 1
So, he will be losing ₹ 1
(iv)For 1 head and 3 tails = 1 – 1.50 – 1.50 – 1.50

= 1 – 4.50

= - ₹ 3.50

So, he will be losing Rs. 3.50

(v) For 4 tails = – 1.50 – 1.50 – 1.50 – 1.50

= - ₹ 6

So, he will be losing Rs. 6

Now the sample space of amounts is

S= {4, 1.50, 1.50, 1.50, 1.50, – 1, – 1, – 1, – 1, – 1, – 1, – 3.50, – 3.50, – 3.50, – 3.50, – 6}

Then, n (S) = 16

P (winning ₹ 4) = 1/16

P (winning ₹ 1.50) = 4/16 … [divide both numerator and denominator by 4]

= ¼

P (winning ₹ 1) = 6/16 … [divide both numerator and denominator by 2]

= 3/8

P (winning ₹ 3.50) = 4/16 … [divide both numerator and denominator by 4]

= ¼

P (winning ₹ 6) = 1/16

= 3/8

**8. Three coins are tossed once. Find the probability of getting**

**(i) 3 heads** **(ii) 2 heads** **(iii) at least 2 heads**

**(iv) at most 2 heads** **(v) no head** **(vi) 3 tails**

**(vii) Exactly two tails** **(viii) no tail** **(ix) at most two tails**

**Solution:-**

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

But, now three coin is tossed so the possible sample space contains,

S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where s is sample space and here n(S) = 8

(i) 3 heads

Let us assume ‘A’ be the event of getting 3 heads

n(A)= 1

∴P(A) = n(A)/n(S)

= 1/8

(ii) 2 heads

Let us assume ‘B’ be the event of getting 2 heads

n (A) = 3

∴P(B) = n(B)/n(S)

= 3/8

(iii) at least 2 heads

Let us assume ‘C’ be the event of getting at least 2 head

n(C) = 4

∴P(C) = n(C)/n(S)

= 4/8

= ½

(iv) at most 2 heads

Let us assume ‘D’ be the event of getting at most 2 heads

n(D) = 7

∴P(D) = n(D)/n(S)

= 7/8

(v) no head

Let us assume ‘E’ be the event of getting no heads

n(E) = 1

∴P(E) = n(E)/n(S)

= 1/8

(vi) 3 tails

Let us assume ‘F’ be the event of getting 3 tails

n(F) = 1

∴P(F) = n(F)/n(S)

= 1/8

(vii) Exactly two tails

Let us assume ‘G’ be the event of getting exactly 2 tails

n(G) = 3

∴P(G) = n(G)/n(S)

= 3/8

(viii) no tail

Let us assume ‘H’ be the event of getting no tails

n(H) = 1

∴P(H) = n(H)/n(S)

= 1/8

(ix) at most two tails

Let us assume ‘I’ be the event of getting at most 2 tails

n(I) = 7

∴P(I) = n(I)/n(S)
= 7/8

**9. If 2/11 is the probability of an event, what is the probability of the event 'not A'.**
**Solution:-**
From the question it is given that, 2/11 is the probability of an event A,
i.e. P (A) = 2/11
Then,
P (not A) = 1 – P (A)
= 1 – (2/11)
= (11 - 2)/11
= 9/11

**10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant**
**Solution:-**
The word given in the question is 'ASSASSINATION'.
Total letters in the given word = 13
Number of vowels in the given word = 6
Number of consonants in the given word = 7
Then, the sample space n(S) = 13
(i) a vowel
Let us assume 'A' be the event of selecting a vowel
n(A) = 6
∴P(A) = n(A)/n(S)
= 6/13
(ii) Let us assume 'B' be the event of selecting the consonant
n(B)= 7
∴P(B) = n(B)/n(S)
= 7/13

**11. In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint order of the numbers is not important.]**
**Solution:-**
From the question it is given that,

Total numbers of numbers in the draw = 20
Numbers to be selected = 6
∴ n (S) = $^{20}C_6$

Let us assume 'A' be the event that six numbers match with the six numbers already fixed by the lottery committee.

