Exercise 2.1

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of *x* and *y*.
Solution:

Given, $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$
As the ordered pairs are equal, the corresponding elements should also be equal.
Thus,
x/3 + 1 = 5/3 and y – 2/3 = 1/3
Solving, we get
x + 3 = 5 and 3y – 2 = 1 [Taking L.C.M and adding]
x = 2 and 3y = 3
Therefore,
x = 2 and y = 1

2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?
Solution:

Given, set A has 3 elements and the elements of set B are {3, 4, and 5}.
So, the number of elements in set B = 3
Then, the number of elements in (A × B) = (Number of elements in A) × (Number of elements in B)
= 3 × 3 = 9
Therefore, the number of elements in (A × B) will be 9.

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.
Solution:

Given, G = {7, 8} and H = {5, 4, 2}
We know that,
The Cartesian product of two non-empty sets P and Q is given as
P × Q = {(*p*, *q*): *p* ∈ P, *q* ∈ Q}
So,
G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}
H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

4. State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {*m*, *n*} and Q = {*n*, *m*}, then P × Q = {(*m*, *n*), (*n*, *m*)}.
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (*x*, *y*) such that *x* ∈ A and *y* ∈ B.
(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.
Solution:

(i) The statement is False. The correct statement is:
If P = {m, n} and Q = {n, m}, then
P × Q = {(m, m), (m, n), (n, m), (n, n)}
(ii) True
(iii) True

5. If A = {–1, 1}, find A × A × A.
Solution:

The A × A × A for a non-empty set A is given by
A × A × A = {(a, b, c): a, b, c ∈ A}
Here, given A = {–1, 1}
So,
A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

6. If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.
Solution:

Given,
A × B = {(a, x), (a, y), (b, x), (b, y)}
We know that the Cartesian product of two non-empty sets P and Q is given by:
P × Q = {(p, q): p ∈ P, q ∈ Q}
Hence, A is the set of all first elements and B is the set of all second elements.
Therefore, A = {a, b} and B = {x, y}

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × C is a subset of B × D
Solution:

Given,
A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}
(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)
Now, B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ
Thus,
L.H.S. = A × (B ∩ C) = A × Φ = Φ
Next,
A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
Thus,
R.H.S. = (A × B) ∩ (A × C) = Φ
Therefore, L.H.S. = R.H.S
- Hence verified

(ii) To verify: A × C is a subset of B × D
First,

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
And,
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}
Now, it's clearly seen that all the elements of set A × C are the elements of set B × D.
Thus, A × C is a subset of B × D.
- Hence verified

8. Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.
Solution:

Given,
A = {1, 2} and B = {3, 4}
So,
A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}
Number of elements in A × B is n(A × B) = 4
We know that,
If C is a set with n(C) = m, then n[P(C)] = 2^m.
Thus, the set A × B has 2^4 = 16 subsets.
And, these subsets are as below:
Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

9. Let A and B be two sets such that *n*(A) = 3 and *n* (B) = 2. If (*x*, 1), (*y*, 2), (*z*, 1) are in A × B, find A and B, where *x*, *y* and *z* are distinct elements.
Solution:

Given,
n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.
We know that,
A = Set of first elements of the ordered pair elements of A × B
B = Set of second elements of the ordered pair elements of A × B.
So, clearly x, y, and z are the elements of A; and
1 and 2 are the elements of B.
As n(A) = 3 and n(B) = 2, it is clear that set A = {x, y, z} and set B = {1, 2}.

10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.
Solution:

We know that,
If n(A) = p and n(B) = q, then n(A × B) = pq.
Also, n(A × A) = n(A) × n(A)
Given,
n(A × A) = 9

So, $n(A) \times n(A) = 9$
Thus, $n(A) = 3$
Also given that, the ordered pairs $(–1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.
And, we know in $A \times A = \{(a, a): a \in A\}$.
Thus, –1, 0, and 1 has to be the elements of A.
As $n(A) = 3$, clearly $A = \{–1, 0, 1\}$.
Hence, the remaining elements of set $A \times A$ are as follows:
$(–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0)$, and $(1, 1)$

Exercise 2.2

1. Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {(x, y): $3x - y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.
Solution:

The relation R from A to A is given as:
R = {(x, y): $3x - y = 0$, where $x, y \in$ A}
= {(x, y): $3x = y$, where $x, y \in$ A}
So,
R = {(1, 3), (2, 6), (3, 9), (4, 12)}
Now,
The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of R = {1, 2, 3, 4}
The whole set A is the codomain of the relation R.
Hence, Codomain of R = A = {1, 2, 3, …, 14}
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of R = {3, 6, 9, 12}

