EXERCISE 3.1

1. Find the radian measures corresponding to the following degree measures:
   (i) 25° (ii) – 47° 30' (iii) 240° (iv) 520°

Solution:

(i) 25°
Here 180° = π radian
It can be written as
25° = \( \frac{\pi}{180} \times 25 \) radian

So we get
= \( \frac{5\pi}{36} \) radian

(ii) – 47° 30'
Here 1° = 60'
It can be written as
–47° 30' = –47 \( \frac{1}{2} \) degree

So we get
= \( \frac{-95}{2} \) degree

Here 180° = π radian
\( \frac{-95}{2} \) degree = \( \frac{\pi}{180} \times \left( \frac{-95}{2} \right) \) radian

It can be written as
= \( \left( \frac{-19}{36 \times 2} \right) \pi \) radian = \( \frac{-19}{72} \) π radian

We get
–47° 30' = \( \frac{-19}{72} \) π radian

(iii) 240°
Here 180° = π radian
It can be written as
240° = \( \frac{\pi}{180} \times 240 \) radian

So we get
= \( \frac{4}{3} \) π radian

(iv) 520°
2. Find the degree measures corresponding to the following radian measures (Use π = \(\frac{22}{7}\))

(i) \(\frac{11}{16}\)

Here \(\pi\) radian = 180°

\[
\frac{11}{16} \text{ radian} = \frac{180}{\pi} \times \frac{11}{16} \text{ degree}
\]

We can write it as

\[
= \frac{45 \times 11}{\pi \times 4} \text{ degree}
\]

So we get

\[
= \frac{45 \times 11 \times 7}{22 \times 4} \text{ degree}
\]

\[
= \frac{315}{8} \text{ degree}
\]

\[
= 39\frac{3}{8} \text{ degree}
\]

Take 1° = 60’

\[
= 39° + \frac{3 \times 60}{8} \text{ minutes}
\]

We get

\[
= 39° + 22' + \frac{1}{2} \text{ minutes}
\]

Consider 1’ = 60’’

\[
= 39° 22' 30''
\]

(ii) -4

Here \(\pi\) radian = 180°
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(iii) $\frac{5\pi}{3}$
Here $\pi$ radian = $180^\circ$
\[
\frac{5\pi}{3} \text{ radian} = \frac{180^\circ \times 5\pi}{\pi} \text{ degree}
\]
We get
\[
= 300^\circ
\]

(iv) $\frac{7\pi}{6}$
Here $\pi$ radian = $180^\circ$
\[
\frac{7\pi}{6} \text{ radian} = \frac{180^\circ \times 7\pi}{\pi} \text{ degree}
\]
We get
\[
= 210^\circ
\]

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

It is given that

No. of revolutions made by the wheel in
1 minute = 360
1 second = $360/6 = 60$

We know that

The wheel turns an angle of $2\pi$ radian in one complete revolution.

In 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian = $12\pi$ radian

Therefore, in one second, the wheel turns an angle of $12\pi$ radian.
4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use π = 22/7).

Solution:

Consider a circle of radius \( r \) unit with 1 unit as the arc length which subtends an angle \( \theta \) radian at the centre.

\[ \theta = \frac{l}{r} \]

Here \( r = 100 \) cm, \( l = 22 \) cm

\[ \theta = \frac{22}{100} \text{ radian} = \frac{180 \times 22}{\pi \times 100} \text{ degree} \]

It can be written as

\[ = \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree} \]

\[ = \frac{126}{10} \text{ degree} \]

So we get

\[ = 12.3 \text{ degree} \]

Here \( 1^\circ = 60' \)

\[ = 12^\circ 36' \]

Therefore, the required angle is \( 12^\circ 36' \).

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution:

The dimensions of the circle are

Diameter = 40 cm

Radius = \( 40/2 = 20 \) cm

Consider AB be as the chord of the circle i.e. length = 20 cm

\[ \text{In } \triangle OAB, \]

Radius of circle = \( OA = OB = 20 \) cm

Similarly \( AB = 20 \) cm

Hence, \( \triangle OAB \) is an equilateral triangle.

\[ \theta = 60^\circ = \pi/3 \text{ radian} \]

In a circle of radius \( r \) unit, if an arc of length \( l \) unit subtends an angle \( \theta \) radian at the centre

We get \( \theta = 1/r \)
\[ \frac{\pi}{3} = \frac{AB}{20} \Rightarrow AB = \frac{20\pi}{3} \text{ cm} \]

Therefore, the length of the minor arc of the chord is \(20\pi/3\) cm.

6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution:

Consider \(r_1\) and \(r_2\) as the radii of the two circles.

Let an arc of length \(l\) subtend an angle of 60° at the centre of the circle of radius \(r_1\) and an arc of length \(l\) subtend an angle of 75° at the centre of the circle of radius \(r_2\).

Here 60° = \(\pi/3\) radian and 75° = \(5\pi/12\) radian

In a circle of radius \(r\) unit, if an arc of length \(l\) unit subtends an angle \(\theta\) radian at the centre

We get \(\theta = 1/r\) or \(l = r \theta\)

We know that

\[ l = \frac{r_1\pi}{3} \quad \text{and} \quad l = \frac{r_2\cdot5\pi}{12} \]

By equating both we get

\[ \frac{r_1\pi}{3} = \frac{r_2\cdot5\pi}{12} \]

On further calculation

\[ r_1 = \frac{r_2\cdot5}{4} \]

So we get

\[ r_1 = \frac{5}{4} \]

\[ r_2 = \frac{5}{4} \]

Therefore, the ratio of the radii is 5:4.

7. Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

In a circle of radius \(r\) unit, if an arc of length \(l\) unit subtends an angle \(\theta\) radian at the centre, then \(\theta = 1/r\)

We know that \(r = 75\) cm

(i) \(l = 10\) cm

So we get

\(\theta = 10/75\) radian

By further simplification

\(\theta = 2/15\) radian
(ii) $l = 15 \text{ cm}$
So we get
\[ \theta = \frac{15}{75} \text{ radian} \]
By further simplification
\[ \theta = \frac{1}{5} \text{ radian} \]

(iii) $l = 21 \text{ cm}$
So we get
\[ \theta = \frac{21}{75} \text{ radian} \]
By further simplification
\[ \theta = \frac{7}{25} \text{ radian} \]
Find the values of other five trigonometric functions in Exercises 1 to 5.

