Exercise 5.1

Express each of the complex number given in the Exercises 1 to 10 in the form a + ib.

1. (5i) (-3/5i)

Solution:

(5i) (-3/5i) = 5 x (-3/5) x i^2
= -3 x -1 [i^2 = -1]
= 3

Hence,
(5i) (-3/5i) = 3 + i0

2. $i^9 + i^{19}$

Solution:

$i^9 + i^{19} = (i^2)^4 . i + (i^2)^9 . i$
$= (-1)^4 . i + (-1)^9 .i$
= 1 x i + -1 x i
= i – i
= 0

Hence,
$i^9 + i^{19}$ = 0 + i0

3. i^{-39}

Solution:

$i^{-39} = 1/ i^{39} = 1/ i^{4 \times 9 + 3} = 1/ (1^9 \times i^3) = 1/ i^3 = 1/ (-i)$ [$i^4 = 1$, i^3 = -I and $i^2 = -1$]

Now, multiplying the numerator and denominator by i we get
i^{-39} = 1 x i / (-i x i)
= i/ 1 = i

Hence,
i^{-39} = 0 + i

4. 3(7 + *i*7) + *i*(7 + *i*7)

Solution:

3(7 + *i*7) + *i*(7 + *i*7) = 21 + *i*21 + *i*7 + i^2 7
= 21 + *i*28 – 7 [i^2 = -1]
= 14 + *i*28

Hence,
3(7 + *i*7) + *i*(7 + *i*7) = 14 + *i*28

5. (1 – *i*) – (–1 + *i*6)

Solution:

(1 – *i*) – (–1 + *i*6) = 1 – *i* + 1 - *i*6
= 2 – *i*7

Hence,
$(1 - i) - (-1 + i6) = 2 - i7$

6. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Solution:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Hence,
$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)=\frac{-19}{5}-\frac{21}{10}i$$

7. $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Solution:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Hence,

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)=\frac{17}{3}+i\frac{5}{3}$$

8. $(1 - i)^4$
Solution:

$(1 - i)^4 = [(1 - i)^2]^2$
$= [1 + i^2 - 2i]^2$
$= [1 - 1 - 2i]^2$ $[i^2 = -1]$
$= (-2i)^2$
$= 4(-1)$
$= -4$
Hence, $(1 - i)^4 = -4 + 0i$

9. $(1/3 + 3i)^3$
Solution:

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27(-i) + i + 9i^2 \qquad [i^3 = -i]$$
$$= \frac{1}{27} - 27i + i - 9 \qquad [i^2 = -1]$$
$$= \left(\frac{1}{27} - 9\right) + i(-27+1)$$
$$= \frac{-242}{27} - 26i$$

Hence, $(1/3 + 3i)^3 = -242/27 - 26i$

10. $(-2 - 1/3i)^3$
Solution:

$$\left(-2-\frac{1}{3}i\right)^3 = (-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$= -\left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$= -\left[8 + \frac{i^3}{27} + 2i\left(2+\frac{i}{3}\right)\right]$$
$$= -\left[8 - \frac{i}{27} + 4i + \frac{2i^2}{3}\right] \qquad [i^3 = -i]$$
$$= -\left[8 - \frac{i}{27} + 4i - \frac{2}{3}\right] \qquad [i^2 = -1]$$

$$= -\left[\frac{22}{3} + \frac{107i}{27}\right]$$

$$= -\frac{22}{3} - \frac{107}{27}i$$

Hence,
$(-2 - 1/3i)^3 = -22/3 - 107/27i$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.
11. 4 – 3i
Solution:

Let's consider $z = 4 - 3i$
Then,
$\bar{z} = 4 + 3i$ and
$|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$
Thus, the multiplicative inverse of $4 - 3i$ is given by z^{-1}

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

12. $\sqrt{5} + 3i$
Solution:

Let's consider $z = \sqrt{5} + 3i$
Then, $\bar{z} = \sqrt{5} - 3i$ and
$|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$
Thus, the multiplicative inverse of $\sqrt{5} + 3i$ is given by z^{-1}

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

13. – i
Solution:

Let's consider $z = -i$
Then, $\bar{z} = i$ and
$|z|^2 = 1^2 = 1$
Thus, the multiplicative inverse of $-i$ is given by z^{-1}

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

14. Express the following expression in the form of a + ib:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Solution:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Hence,

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)} = 0+\frac{-7\sqrt{2}i}{2}$$

Exercise 5.2

Find the modulus and the arguments of each of the complex numbers in Exercises 1 to 2.