2. Define a relation R on the set N of natural numbers by R = {(x, y): $y = x + 5$, x is a natural number less than 4; $x, y \in$ N}. Depict this relationship using roster form. Write down the domain and the range.
Solution:

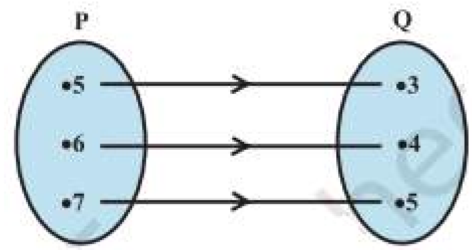
The relation R is given by:
R = {(x, y): $y = x + 5$, x is a natural number less than 4, $x, y \in$ **N**}
The natural numbers less than 4 are 1, 2, and 3.
So,
R = {(1, 6), (2, 7), (3, 8)}
Now,
The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of R = {1, 2, 3}
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of R = {6, 7, 8}

3. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}. Write R in roster form.
Solution:

Given,
A = {1, 2, 3, 5} and B = {4, 6, 9}
The relation from A to B is given as:
R = {(x, y): the difference between x and y is odd; $x \in$ A, $y \in$ B}
Thus,
R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

4. The figure shows a relationship between the sets P and Q. write this relation
(i) in set-builder form (ii) in roster form.

What is its domain and range?
Solution:



From the given figure, it's seen that
P = {5, 6, 7}, Q = {3, 4, 5}
The relation between P and Q:
Set-builder form
(i) R = {(x, y): $y = x – 2$; $x \in$ P} or R = {(x, y): $y = x – 2$ for x = 5, 6, 7}
Roster form
(ii) R = {(5, 3), (6, 4), (7, 5)}
Domain of R = {5, 6, 7}
Range of R = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by
{(*a*, *b*): *a*, *b* ∈ A, *b* is exactly divisible by *a*}.
(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.
Solution:

Given,
A = {1, 2, 3, 4, 6} and relation R = {(a, b): a, $b \in$ A, b is exactly divisible by a}
Hence,
(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}
(ii) Domain of R = {1, 2, 3, 4, 6}
(iii) Range of R = {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by R = {(x, $x + 5$): $x \in$ {0, 1, 2, 3, 4, 5}}.
Solution:

Given,
Relation R = {(x, $x + 5$): $x \in$ {0, 1, 2, 3, 4, 5}}
Thus,
R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
So,
Domain of R = {0, 1, 2, 3, 4, 5} and,
Range of R = {5, 6, 7, 8, 9, 10}

7. Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.
Solution:

Given,
Relation R = {(x, x^3): x is a prime number less than 10}
The prime numbers less than 10 are 2, 3, 5, and 7.
Therefore,

R = {(2, 8), (3, 27), (5, 125), (7, 343)}

8. Let A = {*x*, *y*, z} and B = {1, 2}. Find the number of relations from A to B.
Solution:

Given, A = {*x*, *y*, z} and B = {1, 2}.
Now,
A × B = {(*x*, 1), (*x*, 2), (*y*, 1), (*y*, 2), (*z*, 1), (*z*, 2)}
As *n*(A × B) = 6, the number of subsets of A × B will be 2^6.
Thus, the number of relations from A to B is 2^6.

9. Let R be the relation on Z defined by R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}. Find the domain and range of R.
Solution:

Given,
Relation R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}
We know that the difference between any two integers is always an integer.
Therefore,
Domain of R = Z and Range of R = Z

Exercise 2.3 Page No: 44

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.
(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(1, 3), (1, 5), (2, 5)}
Solution:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
As 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation can be called as a function.
Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
As 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation can be called as a function.
Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}
It's seen that the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation cannot be called as a function.

2. Find the domain and range of the following real function:
(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{(9 - x^2)}$
Solution:

(i) Given,
$f(x) = -|x|, x \in R$
We know that,

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

As $f(x)$ is defined for $x \in R$, the domain of f is R.
It is also seen that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.
Therefore, the range of f is given by $(-\infty, 0]$.

(ii) $f(x) = \sqrt{(9 - x^2)}$
As $\sqrt{(9 - x^2)}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, for $9 - x^2 \geq 0$.
So, the domain of $f(x)$ is $\{x: -3 \leq x \leq 3\}$ or $[-3, 3]$.
Now,
For any value of x in the range $[-3, 3]$, the value of $f(x)$ will lie between 0 and 3.
Therefore, the range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

3. A function f is defined by $f(x) = 2x - 5$. Write down the values of
(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$
Solution:

Given,
Function, $f(x) = 2x - 5$.
Therefore,
(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$
(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$
(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

4. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.
Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$
Solution:

Given function, $t(C) = \frac{9C}{5} + 32$
So,
(i) $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$
(ii) $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$
(iii) $t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$
(iv) Given that, $t(C) = 212$

$$\therefore 212 = \frac{9C}{5} + 32$$
$$\Rightarrow \frac{9C}{5} = 212 - 32$$
$$\Rightarrow \frac{9C}{5} = 180$$
$$\Rightarrow 9C = 180 \times 5$$
$$\Rightarrow C = \frac{180 \times 5}{9} = 100$$

Therefore, the value of t when $t(C) = 212$, is 100.