1. \( \cos x = -\frac{1}{2} \), \( x \) lies in third quadrant.
   Solution:
   
   It is given that
   
   \( \cos x = -\frac{1}{2} \)
   
   \( \sec x = \frac{1}{\cos x} \)
   
   Substituting the values
   
   \[
   = \left( \frac{1}{-\frac{1}{2}} \right) = -2
   \]
   
   Consider
   
   \( \sin^2 x + \cos^2 x = 1 \)
   
   We can write it as
   
   \( \sin^2 x = 1 - \cos^2 x \)
   
   Substituting the values
   
   \( \sin^2 x = 1 - \left(\frac{1}{2}\right)^2 \)
   
   \( \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4} \)
   
   \( \sin^2 x = \pm \sqrt{3}/2 \)
   
   Here \( x \) lies in the third quadrant so the value of \( \sin x \) will be negative
   
   \( \sin x = -\sqrt{3}/2 \)
   
   We can write it as
   
   \[
   \cos ecx = \frac{1}{\sin x} = \left( \frac{-\sqrt{3}}{2} \right) = -\frac{2}{\sqrt{3}}
   \]
   
   So we get
   
   \[
   \tan x = \frac{\sin x}{\cos x} = \left( \frac{-\sqrt{3}}{2} \right) = \sqrt{3}
   \]
   
   Here
   
   \[
   \cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}
   \]

2. \( \sin x = \frac{3}{5} \), \( x \) lies in second quadrant.
   Solution:
   
   It is given that
   
   \( \sin x = \frac{3}{5} \)
   
   We can write it as
   
   \[
   \cosec x = \frac{1}{\sin x} = \left( \frac{3}{\frac{5}{5}} \right) = \frac{5}{3}
   \]
We know that
\[ \sin^2 x + \cos^2 x = 1 \]
We can write it as
\[ \cos^2 x = 1 - \sin^2 x \]
Substituting the values
\[ \cos^2 x = 1 - \left(\frac{3}{5}\right)^2 \]
\[ \cos^2 x = 1 - \frac{9}{25} \]
\[ \cos^2 x = \frac{16}{25} \]
\[ \cos x = \pm \frac{4}{5} \]
Here \( x \) lies in the second quadrant so the value of \( \cos x \) will be negative
\[ \cos x = -\frac{4}{5} \]
We can write it as
\[ \sec x = \frac{1}{\cos x} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4} \]
So we get
\[ \tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{-\frac{4}{5}} = -\frac{3}{4} \]

Here
\[ \cot x = \frac{1}{\tan x} = -\frac{4}{3} \]

3. \( \cot x = \frac{3}{4} \), \( x \) lies in third quadrant.

Solution:

It is given that
\[ \cot x = \frac{3}{4} \]
We can write it as
\[ \tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3} \]

We know that
\[ 1 + \tan^2 x = \sec^2 x \]
We can write it as
\[ 1 + (4/3)^2 = \sec^2 x \]
Substituting the values
\[ 1 + \frac{16}{9} = \sec^2 x \]
\[ \cos^2 x = \frac{25}{9} \]
\[ \sec x = \pm \frac{5}{3} \]
Here \( x \) lies in the third quadrant so the value of \( \sec x \) will be negative
\[ \sec x = -\frac{5}{3} \]
We can write it as
4. sec x = 13/5, x lies in fourth quadrant.
Solution:

It is given that

sec x = 13/5

We can write it as

\[ \cos x = \frac{1}{\sec x} = \frac{1}{\frac{13}{5}} = \frac{5}{13} \]

So we get

\[ \tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{1 - \cos^2 x}}{\cos x} = \frac{\sqrt{1 - \left(\frac{5}{13}\right)^2}}{\frac{5}{13}} = \frac{12}{5} \]

By further calculation

\[ \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5} \]

Here

\[ \csc x = \frac{1}{\sin x} = -\frac{5}{4} \]

4. sec x = 13/5, x lies in fourth quadrant.
Solution:

It is given that

sec x = 13/5

We can write it as

\[ \cos x = \frac{1}{\sec x} = \frac{1}{\frac{13}{5}} = \frac{5}{13} \]

We know that

\[ \sin^2 x + \cos^2 x = 1 \]

We can write it as

\[ \sin^2 x = 1 - \cos^2 x \]

Substituting the values

\[ \sin^2 x = 1 - \left(\frac{5}{13}\right)^2 \]
\[ \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169} \]
\[ \sin^2 x = \pm \frac{12}{13} \]

Here x lies in the fourth quadrant so the value of \( \sin x \) will be negative

\[ \sin x = -\frac{12}{13} \]

We can write it as

\[ \csc x = \frac{1}{\sin x} = -\frac{13}{12} \]

So we get

\[ \tan x = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5} \]
Here

\[ \cot x = \frac{1}{\tan x} = \frac{1}{-\frac{5}{12}} = -\frac{12}{5} \]

5. \( \tan x = -\frac{5}{12} \), \( x \) lies in second quadrant.
Solution:

It is given that
\( \tan x = -\frac{5}{12} \)
We can write it as
\[ \cot x = \frac{1}{\tan x} = \frac{1}{-\frac{5}{12}} = -\frac{12}{5} \]

We know that
\[ 1 + \tan^2 x = \sec^2 x \]
We can write it as
\[ 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x \]
Substituting the values
\[ 1 + 25/144 = \sec^2 x \]
\[ \sec^2 x = 169/144 \]
\[ \sec x = \pm \frac{13}{12} \]
Here \( x \) lies in the second quadrant so the value of \( \sec x \) will be negative
\[ \sec x = -\frac{13}{12} \]
We can write it as

Find the values of the trigonometric functions in Exercises 6 to 10.
6. \( \sin 765^\circ \)
Solution:

We know that values of \( \sin x \) repeat after an interval of \( 2\pi \) or 360°.
So we get
\[
\sin 765° = \sin (2 \times 360° + 45°)
\]
By further calculation
\[
= \sin 45° \\
= 1/ \sqrt{2}
\]

7. cosec \((-1410°)"

Solution:

We know that values of cosec \( x \) repeat after an interval of \( 2\pi \) or 360°.
So we get
\[
cosec (-1410°) = cosec (-1410° + 4 \times 360°)
\]
By further calculation
\[
= cosec (-1410° + 1440°) \\
= cosec 30° = 2
\]

8. \( \tan \frac{19\pi}{3} \)

Solution:

We know that values of \( \tan x \) repeat after an interval of \( \pi \) or 180°.
So we get
\[
\tan \frac{19\pi}{3} = \tan 6\frac{1}{3} \pi
\]
By further calculation
\[
= \tan \left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3}
\]
We get
\[
= \tan 60° \\
= \sqrt{3}
\]

9. \( \sin \left(-\frac{11\pi}{3}\right) \)

Solution:

We know that values of \( \sin x \) repeat after an interval of \( 2\pi \) or 360°.
So we get
10. **Solution:**

We know that values of \( \tan x \) repeat after an interval of \( \pi \) or \( 180^\circ \).