1. z = – 1 – i √3

Solution:

Given,

$z = -1 - i\sqrt{3}$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we get

$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$

$r^2 = 4$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$r = \sqrt{4} = 2$ $\quad$ [Conventionally, $r > 0$]

Thus, modulus = 2

So, we have

$2\cos\theta = -1$ and $2\sin\theta = -\sqrt{3}$

$\cos\theta = \frac{-1}{2}$ and $\sin\theta = \frac{-\sqrt{3}}{2}$

As the values of both sin θ and cos θ are negative, θ lies in III Quadrant,

$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$

Therefore, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

2. z = -√3 + i

Solution:

Given,

$z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding , we get

$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$

$r^2 = 3 + 1 = 4$ $\quad\quad [\cos^2\theta + \sin^2\theta = 1]$

$r = \sqrt{4} = 2$ $\quad\quad$ [Conventionally, $r > 0$]

Thus, modulus = 2

So,

$2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\quad\quad$ [As θ lies in the II quadrant]

Therefore, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$ respectively.

Convert each of the complex numbers given in Exercises 3 to 8 in the polar form:

3. 1 – i

Solution:

Given complex number,

$1 - i$

Let $r\cos\theta = 1$ and $r\sin\theta = -1$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$

$r^2 = 2$

$r = \sqrt{2}$ = Modulus [Conventionally, $r > 0$]

So,

$\sqrt{2}\cos\theta = 1$ and $\sqrt{2}\sin\theta = -1$

$\cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta = -\frac{\pi}{4}$ $\quad\quad$ [As θ lies in the IV quadrant]

So,

$1 - i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

Hence, this is the required polar form.

4. – 1 + i

Solution:

Given complex number,

$-1 + i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$

$$\cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Hence, it can be written as

$$-1 + i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

5. – 1 – i

Solution:

Given complex number,

$-1 - i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$\sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$ [As θ lies in the III quadrant]

Hence, it can be written as

$$-1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$$
$$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

6. – 3
Solution:

Given complex number,

–3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$
$$r^2(\cos^2\theta + \sin^2\theta) = 9$$
$$r^2 = 9$$
$$r = \sqrt{9} = 3 \quad [\text{Conventionally, } r > 0]$$

So,

$3\cos\theta = -3$ and $3\sin\theta = 0$

$\Rightarrow \cos\theta = -1$ and $\sin\theta = 0$

$\therefore \theta = \pi$

Hence, it can be written as

$$-3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$$

This is the required polar form.

7. 3 + *i*
Solution:

Given complex number,

$\sqrt{3} + i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$$

$r^2(\cos^2\theta + \sin^2\theta) = 3 + 1$

$r^2 = 4$

$r = \sqrt{4} = 2$ [Conventionally, $r > 0$]

So,

$2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}$ [As θ lies in the I quadrant]

Hence, it can be written as

$\sqrt{3} + i = r\cos\theta + ir\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6}$

$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

This is the required polar form.

8. ***i***

Solution:

Given complex number, i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1$

$r^2 = 1$

$r = \sqrt{1} = 1$ [Conventionally, $r > 0$]

So,

$\cos\theta = 0$ and $\sin\theta = 1$

$\therefore \theta = \frac{\pi}{2}$

Hence, it can be written as

$i = r\cos\theta + ir\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

This is the required polar form.

Exercise 5.3

Solve each of the following equations:

1. $x^2 + 3 = 0$

Solution:

Given quadratic equation,

$x^2 + 3 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 0$, and $c = 3$

So, the discriminant of the given equation will be

$D = b^2 – 4ac = 0^2 – 4 \times 1 \times 3 = –12$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2\times 1} = \frac{\pm\sqrt{12}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

$$\therefore\ x = \frac{\pm 2\sqrt{3}\, i}{2} = \pm\sqrt{3}\, i$$

2. $2x^2 + x + 1 = 0$

Solution:

Given quadratic equation,

$2x^2 + x + 1 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 2$, $b = 1$, and $c = 1$