∴ $n(A) = {}^{6}C_{6} = 1$

Probability of winning the prize

$$P(A) = \frac{n(A)}{n(S)} = \frac{^{6}C_{6}}{^{20}C_{6}} = \frac{6!14!}{20!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 14!}{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!}$$

$$= \frac{1}{38760}$$

**12. Check whether the following probabilities P(A) and P(B) are consistently defined**
**(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6**
**(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8**
**Solution:-**
(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6
P(A ∩ B) > P(A)
Therefore, the given probabilities are not consistently defined.
(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8
Then,
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
0.8 = 0.5 + 0.4 - P(A ∩ B)
Transposing - P(A ∩ B) to LHS and it becomes P(A ∩ B) and 0.8 to RHS and it becomes – 0.8.
P(A ∩ B) = 0.9 – 0.8
= 0.1
Therefore, P(A ∩ B) < P(A) and P(A ∩ B) < P(B)
So, the given probabilities are consistently defined.

**13. Fill in the blanks in following table:**

| | P(A) | P(B) | P(A ∩ B) | P(A ∪ B) |
|---|---|---|---|---|
| (i) | 1/3 | 1/5 | 1/15 | …. |
| (ii) | 0.35 | ….. | 0.25 | 0.6 |
| (iii) | 0.5 | 0.35 | …. | 0.7 |

**Solution:-**

From the given table,

(i) P(A) = 1/3, P(B) = 1/5, P(A ∩ B) = 1/15, P(A ∪ B) = ?

We know that,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= (1/3) + (1/5) – (1/15)
= ((5 + 3)/15) – (1/15)
= (8/15) – (1/15)
= (8 - 1)/15
= 7/15

(ii) P(A) = 0.35, P(B) = ?, P(A ∩ B) = 0.25, P(A ∪ B) = 0.6

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.6 = 0.35 + P(B) - 0.25

Transposing – 0.25, 0.35 to LHS and it becomes 0.25and – 0.35.

P(B) = 0.6 + 0.25 – 0.35
= 0.5

(iii) P(A) = 0.5, P(B) = 0.35, P(A ∪ B) = 0.7, P(A ∩ B) = ?

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.7 = 0.5 + 0.35 - P(A ∩ B)

Transposing - P(A ∩ B) to LHS and it becomes P(A ∩ B) and 0.7 to RHS and it becomes – 0.7.

P(A ∩ B) = 0.85 – 0.7
= 0.15

**14. Given P(A) = 5/3 and P(B) = 5/1 . Find P(A or B), if A and B are mutually exclusive events.**

**Solution:-**

From the question it is given that,

P(A) = 5/3 and P(B) = 5/1

Then, P(A or B), if A and B are mutually exclusive

P(A∪B) or P(A or B) = P(A) + P(B)

= (3/5) + (1/5)
= 4/5

**15. If E and F are events such that P(E) = ¼ , P(F) = ½ and P(E and F) = 1/8 , find (i) P(E or F), (ii) P(not E and not F)**

**Solution:-**

From the question, we have P(E) = ¼ , P(F) = ½ and P(E ∩ F) = 1/8

(i) P(E or F) i.e. P(E∪F) = P(E) + P(F) – P(E ∩ F)
= ¼ + ½ - (1/8)
= 5/8

(ii) $P(\text{not E and not F}) = P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$
= 1 – (5/8)
= (8 - 5)/8
= 3/8

**16. Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.**

**Solution:-**

From the question it is given that, P(not E and not F) = 0.25

So, $P(\bar{E} \cup \bar{F}) = 0.25$

Then we have,

$\Rightarrow P(\overline{E \cap F}) = 0.25$

⇒1-P(E∩F)=0.25

⇒P(E∩F)= 1 - 0.25
= 0.75

P(E∩F) ≠0

Hence, E and F are not mutually exclusive events.

**17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P(not A), (ii) P(not B) and (iii) P(A or B)**

**Solution:-**

From the question it is given that, P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16.

(i) P(not A) = 1 – P(A)
= 1 – 0.42
= 0.58

(ii) P(not B) = 1 – P(B)
= 1 – 0.48
= 0.52

(iii) P(A not B) = P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 0.42 + 0.48 – 0.16
= 0.74

**18. In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.**

**Solution:-**

Let us assume 'A' be the event that the student is studying mathematics and 'B' be the event that the student is studying biology.