So we get,

\[
\cot \left( -\frac{15\pi}{4} \right) = \cot \left( -\frac{15\pi}{4} + 4\pi \right)
\]

By further calculation,

\[
= \cot \frac{\pi}{4} = 1
\]
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EXERCISE 3.3

Prove that:

1. \[ \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2} \]

Solution:

Consider

L.H.S. = \[ \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} \]

So we get

\[ \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 - (1)^2 \]

By further calculation

= \frac{1}{4} + \frac{1}{4} - 1

= - \frac{1}{2}

= RHS

2. \[ 2 \sin^2 \frac{\pi}{6} + \cos^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2} \]

Solution:

Consider

L.H.S. = \[ 2 \sin^2 \frac{\pi}{6} + \cos^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} \]

By further calculation

\[ = 2 \left( \frac{1}{2} \right)^2 + \cos^2 \left( \frac{\pi}{2} + \frac{\pi}{6} \right) \left( \frac{1}{2} \right)^2 \]

It can be written as

\[ = 2 \cdot \frac{1}{4} + \left( -\cos \frac{\pi}{6} \right)^2 \left( \frac{1}{4} \right) \]

So we get

\[ = \frac{1}{2} + (-2)^2 \left( \frac{1}{4} \right) \]
Here
\[ \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2} = \text{RHS} \]

3. \( \cot^2 \frac{\pi}{6} + \cos \csc \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6 \)

Solution:

Consider

\[ \text{L.H.S.} = \cot^2 \frac{\pi}{6} + \cos \csc \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} \]

So we get

\[ = \left( \sqrt{3} \right)^2 + \cos \csc \left( \frac{\pi - \frac{\pi}{6}}{6} \right) + 3 \left( \frac{1}{\sqrt{3}} \right)^2 \]

By further calculation

\[ = 3 + \cos \csc \frac{\pi}{6} + 3 \times \frac{1}{3} \]

We get

\[ = 3 + 2 + 1 = 6 = \text{RHS} \]

4. \( 2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10 \)

Solution:

Consider

\[ \text{L.H.S} = 2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} \]

So we get

\[ = 2 \left( \sin \left( \frac{\pi - \frac{\pi}{4}}{4} \right) \right)^2 + 2 \left( \frac{1}{\sqrt{2}} \right)^2 + 2 \left( \sec^2 \frac{\pi}{3} \right) \]

By further calculation
5. Find the value of:
(i) \( \sin 75^\circ \)
(ii) \( \tan 15^\circ \)
Solution:

(i) \( \sin 75^\circ \)

It can be written as

\[ = \sin (45^\circ + 30^\circ) \]

Using the formula \( \sin (x + y) = \sin x \cos y + \cos x \sin y \)

\[ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ \]

Substituting the values

\[ = \left( \frac{1}{\sqrt{2}} \right) \left( \frac{\sqrt{3}}{2} \right) + \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{2} \right) \]

By further calculation

\[ = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \]

\[ = \frac{\sqrt{3} + 1}{2\sqrt{2}} \]

(ii) \( \tan 15^\circ \)

It can be written as

\[ = \tan (45^\circ - 30^\circ) \]

Using formula

\[ \tan (x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \]
Prove the following:

6. \[ \cos\left(\frac{\pi}{4} - x\right) \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right) \sin\left(\frac{\pi}{4} - y\right) = \sin(x + y) \]

Solution:

Consider LHS = 
\[ \cos\left(\frac{\pi}{4} - x\right) \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right) \sin\left(\frac{\pi}{4} - y\right) \]

We can write it as
\[ \frac{1}{2} \left[ 2 \cos\left(\frac{\pi}{4} - x\right) \cos\left(\frac{\pi}{4} - y\right) + \frac{1}{2} \left[ -2 \sin\left(\frac{\pi}{4} - x\right) \sin\left(\frac{\pi}{4} - y\right) \right] \right] \]

By further simplification
\[ \frac{1}{2} \left[ \cos\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right) \right] + \cos\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right) \right] \]
7. Solution:

\[
\frac{1}{2} \cos \left( \frac{\pi}{4} - x \right) + \frac{1}{2} \cos \left( \frac{\pi}{4} - y \right) - \cos \left( \frac{\pi}{4} - \frac{x - y}{2} \right)
\]

Using the formula

\[2 \cos A \cos B = \cos (A + B) + \cos (A - B)\]

\[-2 \sin A \sin B = \cos (A + B) - \cos (A - B)\]

\[= 2 \times \frac{1}{2} \cos \left( \frac{\pi}{4} - \frac{x - y}{2} \right)\]

We get

\[= \cos \left( \frac{\pi}{2} - (x + y) \right)\]

\[= \sin (x + y)\]

\[= \text{RHS}\]

7.

\[\frac{\tan \left( \frac{\pi}{4} + x \right)}{\tan \left( \frac{\pi}{4} - x \right)} = \left( \frac{1 + \tan x}{1 - \tan x} \right)^2\]

Solution:

Consider

\[L.H.S. = \frac{\tan \left( \frac{\pi}{4} + x \right)}{\tan \left( \frac{\pi}{4} - x \right)}\]

By using the formula

\[\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\]

\[\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\]

So we get

\[\left( \frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x} \right) = \left( \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \right)^2\]
It can be written as
\[
\frac{1 + \tan x}{1 - \tan x} = \frac{1 - \tan x}{1 + \tan x} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{RHS}
\]

8. \[
\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x
\]

Solution:

Consider

\[
\text{L.H.S.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}
\]

It can be written as
\[
\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}
\]

So we get
\[
= -\cos^2 x
\]
\[
= -\sin^2 x
\]
\[
= \cot^2 x
\]
\[
= \text{RHS}
\]

9. \[
\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1
\]

Solution:

Consider

\[
\text{L.H.S.} = \cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]
\]

It can be written as
\[
= \sin x \cos x (\tan x + \cot x)
\]

