EXERCISE 7.2 PAGE NO: 304

Integrate the functions in Exercises 1 to 37:

1. $2x / 1 + x^2$

Solution:

Let us take $1 + x^2 = t$

So, we get,

$2x\,dx = dt$

$$\int \frac{2x}{1+x^2}dx$$

We get,

$$= \int \frac{1}{t}dt$$

On further calculation, we get,

$$= \log|t| + C$$

Now, substituting $t = 1 + x^2$ we get,

$$= \log|1+x^2| + C$$

$$= \log(1 + x^2) + C$$

2. $(\log x)^2 / x$

Solution:

Let us take,

$\log|x| = t$

On differentiating, we get,

$$\frac{1}{x}dx = dt$$

$$\int \frac{(\log|x|)^2}{x}dx$$

We get,

$$= \int t^2 dt$$

On further calcualtion, we get,

$$= \frac{t^3}{3} + C$$

By substituting $t = \log|x|$ we get,

$$= \frac{(\log|x|)^3}{3} + C$$

3. 1 / (x + x log x)
Solution:

Given

$$\frac{1}{x + x\log x}$$

This can be written as

$$= \frac{1}{x(1+\log x)}$$

Let us take,

$1 + \log x = t$

We get,

1 / x dx = dt

So,

$$\int \frac{1}{x(1+\log x)} dx$$

We get,

$$= \int \frac{1}{t} dt$$

On calcualting further, we get

$$= \log|t| + C$$

Hence, we get,

$$= \log|1+\log x| + C$$

4. sin x sin (cos x)
Solution:

Let us take cos x = t

By differentiating, we get

- sin x dx = dt

Now,

$$\int \sin x \cdot \sin(\cos x)\, dx$$

We obtain,

$$= -\int \sin t \, dt$$

On further calculation, we get

$$= -[-\cos t] + C$$

$$= \cos t + C$$

By substituting t = cos x, we get

$$= \cos(\cos x) + C$$

5. Sin (ax + b) cos (ax + b)
Solution:

Given

$$\sin(ax+b)\cos(ax+b)$$

On integrating the above function, we get

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2}$$

We obtain,

$$= \frac{\sin 2(ax+b)}{2}$$

Let 2 (ax + b) = t
We get,
2a dx = dt
We get,

$$\int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2} \int \frac{\sin t \ dt}{2a}$$

On further calculation, we get,

$$= \frac{1}{4a}[-\cos t] + C$$

By putting t = 2 (ax + b), we get

$$= \frac{-1}{4a} \cos 2(ax+b) + C$$

6. √ax + b
Solution:

Let us take,
ax + b = t
We get,
a dx = dt
Hence,
dx = 1 / a dt
Now,

$$\int (ax+b)^{\frac{1}{2}}\,dx$$

We get,

$$=\frac{1}{a}\int t^{\frac{1}{2}}dt$$

On further calculation, we get

$$=\frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

Hence, we get,

$$=\frac{2}{3a}(ax+b)^{\frac{3}{2}}+C$$

7. x √x + 2
Solution:

Let us take,
(x + 2) = t
We get,
dx = dt
Now,

$$\int x\sqrt{x+2}\,dx$$

We get,

$$=\int (t-2)\sqrt{t}\,dt$$

On further calculating, we get

$$=\int\left(t^{\frac{3}{2}}-2t^{\frac{1}{2}}\right)dt$$

By taking separately, we get

$$=\int t^{\frac{3}{2}}dt-2\int t^{\frac{1}{2}}dt$$

So,

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

By further calculation, we get

$$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

8. x $\sqrt{1 + 2x^2}$

Solution:

Let us take,

$1 + 2x^2 = t$

We get,

$4x\ dx = dt$

$$\int x\sqrt{1+2x^2}\,dx$$

We obtain,

$$= \int \frac{\sqrt{t}\,dt}{4}$$

So,

$$= \frac{1}{4}\int t^{\frac{1}{2}}dt$$

On further calculation, we get

$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$$

9. (4x + 2) $\sqrt{x^2 + x + 1}$

Solution:

Let us take,

$x^2 + x + 1 = t$

We get,

$(2x + 1)\, dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\, dx$$

We obtain,

$$= \int 2\sqrt{t}\, dt$$

$$= 2\int \sqrt{t}\, dt$$

On further calculation, we get

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + C$$

10. 1 / (x - √x)

Solution:

Given

$$\frac{1}{x-\sqrt{x}}$$

This can be written as

$$= \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let us take,

$$\left(\sqrt{x}-1\right) = t$$

We get,

$$\frac{1}{2\sqrt{x}}\, dx = dt$$

$$\int \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)} dx = \int \frac{2}{t}\, dt$$

On further calculation, we get

$= 2\log|t| + C$

Hence, we obtain,

$= 2\log\left|\sqrt{x} - 1\right| + C$

11. x / (√x + 4), x > 0

Solution:

Let us take,

x + 4 = t

We get,

dx = dt

$$\int \frac{x}{\sqrt{x+4}}\,dx = \int \frac{(t-4)}{\sqrt{t}}\,dt$$

So,

$$= \int\left(\sqrt{t} - \frac{4}{\sqrt{t}}\right)dt$$

On further calculation, we get

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$$

$$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t\cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t^{\frac{1}{2}}(t-12) + C$$

By substituting t = x + 4, we obtain

$$= \frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$$

$$= \frac{2}{3}\sqrt{x+4}\,(x-8) + C$$

12. $(x^3 - 1)^{1/3}\, x^5$

Solution:

Let us take,

$x^3 - 1 = t$

We get,

$$3x^2 dx = dt$$

$$\int (x^3 - 1)^{\frac{1}{3}} x^5 dx$$

We get,

$$= \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

By putting $x^3 - 1 = t$, we obtain

$$= \int t^{\frac{1}{3}} (t+1) \frac{dt}{3}$$

$$= \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt$$

On further calculation, we get

$$= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C$$

$$= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C$$

$$= \frac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \frac{1}{4} (x^3 - 1)^{\frac{4}{3}} + C$$

13. $x^2 / (2 + 3x^3)^3$

Solution:

Let us take,

$2 + 3x^3 = t$

We get,

$9x^2\, dx = dt$

$$\int \frac{x^2}{(2 + 3x^3)^3} dx$$

So,

$$= \frac{1}{9} \int \frac{dt}{(t)^3}$$

On further calculation, we get

$$= \frac{1}{9}\left[\frac{t^{-2}}{-2}\right] + C$$

$$= \frac{-1}{18}\left(\frac{1}{t^2}\right) + C$$

$$= \frac{-1}{18\left(2+3x^3\right)^2} + C$$

14. 1 / x (log x) ^{m}, x > 0, m ≠ 1
Solution:

Let us take,
log x = t
We get,

$$\frac{1}{x}dx = dt$$

$$\int \frac{1}{x(\log x)^m}dx$$

We obtain,

$$= \int \frac{dt}{(t)^m}$$

On further calculation, we get

$$= \left(\frac{t^{-m+1}}{1-m}\right) + C$$

$$= \frac{(\log x)^{1-m}}{(1-m)} + C$$

15. x / (9 – 4x^2)
Solution:

Let us take,
$9 - 4x^2 = t$
We get,
$-8x\, dx = dt$
Now take,

$$\int \frac{x}{9-4x^2}dx$$

So,

$$= \frac{-1}{8}\int \frac{1}{t}\, dt$$

By further calculating, we obtain

$$= \frac{-1}{8}\log|t| + C$$

$$= \frac{-1}{8}\log|9-4x^2| + C$$

16. e^{2x+3}

Solution:

Let us take,

2x + 3 = t

We get,

2dx = dt

Now

$$\int e^{2x+3}\, dx$$

We obtain,

$$= \frac{1}{2}\int e^t\, dt$$

On further calculation, we get

$$= \frac{1}{2}\left(e^t\right) + C$$

$$= \frac{1}{2}e^{(2x+3)} + C$$

17. $\dfrac{x}{e^{x^2}}$

Solution:

Let us take,

$x^2 = t$

We get,

2x dx = dt

$$\int \frac{x}{e^{x^2}}\, dx$$

So,

$$=\frac{1}{2}\int\frac{1}{e^t}dt$$

$$=\frac{1}{2}\int e^{-t}dt$$

On further calculation, we get

$$=\frac{1}{2}\left(\frac{e^{-t}}{-1}\right)+C$$

$$=-\frac{1}{2}e^{-x^2}+C$$

$$=\frac{-1}{2e^{x^2}}+C$$

18. $\frac{e^{tan^{-1}x}}{1+x^2}$

Solution:

Let us take,

$\tan^{-1} x = t$

We get,

$$\frac{1}{1+x^2}dx = dt$$

$$\int\frac{e^{\tan^{-1}x}}{1+x^2}dx$$

We obtain,

$$=\int e^t dt$$

By further calculation, we get

$$=e^t+C$$

$$=e^{\tan^{-1}x}+C$$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

Solution:

By dividing numerator and denominator by e^x, we find

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

Let us assume,

$e^x+e^{-x}=t$

So,

$(e^x-e^{-x})dx=dt$

$$\int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx$$

We get,

$$=\int\frac{dt}{t}$$

By calculating further, we get

$=\log|t|+C$

$=\log|e^x+e^{-x}|+C$

20. $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Solution:

Let us assume,

$e^{2x}+e^{-2x}=t$

We get,

$(2e^{2x}-2e^{-2x})dx=dt$

$2(e^{2x}-e^{-2x})dx=dt$

Now

$$\int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx$$

We get,

$$=\int\frac{dt}{2t}$$

$= \frac{1}{2}\int \frac{1}{t}dt$

On calculating further, we get

$= \frac{1}{2}\log|t| + C$

$= \frac{1}{2}\log\left|e^{2x} + e^{-2x}\right| + C$

21.

$\tan^2(2x - 3)$

Solution:

$\tan^2(2x-3) = \sec^2(2x-3) - 1$

Let us take,

$2x - 3 = t$

We get,

$2dx = dt$

Now,

$\int \tan^2(2x-3)dx = \int\left[\left(\sec^2(2x-3)\right) - 1\right]dx$

By separating, we obtain

$= \frac{1}{2}\int\left(\sec^2 t\right)dt - \int 1dx$

$= \frac{1}{2}\int \sec^2 t\, dt - \int 1dx$

On further calculation, we get

$= \frac{1}{2}\tan t - x + C$

$= \frac{1}{2}\tan(2x-3) - x + C$

22.

$\sec^2(7 - 4x)$

Solution:

Let us take,

7 – 4x = t

We get,

$-4dx = dt$

Hence,

$$\int \sec^2(7-4x)\,dx = \frac{-1}{4}\int \sec^2 t\,dt$$

On calculating further, we get

$$= \frac{-1}{4}(\tan t) + C$$

$$= \frac{-1}{4}\tan(7-4x) + C$$

23.

$$\frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

Solution:

Let us take,

$\sin^{-1} x = t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

$$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\,dt$$

We get,

$$= \frac{t^2}{2} + C$$

By substituting $t = \sin^{-1} x$, we get

$$= \frac{(\sin^{-1}x)^2}{2} + C$$

24.