5. Find the range of each of the following functions.
(i) $f(x) = 2 - 3x$, $x \in R$, $x > 0$.
(ii) $f(x) = x^2 + 2$, x is a real number.
(iii) $f(x) = x$, x is a real number.

Solution:

(i) Given,
$f(x) = 2 - 3x$, $x \in R$, $x > 0$.
We have,
$x > 0$
So,
$3x > 0$
$-3x < 0$ [Multiplying by -1 both the sides, the inequality sign changes]
$2 - 3x < 2$
Therefore, the value of $2 - 3x$ is less than 2.
Hence, Range $= (-\infty, 2)$

(ii) Given,
$f(x) = x^2 + 2$, x is a real number
We know that,
$x^2 \geq 0$
So,
$x^2 + 2 \geq 2$ [Adding 2 both the sides]
Therefore, the value of $x^2 + 2$ is always greater or equal to 2 for x is a real number.
Hence, Range $= [2, \infty)$

(iii) Given,
$f(x) = x$, x is a real number
Clearly, the range of f is the set of all real numbers.
Thus,
Range of $f = R$

Miscellaneous Exercise

1. The relation *f* is defined by $f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$

The relation *g* is defined by $g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$

Show that *f* is a function and *g* is not a function.
Solution:

The given relation *f* is defined as:

$$f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$$

It is seen that, for $0 \le x < 3$,
$f(x) = x^2$ and for $3 < x \le 10$,
$f(x) = 3x$
Also, at $x = 3$
$f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$
i.e., at $x = 3$, $f(x) = 9$ [Single image]
Hence, for $0 \le x \le 10$, the images of $f(x)$ are unique.
Therefore, the given relation is a function.
Now,
In the given relation *g* is defined as

$$g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$$

It is seen that, for $x = 2$
$g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$
Thus, element 2 of the domain of the relation *g* corresponds to two different images i.e., 4 and 6.
Therefore, this relation is not a function.

2. If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$
Solution:

Given,
$f(x) = x^2$
Hence,

$$\frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

3. Find the domain of the function $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$
Solution:

Given function,

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}.$$

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It clearly seen that, the function *f* is defined for all real numbers except at $x = 6$ and $x = 2$ as the denominator becomes zero otherwise.
Therefore, the domain of *f* is R – {2, 6}.

4. Find the domain and the range of the real function *f* defined by *f*(x) = √(x - 1).
Solution:

Given real function,
$f(x) = \sqrt{(x - 1)}$
Clearly, $\sqrt{(x - 1)}$ is defined for $(x – 1) \geq 0$.
So, the function $f(x) = \sqrt{(x - 1)}$ is defined for $x \geq 1$.
Thus, the domain of *f* is the set of all real numbers greater than or equal to 1.
Domain of $f = [1, \infty)$.
Now,
As $x \geq 1 \Rightarrow (x – 1) \geq 0 \Rightarrow \sqrt{(x - 1)} \geq 0$
Thus, the range of *f* is the set of all real numbers greater than or equal to 0.
Range of $f = [0, \infty)$.

5. Find the domain and the range of the real function *f* defined by *f* (*x*) = |*x* – 1|.
Solution:

Given real function,
$f(x) = |x – 1|$
Clearly, the function $|x – 1|$ is defined for all real numbers.
Hence,
Domain of f = R
Also, for $x \in$ R, $|x – 1|$ assumes all real numbers.
Therefore, the range of *f* is the set of all non-negative real numbers.

6. Let $f=\left\{\left(x,\ \frac{x^2}{1+x^2}\right): x\in \mathbf{R}\right\}$ **be a function from R into R. Determine the range of *f*.**

Solution:

Given function,

$$f=\left\{\left(x,\ \frac{x^2}{1+x^2}\right): x\in \mathbf{R}\right\}$$

Substituting values and determining the images, we have

$$= \left\{(0,\ 0),\ \left(\pm 0.5,\ \frac{1}{5}\right),\ \left(\pm 1,\ \frac{1}{2}\right),\ \left(\pm 1.5,\ \frac{9}{13}\right),\ \left(\pm 2,\ \frac{4}{5}\right),\ \left(3,\ \frac{9}{10}\right),\ \left(4,\ \frac{16}{17}\right),\ \ldots\right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[As the denominator is greater than the numerator.]