So we get
10. \( \sin (n + 1)x \sin (n + 2)x + \cos (n + 1)x \cos (n + 2)x = \cos x \)

\[ \text{Solution:} \]

\[ \text{LHS} = \sin (n + 1)x \sin (n + 2)x + \cos (n + 1)x \cos (n + 2)x \]

By multiplying and dividing by 2

\[ = \frac{1}{2} \left[ 2 \sin (n + 1)x \sin (n + 2)x + 2 \cos (n + 1)x \cos (n + 2)x \right] \]

Using the formula

\[ -2 \sin A \sin B = \cos (A + B) - \cos (A - B) \]

\[ 2 \cos A \cos B = \cos (A + B) + \cos (A - B) \]

\[ = \frac{1}{2} \left[ \cos \left( (n + 1)x - (n + 2)x \right) \right] - \cos \left( (n + 1)x + (n + 2)x \right) \]

By further calculation

\[ = \frac{1}{2} \cdot 2 \cos \left( (n + 1)x - (n + 2)x \right) \]

\[ = \cos (-x) \]

\[ = \cos x \]

\[ = \text{RHS} \]

11. \( \cos \left( \frac{3 \pi}{4} + x \right) - \cos \left( \frac{3 \pi}{4} - x \right) = -\sqrt{2} \sin x \)

\[ \text{Solution:} \]

\[ \text{Consider} \]

\[ \text{L.H.S.} = \cos \left( \frac{3 \pi}{4} + x \right) - \cos \left( \frac{3 \pi}{4} - x \right) \]

Using the formula
12. \( \sin^2 6x - \sin^2 4x = \sin 2x \sin 10x \)

Solution:

Consider

L.H.S. = \( \sin^2 6x - \sin^2 4x \)

Using the formula

\[
\sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)
\]

\[
\sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)
\]

So we get

\[
= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x)
\]

By further calculation
13. \( \cos^2 2x - \cos^2 6x = \sin 4x \sin 8x \)

Solution:

Consider

L.H.S. = \( \cos^2 2x - \cos^2 6x \)

Using the formula

\[
\cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)
\]

\[
\cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)
\]

So we get

\[
= (\cos 2x + \cos 6x) (\cos 2x - 6x)
\]

By further calculation

\[
= \left[ 2 \cos \left( \frac{2x+6x}{2} \right) \cos \left( \frac{2x-6x}{2} \right) \right] \left[ -2 \sin \left( \frac{2x+6x}{2} \right) \sin \left( \frac{2x-6x}{2} \right) \right]
\]

We get

\[
= [2 \cos 4x \cos (2x)] [-2 \sin 4x \sin (-2x)]
\]

It can be written as

\[
= [2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]
\]

So we get

\[
= (2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)
\]

\[
= \sin 8x \sin 4x
\]

= RHS

14. \( \sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x \)

Solution:
Consider

L.H.S. = \sin 2x + 2 \sin 4x + \sin 6x

= [\sin 2x + \sin 6x] + 2 \sin 4x

Using the formula

\sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right).

= \left[ 2 \sin \left( \frac{2x+6x}{2} \right) \cos \left( \frac{2x-6x}{2} \right) \right] + 2 \sin 4x

By further simplification

= 2 \sin 4x \cos (-2x) + 2 \sin 4x

It can be written as

= 2 \sin 4x \cos 2x + 2 \sin 4x

Taking common terms

= 2 \sin 4x (\cos 2x + 1)

Using the formula

= 2 \sin 4x (2 \cos^2 x - 1 + 1)

We get

= 2 \sin 4x (2 \cos^2 x)

= 4 \cos^2 x \sin 4x

= \text{R.H.S.}

15. \cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)

Solution:

Consider

L.H.S. = \cot 4x (\sin 5x + \sin 3x)

It can be written as
Using the formula
\[ \sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right) \]

So we get
\[ = 2 \cos 4x \cos x \]

Similarly

R.H.S. = \( \cot x (\sin 5x - \sin 3x) \)

It can be written as
\[ \frac{\cos x}{\sin x} \left[ 2\cos\left(\frac{5x + 3x}{2}\right)\sin\left(\frac{5x - 3x}{2}\right) \right] \]

Using the formula
\[ \sin A - \sin B = 2\cos\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right) \]

So we get
\[ = 2 \cos 4x \cos x \]

Hence, LHS = RHS.

16. \[ \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{\sin 2x}{\cos 10x} \]

Solution:

Consider

\( \text{L.H.S} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \)

Using the formula
17. Solution:

\[
\cos A - \cos B = -2 \sin \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right)
\]

\[
\sin A - \sin B = 2 \cos \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right)
\]

\[
= \frac{-2 \sin \left( \frac{9x + 5x}{2} \right) \sin \left( \frac{9x - 5x}{2} \right)}{2 \cos \left( \frac{17x + 3x}{2} \right) \sin \left( \frac{17x - 3x}{2} \right)}
\]

By further calculation

\[
= \frac{-2 \sin 7x \sin 2x}{2 \cos 10x \sin 7x}
\]

So we get

\[
= -\frac{\sin 2x}{\cos 10x}
\]

= RHS

17. Solution:

\[
\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x
\]

Consider

\[
\text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}
\]

Using the formula

\[
\sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]

\[
\cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]

\[
= \frac{2 \sin \left( \frac{5x + 3x}{2} \right) \cos \left( \frac{5x - 3x}{2} \right)}{2 \cos \left( \frac{5x + 3x}{2} \right) \cos \left( \frac{5x - 3x}{2} \right)}
\]

By further calculation
\[
\frac{2 \sin 4x \cdot \cos x}{2 \cos 4x \cdot \cos x}
\]

So we get
\[
\frac{\sin 4x}{\cos 4x}
\]
\[
= \tan 4x
\]
\[
= \text{RHS}
\]

18.
\[
\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x - y}{2}
\]
**Solution:**

Consider

L.H.S. = \[
\frac{\sin x - \sin y}{\cos x + \cos y}
\]

Using the formula

\[
\sin A - \sin B = 2 \cos \left(\frac{A + B}{2}\right) \sin \left(\frac{A - B}{2}\right)
\]

\[
\cos A + \cos B = 2 \cos \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right)
\]

\[
= \frac{2 \cos \left(\frac{x + y}{2}\right) \cdot \sin \left(\frac{x - y}{2}\right)}{2 \cos \left(\frac{x + y}{2}\right) \cdot \cos \left(\frac{x - y}{2}\right)}
\]