So, the discriminant of the given equation will be

$D = b^2 – 4ac = 1^2 – 4 \times 2 \times 1 = 1 – 8 = –7$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\times 2} = \frac{-1 \pm \sqrt{7}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$

3. $x^2 + 3x + 9 = 0$

Solution:

Given quadratic equation,

$x^2 + 3x + 9 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 3$, and $c = 9$

So, the discriminant of the given equation will be

$D = b^2 – 4ac = 3^2 – 4 \times 1 \times 9 = 9 – 36 = –27$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

4. $-x^2 + x - 2 = 0$
Solution:

Given quadratic equation,
$-x^2 + x - 2 = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = -1$, $b = 1$, and $c = -2$
So, the discriminant of the given equation will be
$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$
Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm \sqrt{7}\,i}{-2} \qquad \left[\sqrt{-1} = i\right]$$

5. $x^2 + 3x + 5 = 0$
Solution:

Given quadratic equation,
$x^2 + 3x + 5 = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = 1$, $b = 3$, and $c = 5$
So, the discriminant of the given equation will be
$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$
Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

6. $x^2 - x + 2 = 0$
Solution:

Given quadratic equation,
$x^2 - x + 2 = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = 1$, $b = -1$, and $c = 2$
So, the discriminant of the given equation is
$D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7$
Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1} = \frac{1 \pm \sqrt{7}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

7. $\sqrt{2}x^2 + x + \sqrt{2} = 0$
Solution:

Given quadratic equation,
$\sqrt{2}x^2 + x + \sqrt{2} = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$
So, the discriminant of the given equation is
$D = b^2 - 4ac = (1)^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$
Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times \sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

8. $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$
Solution:

Given quadratic equation,
$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = \sqrt{3}$, $b = -\sqrt{2}$, and $c = 3\sqrt{3}$
So, the discriminant of the given equation is
$D = b^2 - 4ac = (-\sqrt{2})^2 - 4 \times \sqrt{3} \times 3\sqrt{3} = 2 - 36 = -34$
Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\left(-\sqrt{2}\right) \pm \sqrt{-34}}{2 \times \sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}\,i}{2\sqrt{3}} \qquad \left[\sqrt{-1} = i\right]$$

9. $x^2 + x + 1/\sqrt{2} = 0$
Solution:

Given quadratic equation,
$x^2 + x + 1/\sqrt{2} = 0$
It can be rewritten as,
$\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = \sqrt{2}$, $b = \sqrt{2}$, and $c = 1$
So, the discriminant of the given equation is
$D = b^2 - 4ac = (\sqrt{2})^2 - 4 \times \sqrt{2} \times 1 = 2 - 4\sqrt{2} = 2(1 - 2\sqrt{2})$
Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\sqrt{2} \pm \sqrt{2 - 4\sqrt{2}}}{2 \times \sqrt{2}} = \frac{-\sqrt{2} \pm \sqrt{2\left(1 - 2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$= \left(\frac{-\sqrt{2} \pm \sqrt{2}\left(\sqrt{2\sqrt{2} - 1}\right)i}{2\sqrt{2}}\right) \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{-1 \pm \left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

10. $x^2 + x/\sqrt{2} + 1 = 0$
Solution:

Given quadratic equation,
$x^2 + x/\sqrt{2} + 1 = 0$
It can be rewritten as,
$\sqrt{2}x^2 + x + \sqrt{2} = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$
So, the discriminant of the given equation is
$D = b^2 - 4ac = (1)^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$
Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

Miscellaneous Exercise

1. Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Solution:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$

$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$

$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$

$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$

$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$

$$=\left[-1+\frac{i}{i^2}\right]^3$$

$$=\left[-1-i\right]^3$$

$$=\left(-1\right)^3\left[1+i\right]^3$$

$$=-\left[1^3+i^3+3\cdot1\cdot i\left(1+i\right)\right]$$

$$=-\left[1+i^3+3i+3i^2\right]$$

$$=-\left[1-i+3i-3\right]$$

$$=-\left[-2+2i\right]$$

$$=2-2i$$

2. For any two complex numbers z_1 and z_2, prove that
Re (z_1z_2) = Re z_1 Re z_2 – Im z_1 Im z_2
Solution:

Lets's assume $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ as two complex numbers

Product of these complex numbers, z_1z_2

$$z_1z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

Now,

$$\text{Re}(z_1z_2) = x_1x_2 - y_1y_2$$
$$\Rightarrow \text{Re}(z_1z_2) = \text{Re}\,z_1\,\text{Re}\,z_2 - \text{Im}\,z_1\,\text{Im}\,z_2$$

Hence, proved.