Or,
We know that, for $x \in R$,
$x^2 \geq 0$
Then,
$x^2 + 1 \geq x^2$
$1 \geq x^2 / (x^2 + 1)$

Therefore, the range of $f = [0, 1)$

7. Let f, g: R → R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x – 3$. Find $f + g$, $f – g$ and f/g.
Solution:

Given, the functions f, g: R → R is defined as
$f(x) = x + 1$, $g(x) = 2x – 3$
Now,
$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x – 3) = 3x – 2$
Thus, $(f + g)(x) = 3x – 2$
$(f – g)(x) = f(x) – g(x) = (x + 1) – (2x – 3) = x + 1 – 2x + 3 = – x + 4$
Thus, $(f – g)(x) = –x + 4$
$f/g(x) = f(x)/g(x)$, $g(x) \neq 0$, $x \in R$
$f/g(x) = x + 1/ 2x – 3$, $2x – 3 \neq 0$
Thus, $f/g(x) = x + 1/ 2x – 3$, $x \neq 3/2$

8. Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a function from Z to Z defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.
Solution:

Given, f = {(1, 1), (2, 3), (0, –1), (–1, –3)}
And the function defined as, $f(x) = ax + b$
For $(1, 1) \in f$
We have, $f(1) = 1$
So, $a \times 1 + b = 1$
$a + b = 1$ …. (i)
And for $(0, –1) \in f$
We have $f(0) = –1$
$a \times 0 + b = –1$
$b = –1$
On substituting $b = –1$ in (i), we get
$a + (–1) = 1 \Rightarrow a = 1 + 1 = 2.$

Therefore, the values of a and b are 2 and –1 respectively.

9. Let R be a relation from N to N defined by R = {(a, b): a, b ∈ N and $a = b^2$}. Are the following true?
(i) (a, a) ∈ R, for all a ∈ N
(ii) (a, b) ∈ R, implies (b, a) ∈ R
(iii) (a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R.
Justify your answer in each case.
Solution:

Given relation R = {(a, b): a, b ∈ N and $a = b^2$}
(i) It can be seen that 2 ∈ N; however, $2 \neq 2^2 = 4$.
Thus, the statement "(a, a) ∈ R, for all a ∈ N" is not true.

(ii) Its clearly seen that (9, 3) ∈ N because 9, 3 ∈ N and $9 = 3^2$.
Now, $3 \neq 9^2 = 81$; therefore, (3, 9) ∉ N
Thus, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.

(iii) Its clearly seen that (16, 4) ∈ R, (4, 2) ∈ R because 16, 4, 2 ∈ N and $16 = 4^2$ and $4 = 2^2$.
Now, $16 \neq 2^2 = 4$; therefore, (16, 2) ∉ N
Thus, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true.

10. Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?
(i) f is a relation from A to B (ii) f is a function from A to B.
Justify your answer in each case.
Solution:

Given,
A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}
So,
A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}
Also given that,
f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}
(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.
It's clearly seen that f is a subset of A × B.
Therefore, f is a relation from A to B.

(ii) As the same first element i.e., 2 corresponds to two different images (9 and 11), relation f is not a function.

11. Let f be the subset of Z × Z defined by f = {(ab, $a + b$): a, b ∈ Z}. Is f a function from Z to Z: justify your answer.
Solution:

Given relation f is defined as
$f = \{(ab, a + b): a, b \in Z\}$
We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.
As 2, 6, –2, –6 ∈ Z, $(2 \times 6, 2 + 6), (-2 \times -6, -2 + (-6)) \in f$
i.e., $(12, 8), (12, -8) \in f$
It's clearly seen that, the same first element, 12 corresponds to two different images (8 and –8).
Therefore, the relation f is not a function.

12. Let A = {9, 10, 11, 12, 13} and let f: A → N be defined by $f(n)$ = the highest prime factor of n. Find the range of f.
Solution:

Given,
A = {9, 10, 11, 12, 13}
Now, f: A → **N** is defined as
$f(n)$ = The highest prime factor of n
So,
Prime factor of 9 = 3
Prime factors of 10 = 2, 5
Prime factor of 11 = 11
Prime factors of 12 = 2, 3
Prime factor of 13 = 13
Thus, it can be expressed as
$f(9)$ = The highest prime factor of 9 = 3
$f(10)$ = The highest prime factor of 10 = 5
$f(11)$ = The highest prime factor of 11 = 11
$f(12)$ = The highest prime factor of 12 = 3
$f(13)$ = The highest prime factor of 13 = 13
The range of f is the set of all $f(n)$, where $n \in$ A.
Therefore,
Range of f = {3, 5, 11, 13}