By further calculation

\[
= \tan \left(\frac{x - y}{2}\right)
\]

So we get
\[
= \tan \left(\frac{x - y}{2}\right)
\]
\[
= \text{RHS}
\]
19. \[ \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x \]
Solution:

Consider

L.H.S. = \[ \frac{\sin x + \sin 3x}{\cos x + \cos 3x} \]

Using the formula

\[ \sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right) \]
\[ \cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right) \]

\[ = \frac{2 \sin \left( \frac{x + 3x}{2} \right) \cos \left( \frac{x - 3x}{2} \right)}{2 \cos \left( \frac{x + 3x}{2} \right) \cos \left( \frac{x - 3x}{2} \right)} \]

By further calculation

= \frac{\sin 2x}{\cos 2x}

So we get

= \tan 2x

= RHS

20. \[ \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x \]
Solution:

Consider

L.H.S. = \[ \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \]

Using the formula

\[ \sin A - \sin B = 2 \cos \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right) \]
21. Solution:

\[
\cos^2 A - \sin^2 A = \cos 2A
\]

\[
= \frac{2 \cos \left( \frac{x + 3x}{2} \right) \sin \left( \frac{x - 3x}{2} \right)}{-\cos 2x}
\]

By further calculation

\[
= \frac{2 \cos 2x \sin (-x)}{-\cos 2x}
\]

So we get

\[
= 2 \sin x
\]

= RHS

21. Solution:

\[
\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x
\]

Consider

L.H.S. = \[
\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}
\]

It can be written as

\[
= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}
\]

Using the formula

\[
\cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]

\[
\sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right)
\]

\[
= \frac{2 \cos \left( \frac{4x + 2x}{2} \right) \cos \left( \frac{4x - 2x}{2} \right) + \cos 3x}{2 \sin \left( \frac{4x + 2x}{2} \right) \cos \left( \frac{4x - 2x}{2} \right) + \sin 3x}
\]

By further calculation
\[
\frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x}
\]

So we get
\[
\frac{\cos 3x (2 \cos x + 1)}{\sin 3x (2 \cos x + 1)}
\]
\[
= \cot 3x
\]
\[
= \text{RHS}
\]

22. \(\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1\)

Solution:

Consider

LHS = \(\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x\)

It can be written as
\[
= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)
\]
\[
= \cot x \cot 2x - (\cot 2x + x) (\cot 2x + \cot x)
\]

Using the formula
\[
\cot (A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}
\]
\[
= \cot x \cot 2x - \left(\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right) (\cot 2x + \cot x)
\]

So we get
\[
= \cot x \cot 2x - (\cot 2x \cot x - 1)
\]
\[
= 1
\]
\[
= \text{RHS}
\]

23.
\[
\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}
\]

Solution:

Consider

LHS = \(\tan 4x = \tan 2(2x)\)

By using the formula
\[ \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \]

\[ = \frac{2 \tan 2x}{1 - \tan^2 (2x)} \]

It can be written as

\[ = \frac{2 \left( \frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left( \frac{2 \tan x}{1 - \tan^2 x} \right)^2} \]

By further simplification

\[ \left( \frac{4 \tan x}{1 - \tan^2 x} \right) \]

\[ = \left[ \frac{1 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \right] \]

Taking LCM

\[ \frac{4 \tan x}{1 - \tan^2 x} \]

\[ = \frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \]

On further simplification

\[ = \frac{4 \tan x (1 - \tan^2 x)}{(1 - \tan^2 x)^2 - 4 \tan^2 x} \]

We get

\[ = \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x} \]

It can be written as

\[ = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x} \]

\[ = \text{RHS} \]

24. \( \cos 4x = 1 - 8\sin^2 x \cos^2 x \)
Solution:

Consider
LHS = \cos 4x

We can write it as
= \cos 2(2x)

Using the formula \( \cos 2A = 1 - 2 \sin^2 A \)

= 1 - 2 \sin^2 2x

Again by using the formula \( \sin 2A = 2 \sin A \cos A \)

= 1 - 2(2 \sin x \cos x)^2

So we get

= 1 - 8 \sin^2 x \cos^2 x

= R.H.S.

25. \cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1

Solution:

Consider
L.H.S. = \cos 6x

It can be written as

= \cos 3(2x)

Using the formula \( \cos 3A = 4 \cos^3 A - 3 \cos A \)

= 4 \cos^3 2x - 3 \cos 2x

Again by using formula \( \cos 2x = 2 \cos^2 x - 1 \)

= 4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1)]

By further simplification

= 4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)]

We get

= 4 [8 \cos^6 x - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3

By multiplication

= 32 \cos^6 x - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3

On further calculation

= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1

= R.H.S.
EXERCISE 3.4

Find the principal and general solutions of the following equations:

1. \( \tan x = \sqrt{3} \)
   
   Solution:
   
   It is given that
   
   \( \tan x = \sqrt{3} \)
   
   We know that
   
   \( \tan \frac{\pi}{3} = \sqrt{3} \)
   
   It can be written as
   
   \( \tan \left( \frac{4\pi}{3} \right) = \tan \left( \pi + \frac{\pi}{3} \right) \)
   
   So we get
   
   \( = \tan \frac{\pi}{3} = \sqrt{3} \)
   
   Hence, the principal solutions are \( x = \pi/3 \) and \( 4\pi/3 \)

   \( \tan x = \tan \frac{\pi}{3} \)
   
   We get
   
   \( x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z} \)
   
   Hence, the general solution is
   
   \( x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z} \)

2. \( \sec x = 2 \)
   
   Solution:
   
   It is given that
   
   \( \sec x = 2 \)
   
   We know that
   
   \( \sec \frac{\pi}{3} = 2 \)
   
   It can be written as
3. cot \( x = -\sqrt{3} \)

**Solution:**

It is given that
\[ \cot x = -\sqrt{3} \]

We know that
\[ \cot \frac{\pi}{6} = \sqrt{3} \]

It can be written as
\[ \cot \left( \frac{\pi}{6} - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3} \]

And
\[ \cot \left( 2\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3} \]

So we get
\[ \cot \frac{5\pi}{6} = -\sqrt{3} \text{ and } \cot \frac{11\pi}{6} = -\sqrt{3} \]
Hence, the principal solutions are \( x = \frac{5\pi}{6} \) and \( 11\pi/6 \).

\[
\cot x = \cot \frac{5\pi}{6}
\]