3. Reduce $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ **to the standard form**

Solution:

$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$

$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad [\text{On multiplying numerator and denominator by } (14+5i)]$$
$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$
$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442}+\frac{599i}{442}$$

Hence, this is the required standard form.

4. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Solution:

Given,

$$x - iy = \sqrt{\frac{a - ib}{c - id}}$$

$$= \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}} \left[\text{On multiplying numerator and denominator by } (c + id)\right]$$

$$= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}}$$

So,

$$(x - iy)^2 = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

$$x^2 - y^2 - 2ixy = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

On comparing real and imaginary parts, we get

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2}, \quad -2xy = \frac{ad - bc}{c^2 + d^2} \qquad (1)$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \qquad [\text{Using } (1)]$$

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}$$

$$= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}$$

$$= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}$$

$$= \frac{a^2 + b^2}{c^2 + d^2}$$

- Hence Proved

5. Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$, **(ii)** $\frac{1+3i}{1-2i}$

Solution:

(i) Here, $z=\frac{1+7i}{(2-i)^2}$

$=\frac{1+7i}{(2-i)^2}=\frac{1+7i}{4+i^2-4i}=\frac{1+7i}{4-1-4i}$

$=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}=\frac{3+4i+21i+28i^2}{3^2+4^2}$ [Multiplying by its conjugate in the numerator and denominator]

$=\frac{3+4i+21i-28}{3^2+4^2}=\frac{-25+25i}{25}$

$=-1+i$

Let $r\cos\theta=-1$ and $r\sin\theta=1$

On squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=1+1$

$r^2(\cos^2\theta+\sin^2\theta)=2$

$r^2=2$ $[\cos^2\theta+\sin^2\theta=1]$

$r=\sqrt{2}$ [Conventionally, $r>0$]

So,

$\sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow\cos\theta=\frac{-1}{\sqrt{2}}$ and $\sin\theta=\frac{1}{\sqrt{2}}$

$\therefore\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$ [As θ lies in II quadrant]

Expressing as, $z=r\cos\theta+i\,r\sin\theta$

$=\sqrt{2}\cos\frac{3\pi}{4}+i\sqrt{2}\sin\frac{3\pi}{4}=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

Therefore, this is the required polar form.

(ii) Let, $z = \frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i+3i-6}{1+4}$$

$$= \frac{-5+5i}{5} = -1+i$$

Now,

Let $r \cos \theta = -1$ and $r \sin \theta = 1$

On squaring and adding, we get

$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$

$r^2 (\cos^2 \theta + \sin^2 \theta) = 2$

$r^2 = 2$ [$\cos^2 \theta + \sin^2 \theta = 1$]

$\Rightarrow r = \sqrt{2}$ [Conventionally, $r > 0$]

$\therefore \sqrt{2} \cos \theta = -1$ and $\sqrt{2} \sin \theta = 1$

$$\cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in II quadrant]

Expressing as, $z = r \cos \theta + i\, r \sin \theta$

$$z = \sqrt{2} \cos \frac{3\pi}{4} + i\sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$

Therefore, this is the required polar form.

Solve each of the equation in Exercises 6 to 9.

6. $3x^2 - 4x + 20/3 = 0$

Solution:

Given quadratic equation, $\mathbf{3x^2 - 4x + 20/3 = 0}$

It can be re-written as: $9x^2 - 12x + 20 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 9$, $b = -12$, and $c = 20$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18}$$

$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3} i$$

7. $x^2 – 2x + 3/2 = 0$
Solution:

Given quadratic equation, $x^2 – 2x + 3/2 = 0$
It can be re-written as $2x^2 – 4x + 3 = 0$
On comparing it with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = –4$, and $c = 3$
So, the discriminant of the given equation will be
$D = b^2 – 4ac = (–4)^2 – 4 \times 2 \times 3 = 16 – 24 = –8$
Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2\times 2} = \frac{4 \pm 2\sqrt{2}\,i}{4} \qquad [\sqrt{-1} = i]$$

$$= \frac{2 \pm \sqrt{2}\,i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

8. $27x^2 – 10x + 1 = 0$
Solution:

Given quadratic equation, $27x^2 – 10x + 1 = 0$
On comparing it with $ax^2 + bx + c = 0$, we get
$a = 27$, $b = –10$, and $c = 1$
So, the discriminant of the given equation will be
$D = b^2 – 4ac = (–10)^2 – 4 \times 27 \times 1 = 100 – 108 = –8$
Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2\times 27} = \frac{10 \pm 2\sqrt{2}\,i}{54}$$

$$= \frac{5 \pm \sqrt{2}\,i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27} i$$

9. $21x^2 – 28x + 10 = 0$
Solution:

Given quadratic equation, $21x^2 – 28x + 10 = 0$
On comparing it with $ax^2 + bx + c = 0$, we have
$a = 21$, $b = –28$, and $c = 10$
So, the discriminant of the given equation will be
$D = b^2 – 4ac = (–28)^2 – 4 \times 21 \times 10 = 784 – 840 = –56$
Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2\times 21} = \frac{28 \pm \sqrt{56}\,i}{42}$$

$$= \frac{28 \pm 2\sqrt{14}\,i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42} i = \frac{2}{3} \pm \frac{\sqrt{14}}{21} i$$

10. If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$

Solution:

Given, $z_1 = 2 - i$, $z_2 = 1 + i$

So,

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad [i^2 = -1]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Hence, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

11. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Solution:

Given,

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$

$$= \frac{x^2 + i^2 + 2xi}{2x^2+1}$$

$$= \frac{x^2 - 1 + i2x}{2x^2+1}$$

$$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$

Comparing the real and imaginary parts, we have

$$a = \frac{x^2-1}{2x^2+1} \text{ and } b = \frac{2x}{2x^2+1}$$

$$\therefore a^2+b^2=\left(\frac{x^2-1}{2x^2+1}\right)^2+\left(\frac{2x}{2x^2+1}\right)^2$$

$$=\frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$$

$$=\frac{x^4+1+2x^2}{(2x^2+1)^2}$$

$$=\frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence proved,

$$a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$$

12. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find

(i) $\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, **(ii)** $\operatorname{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$

Solution:

Given,

$z_1=2-i,\ z_2=-2+i$

(i) $z_1z_2=(2-i)(-2+i)=-4+2i+2i-i^2=-4+4i-(-1)=-3+4i$

$\bar{z}_1=2+i$

$$\therefore \frac{z_1z_2}{\bar{z}_1}=\frac{-3+4i}{2+i}$$

On multiplying numerator and denominator by (2 - *i*), we get

$$\frac{z_1z_2}{\bar{z}_1}=\frac{(-3+4i)(2-i)}{(2+i)(2-i)}=\frac{-6+3i+8i-4i^2}{2^2+1^2}=\frac{-6+11i-4(-1)}{2^2+1^2}$$

$$=\frac{-2+11i}{5}=\frac{-2}{5}+\frac{11}{5}i$$

Comparing the real parts, we have

$$\operatorname{Re}\left(\frac{z_1z_2}{\bar{z}_1}\right)=\frac{-2}{5}$$

(ii) $\frac{1}{z_1\overline{z}_1}=\frac{1}{(2-i)(2+i)}=\frac{1}{(2)^2+(1)^2}=\frac{1}{5}$

On comparing the imaginary part, we get

$\text{Im}\left(\frac{1}{z_1\overline{z}_1}\right)=0$

13. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$
Solution:

Let $z=\frac{1+2i}{1-3i}$, then

$$z=\frac{1+2i}{1-3i}\times\frac{1+3i}{1+3i}=\frac{1+3i+2i+6i^2}{1^2+3^2}=\frac{1+5i+6(-1)}{1+9}$$

$$=\frac{-5+5i}{10}=\frac{-5}{10}+\frac{5i}{10}=\frac{-1}{2}+\frac{1}{2}i$$

Let $z=r\cos\theta+ir\sin\theta$

So,

$r\cos\theta=\frac{-1}{2}$ and $r\sin\theta=\frac{1}{2}$

On squaring and adding, we get

$$r^2\left(\cos^2\theta+\sin^2\theta\right)=\left(\frac{-1}{2}\right)^2+\left(\frac{1}{2}\right)^2$$