So, P(A) = 40/100
= 2/5

And, P(B) = 30/100
= 3/10

Then, P(A ∩ B) = (10/100)
= 1/10, P(A ∩ B) is probability of studying both mathematics and biology.

Here, Probability of studying mathematics or biology will be given by P (A∪B)

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
= (2/5) + (3/10) – (1/10)
= 6/10
= 3/5

Hence, (3/5) is the probability that the student will studying mathematics or biology.

**19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?**

**Solution:-**

Let us assume probability of a randomly chosen student passing the first examination is 0.8 be P(A).

And also assume the probability of passing the second examination is 0.7 be P(B)

Then,

$P(A \cup B)$ is probability of passing at least one of the examination

Now,

$P(A \cup B) = 0.95$ , $P(A)=0.8$, $P(B)=0.7$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.95 = 0.8 + 0.7 - P(A \cap B)$

Transposing $- P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.95 to RHS and it becomes - 0.95

$P(A \cap B) = 1.5 - 0.95$

$= 0.55$

Hence, 0.55 is the probability that student will pass both the examinations.

**20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?**

**Solution:-**

Let us assume probability of passing the English examination is 0.75 be P(A).

And also assume the probability of passing the Hindi examination is be P(B).

Here given, $P(A) = 0.75$, $P(A \cap B) – 0.5$, $P(A' \cap B') = 0.1$

We know that, $P(A' \cap B') = 1 – P(A \cup B)$

Then, $P(A \cup B) = 1 - P(A' \cap B')$

$= 1 – 0.1$

$= 0.9$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.9 = 0.75 + P(B) – 0.5$

Transposing 0.75, - 0.5 to LHS and it becomes - 0.75, 0.5.

$P(B) = 0.9 + 0.5 - 0.75$

$= 0.65$

**21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that**
**(i) The student opted for NCC or NSS.**
**(ii) The student has opted neither NCC nor NSS.**
**(iii) The student has opted NSS but not NCC.**
**Solution:-**
From the question it is given that,
The total number of students in class = 60
Thus, the sample space consist of n(S) = 60
Let us assume that the students opted for NCC be 'A'
And also assume that the students opted for NSS be 'B'
So, n(A) = 30, n(B) = 32 , $n(A\cap B) = 24$
We know that, P(A) = n(A)/n(S)
= 30/60
= ½
P(B) = n(B)/n(S)
= 32/60
= 8/15
$P(A\cap B) = n(A\cap B)/n(S)$
= 24/60
= 2/5
Therefore, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
(i) The student opted for NCC or NSS.
P (A or B) = P(A) + P(B) –P(A and B)
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$
= ½ + (8/15) – (2/5)
= 19/30
(ii) P(student opted neither NCC nor NSS)
P(not A and not B) = $P(A^{I}\cap B^{I})$
We know that, $P(A^{I}\cap B^{I}) = 1 - P(A\cup B)$
= 1 – (19/30)
= 11/30
(iii) P(student opted NSS but not NCC)
$n(B - A) = n(B) - n(A\cap B)$
⇒ 32 – 24 = 8
The probability that the selected student has opted for NSS and not NCC is
= (8/60) = 2/15

## MISCELLANEOUS EXERCISE PAGE: 409

**1. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that**
**(i) all will be blue? (ii) at least one will be green?**
**Solution:-**
From the question it is given that,
Number of red marbles in the box = 10
Number of blue marbles in the box = 20
Number of green marbles in the box = 30
So, Total number of marbles in the box = 10 + 20 + 30 = 60
Number of ways of drawing 5 marbles from 60 marbles = $^{60}C_5$
(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.
We have,
Number of ways of drawing 5 blue marbles from 20 blue marbles = $^{20}C_5$
Then,
Probability that all marbles will be blue = $^{20}C_5$ / $^{60}C_5$
(ii) Number of ways in which the drawn marble is not green = $^{(20+10)}C_5$
We have,
Probability that no marble is green = $^{30}C_5$ / $^{60}C_5$
Then,
Probability that at least one marble is green = 1 - $^{30}C_5$ / $^{60}C_5$