We know that \( \cot x = 1/\tan x \)

\[
\tan x = \tan \frac{5\pi}{6}
\]

So we get

\[
x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}
\]

Hence, the general solution is

\[
x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}
\]

4. \( \csc x = -2 \)

Solution:

It is given that

\( \csc x = -2 \)

We know that

\( \csc \frac{\pi}{6} = 2 \)

It can be written as

\[
\csc \left( \pi + \frac{\pi}{6} \right) = -\csc \frac{\pi}{6} = -2
\]

And

\[
\csc \left( 2\pi - \frac{\pi}{6} \right) = -\csc \frac{\pi}{6} = -2
\]

So we get

\[
\csc \frac{7\pi}{6} = -2 \text{ and } \csc \frac{11\pi}{6} = -2
\]

Hence, the principal solutions are \( x = 7\pi/6 \) and \( 11\pi/6 \).

\[
\csc x = \csc \frac{7\pi}{6}
\]
We know that cosec \( x = \frac{1}{\sin x} \)

\[ \sin x = \sin \frac{7\pi}{6} \]

So we get

\[ x = n\pi + (-1)^n \frac{7\pi}{6} \text{, where } n \in \mathbb{Z} \]

Hence, the general solution is

\[ x = n\pi + (-1)^n \frac{7\pi}{6} \text{, where } n \in \mathbb{Z} \]

Find the general solution for each of the following equations:

5. \( \cos 4x = \cos 2x \)

Solution:

It is given that

\( \cos 4x = \cos 2x \)

We can write it as

\( \cos 4x - \cos 2x = 0 \)

Using the formula

\[ \cos A - \cos B = -2\sin \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right) \]

We get

\[ -2 \sin \left( \frac{4x + 2x}{2} \right) \sin \left( \frac{4x - 2x}{2} \right) = 0 \]

By further simplification

\[ \sin 3x \sin x = 0 \]

We can write it as

\[ \sin 3x = 0 \text{ or } \sin x = 0 \]

By equating the values

\[ 3x = n\pi \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z} \]

We get

\[ x = \frac{n\pi}{3} \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z} \]
6. \( \cos 3x + \cos x - \cos 2x = 0 \)

Solution:

It is given that

\[ \cos 3x + \cos x - \cos 2x = 0 \]

We can write it as

\[ 2 \cos \left( \frac{3x + x}{2} \right) \cos \left( \frac{3x - x}{2} \right) - \cos 2x = 0 \]

Using the formula

\[ \cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right) \]

We get

\[ 2 \cos 2x \cos x - \cos 2x = 0 \]

By further simplification

\[ \cos 2x (2 \cos x - 1) = 0 \]

We can write it as

\[ \cos 2x = 0 \text{ or } 2 \cos x - 1 = 0 \]

\[ \cos 2x = 0 \text{ or } \cos x = 1/2 \]

By equating the values

\[ 2x = (2n + 1) \frac{\pi}{2} \quad \text{or} \quad \cos x = \cos \frac{\pi}{3}, \text{ where } n \in \mathbb{Z} \]

We get

\[ x = (2n + 1) \frac{\pi}{4} \quad \text{or} \quad x = 2n \pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z} \]

7. \( \sin 2x + \cos x = 0 \)

Solution:

It is given that

\[ \sin 2x + \cos x = 0 \]

We can write it as

\[ 2 \sin x \cos x + \cos x = 0 \]

\[ \cos x (2 \sin x + 1) = 0 \]

\[ \cos x = 0 \text{ or } 2 \sin x + 1 = 0 \]

Let \( \cos x = 0 \)
sec^2 2x = 1 - tan 2x

Solution:

It is given that sec^2 2x = 1 - tan 2x

We can write it as

1 + tan^2 2x = 1 - tan 2x

Taking common terms

tan 2x (tan 2x + 1) = 0

If tan 2x = 0

tan 2x = tan 0

We get

2x = nπ, where n ∈ Z

x = nπ/2, where n ∈ Z

If tan 2x + 1 = 0

We can write it as

tan 2x = -1

So we get

x = -7π/6, where n ∈ Z

Hence, the general solution is

(2n + 1)π/2 or nπ + (-1)^n 7π/6, n ∈ Z
Here
2x = nπ + 3π/4, where n ∈ Z
x = nπ/2 + 3π/8, where n ∈ Z
Hence, the general solution is nπ/2 or nπ/2 + 3π/8, n ∈ Z.

9. sin x + sin 3x + sin 5x = 0
Solution:
It is given that
sin x + sin 3x + sin 5x = 0
We can write it as
(sin x + sin 5x) + sin 3x = 0
Using the formula
2 sin \( \frac{A + B}{2} \) cos \( \frac{A - B}{2} \)
By further calculation
2 sin 3x cos (-2x) + sin 3x = 0
It can be written as
2 sin 3x cos 2x + sin 3x = 0
By taking out the common terms
sin 3x (2 cos 2x + 1) = 0
Here
sin 3x = 0 or 2 cos 2x + 1 = 0
If sin 3x = 0
3x = nπ, where n ∈ Z
We get
x = nπ/3, where n ∈ Z
If 2 cos 2x + 1 = 0
cos 2x = -1/2
By further simplification
= - cos π/3
= cos (π − π/3)
So we get
cos 2x = cos 2π/3
Here
\[2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in \mathbb{Z}\]

Dividing by 2

\[x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}\]

Hence, the general solution is

\[\frac{n\pi}{3} \text{ or } n\pi \pm \frac{\pi}{3}, \text{ } n \in \mathbb{Z}\]
MISCELLANEOUS EXERCISE

Prove that:
1. \(2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0\)

Solution:

L.H.S. = \(2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13}\)

Using the formula

\[\cos x + \cos y = 2 \cos \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)\]

so we get

\[= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \left(\frac{3\pi + 5\pi}{2}\right) \cos \left(\frac{3\pi - 5\pi}{2}\right)\]

By further calculation

\[= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \left(\frac{-\pi}{13}\right)\]

We get

\[= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \frac{\pi}{13}\]

Taking out the common terms

\[= 2 \cos \frac{\pi}{13} \left[ \cos \frac{9\pi}{13} + \cos \frac{4\pi}{13} \right]\]

It can be written as

\[= 2 \cos \frac{\pi}{13} \left[ 2 \cos \frac{9\pi + 4\pi}{2} \cos \frac{9\pi - 4\pi}{2} \right]\]

On further calculation

\[= 2 \cos \frac{\pi}{13} \left[ 2 \cos \frac{\pi}{2} \cos \frac{5\pi}{26} \right]\]

We get

\[= 2 \cos \frac{\pi}{13} \times 2 \times 0 \times \cos \frac{5\pi}{26}\]

\[= 0\]
2. $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

Solution:

Consider LHS = $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$

By further calculation

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

Taking out the common terms

$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$

Using the formula

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$= \cos (3x - x) - \cos 2x$

So we get

$= \cos 2x - \cos 2x$

$= 0$

= RHS

3.