$r^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ [Conventionally, $r>0$]

$r=\frac{1}{\sqrt{2}}$

Now,

$\frac{1}{\sqrt{2}}\cos\theta=\frac{-1}{2}$ and $\frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2}$

$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}}$ and $\sin\theta=\frac{1}{\sqrt{2}}$

$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$ [As θ lies in the II quadrant]

14. Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.
Solution:

Let's assume z = $(x-iy)(3+5i)$

$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$

$\therefore \bar{z} = (3x + 5y) - i(5x - 3y)$

Also given, $\bar{z} = -6 - 24i$

And,
(3x + 5y) – i(5x – 3y) = -6 -24i
On equating real and imaginary parts, we have
3x + 5y = -6 …… (i)
5x – 3y = 24 …… (ii)
Performing (i) x 3 + (ii) x 5, we get
(9x + 15y) + (25x – 15y) = -18 + 120
34x = 102
x = 102/34 = 3
Putting the value of x in equation (i), we get
3(3) + 5y = -6
5y = -6 – 9 = -15
y = -3
Therefore, the values of x and y are 3 and –3 respectively.

15. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$
Solution:

$$\frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$$

$$= \frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$

$$= \frac{4i}{2} = 2i$$

$$\therefore \left|\frac{1+i}{1-i} - \frac{1-i}{1+i}\right| = |2i| = \sqrt{2^2} = 2$$

16. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$
Solution:

Given,

$(x + iy)^3 = u + iv$

$x^3 + (iy)^3 + 3 \cdot x \cdot iy(x + iy) = u + iv$

$x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u + iv$

$x^3 - iy^3 + 3x^2yi - 3xy^2 = u + iv$

$(x^3 - 3xy^2) + i(3x^2y - y^3) = u + iv$

On equating real and imaginary parts, we get

$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$

$$\frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$
$$= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$$
$$= x^2 - 3y^2 + 3x^2 - y^2$$
$$= 4x^2 - 4y^2$$
$$= 4(x^2 - y^2)$$
$$\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

Hence proved.

17. If α and β are different complex numbers with |β| = 1, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$
Solution:

Let α = *a* + *ib* and β = *x* + *iy*

Given, $|\beta| = 1$

So, $\sqrt{x^2 + y^2} = 1$

$\Rightarrow x^2 + y^2 = 1 \qquad \text{... (i)}$

$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \left|\frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)}\right|$$
$$= \left|\frac{(x - a) + i(y - b)}{1 - (ax + aiy - ibx + by)}\right|$$
$$= \left|\frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)}\right|$$
$$= \frac{|(x - a) + i(y - b)|}{|(1 - ax - by) + i(bx - ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$
$$= \frac{\sqrt{(x - a)^2 + (y - b)^2}}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}}$$
$$= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}}$$

$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \quad [\text{Using } (1)]$$

$$= 1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$

18. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$
Solution:

$$|1-i|^x = 2^x$$

$$\left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$

$$\left(\sqrt{2}\right)^x = 2^x$$

$$2^{\frac{x}{2}} = 2^x$$

$$\frac{x}{2} = x$$

$$x = 2x$$

$$2x - x = 0$$

$$x = 0$$

Therefore, 0 is the only integral solution of the given equation.
Hence, the number of non-zero integral solutions of the given equation is 0.

19. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.
Solution:

Given,

$$(a+ib)(c+id)(e+if)(g+ih) = A+iB$$

$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$

$$\Rightarrow |(a+ib)|\times|(c+id)|\times|(e+if)|\times|(g+ih)| = |A+iB| \quad [|z_1z_2| = |z_1||z_2|]$$

$$\sqrt{a^2+b^2}\times\sqrt{c^2+d^2}\times\sqrt{e^2+f^2}\times\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

On squaring both sides, we get

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

Hence proved.

20. If, $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of *m*.

Solution:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\left(\frac{2i}{2}\right)^m = 1$$

$$i^m = 1$$

Hence, $m = 4k$, where k is some integer.

Thus, the least positive integer is 1.

Therefore, the least positive integral value of *m* is 4 (= 4 × 1).