**2. 4 cards are drawn from a well – shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?**
**Solution:-**
From the question it is given that,
4 cards are drawn from a well – shuffled deck of 52 cards
Number of ways of drawing 4 cards from 52 cards = $^{52}C_4$
In a deck of 52 cards, there are 13 diamonds and 13 spades.
Number of ways of drawing 3 diamonds and one spade = $^{13}C_3 \times {}^{13}C_1$
Therefore, the probability of obtaining 3 diamonds and one spade
= $(^{13}C_3 \times {}^{13}C_1)/ {}^{52}C_4$

**3. A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine**
**(i) P(2) (ii) P(1 or 3) (iii) P(not 3)**
**Solution:-**

From the question it is given that,
Die has two faces each with number '1',
Three faces each with number '2'
And one face with number '3'
Then, Total number of faces of die = 6
(i) P(2)
Number faces with number '2' = 3 … [given]
So, P(2) = 3/6
= ½
(ii) P(1 or 3)
We know that, P (1 or 3) = P (not 2) = 1 − P (2)
So, P(1 or 3) = 1 – ½
= (2 -1)/2
= ½
(iii) P(not 3)
Number of faces with number '3' = 1
P(3) = 1/6
P(not 3) = 1 – P(3)
= 1 – 1/6
= (6 - 1)/6
= 5/6

**4. In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets.**

**Solution:-**

From the question it is given that,
Number of lottery tickets sold = 10,000
Number prizes awarded = 10
(a) The probability of not getting a prize if we buy one ticket,
P (getting a prize) = 10/10000
= 1/1000
Then,
P (not getting a prize) = 1 – (1/1000)
= (1000 – 1)/1000
= 999/1000
(b) The probability of not getting a prize if we buy two tickets,

Then,
Number of tickets not awarded = 10,000 − 10 = 9990
Therefore, P (not getting a prize) = ${}^{9990}C_2 / {}^{10000}C_2$
(iii) The probability of not getting a prize if we buy 10 tickets, then
Number of tickets not awarded = 10,000 − 10 = 9990
Therefore, P (not getting a prize) = ${}^{9990}C_{10} / {}^{10000}C_{10}$

**5. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that**
**(a) you both enter the same section?**
**(b) you both enter the different sections?**
**Solution:-**
From the question,
Total number of students = 100
I and my friend are among the 100 students.
Then, two sections of 40 and 60 are formed.
Total number of ways of selecting 2 students out of 100 students =
(a) Let S = the two of us will enter the same section if both of us are among 40 students or among 60 students.
Number of ways in which both of us enter the same section = P(S)
P(S) = $({}^{40}C_2 + {}^{60}C_2) / {}^{100}C_2$

$$P(S) = \frac{\frac{40!}{2!\times 38!}+\frac{60!}{2!\times 58!}}{\frac{100!}{2!\times 98!}} = \frac{39\times 40+59\times 60}{99\times 100} = \frac{5100}{9900} = \frac{17}{33}$$

(b) P(we enter different sections)
= 1 − P(we enter the same section)
P(we enter different sections) = 1 – (17/33)
= (33 - 17)/33
= 16/33

**6. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.**
**Solution:-**
Let us assume L1, L2, L3 be three letters and E1, E2, and E3 be their corresponding envelops respectively.

Then, Sample space is
$L_1E_1$, $L_2E_3$, $L_3E_2$,
$L_2E_2$, $L_1E_3$, $L_3E_1$,
$L_3E_3$, $L_1E_2$, $L_2E_1$,
$L_1E_1$, $L_2E_2$, $L_3E_3$,
$L_1E_2$, $L_2E_3$, $L_3E_1$,
$L_1E_3$, $L_2E_1$, $L_3E_2$,
Hence, there are 6 ways of inserting 3 letters in 3 envelops.
And there are 4 ways in which at least one letter is inserted in proper envelope. (first 4 rows of sample space)
Probability that at least one letter is inserted in proper envelope,
= 4/6
= 2/3