$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$

Solution:

Consider LHS = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

By expanding using formula we get

$= \cos^2 x + \cos^2 y + 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y$

Grouping the terms

$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2 (\cos x \cos y - \sin x \sin y)$

Using the formula

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

$= 1 + 1 + 2 \cos (x + y)$

By further calculation

$= 2 + 2 \cos (x + y)$

Taking 2 as common

$= 2 [1 + \cos (x + y)]$

From the formula

$\cos 2A = 2 \cos^2 A - 1$

$= 2 \left[ 1 + 2 \cos^2 \left( \frac{x+y}{2} \right) - 1 \right]$

We get

$= 4 \cos^2 \left( \frac{x+y}{2} \right)$

= RHS

4.

$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \frac{x-y}{2}$
Solution:

LHS = \((\cos x - \cos y)^2 + (\sin x - \sin y)^2\)

By expanding using formula

= \(\cos^2 x + \cos^2 y - 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y\)

Grouping the terms

= \((\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2 (\cos x \cos y + \sin x \sin y)\)

Using the formula \(\cos (A - B) = \cos A \cos B + \sin A \sin B\)

= \(1 + 1 - 2 \cos (x - y)\)

By further calculation

= \(2 \left[ 1 - \cos (x - y) \right]\)

From formula \(\cos 2A = 1 - 2 \sin^2 A\)

= \(2 \left[ 1 - \left(1 - 2 \sin^2 \left(\frac{x-y}{2}\right)\right)\right]\)

We get

= \(4 \sin^2 \left(\frac{x-y}{2}\right)\)

= RHS

5. \(\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x\)

Solution:

Consider

LHS = \(\sin x + \sin 3x + \sin 5x + \sin 7x\)

Grouping the terms

= \((\sin x + \sin 5x) + (\sin 3x + \sin 7x)\)

Using the formula

\(\sin A + \sin B = 2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)\)

So we get

= \(2 \sin \left(\frac{x+5x}{2}\right) \cos \left(\frac{x-5x}{2}\right) + 2 \sin \left(\frac{3x+7x}{2}\right) \cos \left(\frac{3x-7x}{2}\right)\)

By further calculation

= \(2 \sin 3x \cos (-2x) + 2 \sin 5x \cos (-2x)\)

We get

= \(2 \sin 3x \cos 2x + 2 \sin 5x \cos 2x\)

Taking out the common terms
6. \[ \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x \]

**Solution:**

\[ L.H.S. = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} \]

Using the formula

\[ \sin A + \sin B = 2 \sin \left( \frac{A + B}{2} \right) \cdot \cos \left( \frac{A - B}{2} \right), \quad \cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cdot \cos \left( \frac{A - B}{2} \right) \]

\[ = \frac{2 \sin \left( \frac{7x + 5x}{2} \right) \cdot \cos \left( \frac{7x - 5x}{2} \right)}{2 \cos \left( \frac{7x + 5x}{2} \right) \cdot \cos \left( \frac{7x - 5x}{2} \right)} + \frac{2 \sin \left( \frac{9x + 3x}{2} \right) \cdot \cos \left( \frac{9x - 3x}{2} \right)}{2 \cos \left( \frac{9x + 3x}{2} \right) \cdot \cos \left( \frac{9x - 3x}{2} \right)} \]

By further calculation

\[ = \frac{[2 \sin 6x \cdot \cos x] + [2 \sin 6x \cdot \cos 3x]}{[2 \cos 6x \cdot \cos x] + [2 \cos 6x \cdot \cos 3x]} \]

Taking out the common terms

\[ = \frac{2 \sin 6x [\cos x + \cos 3x]}{2 \cos 6x [\cos x + \cos 3x]} \]

We get

\[ = \tan 6x \]

\[ = \text{RHS} \]
7. \[ \sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2} \]

Solution:

LHS = \( \sin 3x + \sin 2x - \sin x \)

It can be written as

\[ = \sin 3x + (\sin 2x - \sin x) \]

Using the formula

\[ \sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \]

\[ = \sin 3x + 2 \cos \left( \frac{2x + x}{2} \right) \sin \left( \frac{2x - x}{2} \right) \]

By further simplification

\[ = \sin 3x + 2 \cos \frac{3x}{2} \sin \frac{x}{2} \]

\[ = \sin 3x + 2 \cos \frac{3x}{2} \sin \frac{x}{2} \]

Using formula \( \sin 2A = 2 \sin A \cos B \)

\[ = 2 \sin \frac{3x}{2} \cos \frac{3x}{2} + 2 \cos \frac{3x}{2} \sin \frac{x}{2} \]

Taking out the common terms

\[ = 2 \cos \left( \frac{3x}{2} \right) \sin \left( \frac{3x}{2} \right) + \sin \left( \frac{x}{2} \right) \]

From the formula

\[ \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \]

\[ = 2 \cos \left( \frac{3x}{2} \right) \left( 2 \sin \frac{3x}{2} \right) \cos \left( \frac{3x}{2} \right) \]

By further simplification

\[ = 2 \cos \left( \frac{3x}{2} \right) \cdot 2 \sin x \cos \left( \frac{x}{2} \right) \]
Find \( \sin x/2, \cos x/2 \) and \( \tan x/2 \) in each of the following:

8.

Solution:

We get

\[
4 \sin x \cos \left( \frac{x}{2} \right) \cos \left( \frac{3x}{2} \right) = \text{RHS}
\]

Find \( \sin x/2, \cos x/2 \) and \( \tan x/2 \) in each of the following:

8.

\( \tan x = -\frac{4}{3} \), \( x \) in quadrant II

Solution:

It is given that

\( x \) is in quadrant II

\( \frac{\pi}{2} < x < \pi \)

Dividing by 2

\( \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \)

Hence, \( \sin x/2, \cos x/2 \) and \( \tan x/2 \) are all positive.

\( \tan x = -\frac{4}{3} \)

From the formula \( \sec^2 x = 1 + \tan^2 x \)

Substituting the values

\( \sec^2 x = 1 + \left( -\frac{4}{3} \right)^2 \)

We get

\( = 1 + \frac{16}{9} = \frac{25}{9} \)

Here

\( \cos^2 x = \frac{9}{25} \)

\( \cos x = \pm \frac{3}{5} \)

Here \( x \) is in quadrant II, \( \cos x \) is negative.

\( \cos x = -\frac{3}{5} \)

From the formula
\[
\cos x = 2 \cos^2 \frac{x}{2} - 1
\]

Substituting the values

\[
\frac{-3}{5} = 2 \cos^2 \frac{x}{2} - 1
\]

By further calculation

\[
2 \cos^2 \frac{x}{2} = 1 - \frac{3}{5}
\]

\[
2 \cos^2 \frac{x}{2} = \frac{2}{5}
\]

\[
\cos^2 \frac{x}{2} = \frac{1}{5}
\]

We get

\[
\cos \frac{x}{2} = \frac{1}{\sqrt{5}}
\]

\[
\cos \frac{x}{2} = \frac{\sqrt{5}}{5}
\]

From the formula

\[
\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1
\]

Substituting the value

\[
\sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1
\]

By further calculation

\[
\sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}
\]

We get

\[
\sin \frac{x}{2} = \frac{2}{\sqrt{5}}
\]

\[
\sin \frac{x}{2} = \frac{2\sqrt{5}}{5}
\]
Here

\[ \tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = 2 \]

Hence, the respective values of \( \sin x/2 \), \( \cos x/2 \) and \( \tan x/2 \) are

\[ \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \text{, and } 2 \]

9. \( \cos x = -1/3 \), \( x \) in quadrant III

Solution:

It is given that

\( x \) is in quadrant III

\[ \pi < x < \frac{3\pi}{2} \]

Dividing by 2

\[ \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \]

Hence, \( \cos x/2 \) and \( \tan x/2 \) are negative where \( \sin x/2 \) is positive.

\[ \cos x = -\frac{1}{3} \]

From the formula \( \cos x = 1 - 2 \sin^2 x/2 \)

We get

\[ \sin^2 x/2 = \frac{1 - (-\frac{1}{3})}{2} = \frac{1 + \frac{1}{3}}{2} \]

Substituting the values

\[ \sin^2 \frac{x}{2} = \frac{1 - \frac{1}{3}}{2} = \frac{1 + \frac{1}{3}}{2} \]

We get

\[ \frac{4}{2} = \frac{3}{3} = \frac{2}{3} \]

https://byjus.com
Here

\[ \sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \]

\[ \sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3} \]

Using the formula

\[ \cos x = 2 \cos^2 \frac{x}{2} - 1 \]

We get

\[ \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} \]

Substituting the values

\[ 1 + \left( \frac{-1}{3} \right) = \left( \frac{3 - 1}{3} \right) \]

\[ = \frac{2}{3} \]

\[ = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} \]

We get

\[ \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \]

By further calculation

\[ \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \]

Here

\[ \tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left( \frac{\sqrt{2}}{\sqrt{3}} \right)}{\left( -\frac{1}{\sqrt{3}} \right)} = -\sqrt{2} \]

Therefore, the respective values of \( \sin x/2 \), \( \cos x/2 \) and \( \tan x/2 \) are

\[ \frac{\sqrt{6}}{3}, \frac{-\sqrt{3}}{3}, \text{ and } -\sqrt{2} \]
10. \( \sin x = \frac{1}{4} \), \( x \) in quadrant II

Solution:

It is given that

\( x \) is in quadrant II

\[ \frac{\pi}{2} < x < \pi \]

Dividing by 2

\[ \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \]

Hence, \( \sin \frac{x}{2}, \cos \frac{x}{2} \) and \( \tan \frac{x}{2} \) are positive.

\[ \sin x = \frac{1}{4} \]

From the formula \( \cos^2 x = 1 - \sin^2 x \)

We get

\[ \cos^2 x = 1 - \left(\frac{1}{4}\right)^2 \]

Substituting the values

\[ \cos^2 x = 1 - \frac{1}{16} = \frac{15}{16} \]

We get

\[ \cos x = -\frac{\sqrt{15}}{4} \]

Here

\[ \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} \]

Substituting the values

\[ 1 - \left( -\frac{\sqrt{15}}{4} \right) = \frac{4 + \sqrt{15}}{8} \]

We get

\[ \sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}} \]

Multiplying and dividing by 2
\[\sqrt{\frac{4 + \sqrt{15}}{8} \times \frac{2}{2}}\]

By further calculation

\[= \sqrt{\frac{8 + 2\sqrt{15}}{16}}\]

\[= \frac{\sqrt{8 + 2\sqrt{15}}}{4}\]

Here

\[\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}\]

By substituting the values

\[= \frac{1 + \left(\frac{-\sqrt{15}}{4}\right)}{2} = \frac{4 - \sqrt{15}}{8}\]

We get

\[\cos \frac{x}{2} = \frac{\sqrt{4 - \sqrt{15}}}{8}\]

By multiplying and dividing by 2

\[= \sqrt{\frac{4 - \sqrt{15}}{8} \times \frac{2}{2}}\]

It can be written as

\[= \sqrt{\frac{8 - 2\sqrt{15}}{16}}\]

\[= \frac{\sqrt{8 - 2\sqrt{15}}}{4}\]

We know that

\[\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}\]
Substituting the values

\[
\frac{\sqrt{8 + 2\sqrt{15}}}{4} \cdot \frac{\sqrt{8 - 2\sqrt{15}}}{4} = \frac{\sqrt{8 + 2\sqrt{15}}}{\sqrt{8 - 2\sqrt{15}}}
\]

By multiplying and dividing the terms

\[
\frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}} \times \frac{8 + 2\sqrt{15}}{8 + 2\sqrt{15}}
\]

We get

\[
\frac{(8 + 2\sqrt{15})^2}{64 - 60} = \frac{8 + 2\sqrt{15}}{2}
\]

\[
= 4 + \sqrt{15}
\]

Therefore, the respective values of \(\sin x/2\), \(\cos x/2\) and \(\tan x/2\) are

\[
\frac{\sqrt{8 + 2\sqrt{15}}}{4}, \frac{\sqrt{8 - 2\sqrt{15}}}{4} \quad \text{and} \quad 4 + \sqrt{15}
\]