**7. A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) = 0.35. Find (i) P(A ∪ B) (ii) P(A$^l$ ∩ B$^l$) (iii) P(A ∩ B$^l$) (iv) P(B ∩ A$^l$)**
**Solution:-**
From the question it is given that, A and B are two events such that,
P(A) = 0.54, P(B) = 0.69, P(A ∩ B) = 0.35
(i) P (A ∪ B)
We know that, P (A ∪ B) = P(A) + P(B) − P(A ∩ B)
⇒ P (A ∪ B) = 0.54 + 0.69 − 0.35 = 0.88
∴ P (A ∪ B) = 0.88

(ii) P(A$^l$ ∩ B$^l$)
We know that, A$^l$ ∩ B$^l$ = (A ∪ B)$^l$ [by De Morgan's law]
So, P (A$^l$ ∩ B$^l$) = P(A ∪ B)$^l$
= 1 − P(A ∪ B)
= 1 − 0.88 = 0.12
∴ P (A$^l$ ∩ B$^l$) = 0.12

(iii) P(A ∩ B$^l$)
We have,
P (A ∩ B$^l$) = P(A) − P(A ∩ B)
= 0.54 − 0.35
= 0.19
Therefore, P (A ∩ B$^l$) = 0.19

(iv) $P(B \cap A')$

We know that: $P(B \cap A') = P(B) - P(A \cap B)$

$\Rightarrow P(B \cap A') = 0.69 - 0.35$

Hence, $P(B \cap A') = 0.34$

**8. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:**

| S.No. | Name | Sex | Age in years |
|---|---|---|---|
| 1 | Harish | M | 30 |
| 2 | Rohan | M | 33 |
| 3 | Sheetal | F | 46 |
| 4 | Alis | F | 28 |
| 5 | Salim | M | 41 |

**A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?**

**Solution:-**

From the given table,

The number of person = 5

Out of 5 persons 3 are Male

Out of 5 persons 2 are 35 years of age.

Let us assume 'A' be the event in which the spokesperson will be a male and B be the event in which the spokesperson will be over 35 years of age.

Accordingly, P(A) = 3/5 and P(B) = 2/5

Since there is only one male who is over 35 years of age,

$P(A \cap B) = 1/5$

We know that: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= (3/5) + (2/5) - (1/5)$

$= 4/5$

Hence, the probability that the spokesperson will either be a male or over 35 years of age is 4/5.

**9. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) the repetition of digits is not allowed?**

**Solution:-**

(i) When the digits are repeated

Since four-digit numbers greater than 5000 are formed,
The thousand's place digit is either 7 or 5.
Total number of 4-digit numbers greater than 5000 = $2 \times 5 \times 5 \times 5 - 1$
= 250 − 1
= 249
A number is divisible by 5 if the digit at its unit's place is either 0 or 5.
∴ Total number of 4-digit numbers greater than 5000 that are divisible by 5
= $2 \times 5 \times 5 \times 2 - 1$
= 100 − 1 = 99
Hence, the probability of forming a number divisible by 5 when the digits are repeated is
= P (number divisible by 5 when digits repeated)
P(number divisible by 5 when digits repeated) = 99/249
= 33/83

(ii) When repetition of digits is not allowed
The thousands place can be filled with either of the two digits 5 or 7 i.e. by 2 ways.
The remaining 3 places can be filled with any of the remaining 4 digits.
Total number of 4-digit numbers greater than 5000 = $2 \times 4 \times 3 \times 2 = 48$
Here, number of 4-digit numbers starting with 5 and divisible by 5
= $1 \times 3 \times 2 \times 1 = 6$
Here, number of 4-digit numbers starting with 7 and divisible by 5
= $1 \times 2 \times 3 \times 2 = 12$
Total number of 4-digit numbers greater than 5000 that are divisible by 5
= 6 + 12 = 18
Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed
= P (number divisible by 5 when digits are not repeated)
P (number divisible by 5 when digits are not repeated) = 18/48
= 3/8

**10. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?**
**Solution:-**
From the question it is given that,
The number lock has 4 wheels, each labelled with ten digits i.e., from 0 to 9.

Then,

The Number of ways of selecting 4 different digits out of the 10 digits = ${}^{10}C_4$

Now, each combination of 4 different digits can be arranged in 4! Ways.

We have,

Number of four digits with no repetitions = $4! \times {}^{10}C_4 = 5040$

There is only one number that can open the suitcase.

Hence, the required probability is 1/5040