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

Solution:

$$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}$$

This can be written as

$$=\frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$$

Let us assume,

$$3\cos x+2\sin x=t$$

$$(-3\sin x+2\cos x)dx=dt$$

$$\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx=\int\frac{dt}{2t}$$

On further calculation, we get

$$=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

Therefore, we get

$$=\frac{1}{2}\log|2\sin x+3\cos x|+C$$

25. $$\frac{1}{\cos^2 x(1-\tan x)^2}$$

Solution:

$$\frac{1}{\cos^2 x(1-\tan x)^2}=\frac{\sec^2 x}{(1-\tan x)^2}$$

Let us assume,

$$(1-\tan x)=t$$

$$-\sec^2 xdx=dt$$

$$\int\frac{\sec^2 x}{(1-\tan x)^2}dx=\int\frac{-dt}{t^2}$$

We get,

$$=-\int t^{-2}dt$$

$$=+\frac{1}{t}+C$$

Therefore, we get

$$= \frac{1}{(1-\tan x)} + C$$

26. $\dfrac{\cos\sqrt{x}}{\sqrt{x}}$

Solution:

Let us take,

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt$

$\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2\int \cos t \, dt$

By further calculation, we get

$= 2\sin t + C$

$= 2\sin\sqrt{x} + C$

27. $\sqrt{\sin 2x} \cos 2x$

Solution:

Let us take,

$\sin 2x = t$

$2\cos 2x \, dx = dt$

$\Rightarrow \int \sqrt{\sin 2x} \, \cos 2x \, dx = \frac{1}{2}\int \sqrt{t} \, dt$

On further calculation, we get

$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{1}{3} t^{\frac{3}{2}} + C$

By substituting t = sin 2x, we get

$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$

28. $\frac{\cos x}{\sqrt{1+\sin x}}$

Solution:

Let us take,

$1+\sin x = t$

$\cos x\, dx = dt$

$$\int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$

By further calculation, we get

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1+\sin x} + C$$

29. $\cot x \log \sin x$

Solution:

Take

log sin x = t

By differentiation we get

$\frac{1}{\sin x} \cdot \cos x\ dx = dt$

So we get

cot x dx = dt

Integrating both sides

$\int \cot x\ \log \sin x\ dx = \int t\, dt$

We get

$= \frac{t^2}{2} + C$

Substituting the value of t

$= \frac{1}{2}(\log \sin x)^2 + C$

30.

$\frac{\sin x}{1+\cos x}$

Solution:

Take 1 + cos x = t
By differentiation
- sin x dx = dt
By integrating both sides

$$\int \frac{\sin x}{1+\cos x}\,dx = \int -\frac{dt}{t}$$

So we get
= - log |t| + C
Substituting the value of t
= - log |1 + cos x| + C

31.

$$\frac{\sin x}{(1+\cos x)^2}$$

Solution:

Take 1 + cos x = t
By differentiation
- sin x dx = dt
Integrating both sides

$$\int \frac{\sin x}{(1+\cos x)^2}\,dx = \int -\frac{dt}{t^2}$$

We get

$$= -\int t^{-2}dt$$

It can be written as

$$= \frac{1}{t} + C$$

Substituting the value of t

$$= \frac{1}{1+\cos x} + C$$

32.

$$\frac{1}{1+\cot x}$$

Solution:

It is given that

$$I = \int \frac{1}{1+\cot x} dx$$

We can write it as

$$= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$$

By taking LCM

$$= \int \frac{\sin x}{\sin x + \cos x} dx$$

Multiply and divide by 2 in numerator and denominator

$$= \frac{1}{2} \int \frac{2\sin x}{\sin x + \cos x} dx$$

It can be written as

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$

On further calculation

$$= \frac{1}{2} \int 1\, dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

We get

$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Take sin x + cos x = t

By differentiation

(cos x – sin x) dx = dt

We get

$$I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

By integration

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

Substituting the value of t

$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

33.

$$\frac{1}{1-\tan x}$$

Solution:

It is given that

$$I = \int \frac{1}{1-\tan x} dx$$

We can write it as

$$= \int \frac{1}{1-\frac{\sin x}{\cos x}} dx$$

By taking LCM

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$

Multiply and divide by 2 in numerator and denominator

$$= \frac{1}{2} \int \frac{2\cos x}{\cos x - \sin x} dx$$

It can be written as

$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$

On further calculation

$$= \frac{1}{2} \int 1\, dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

We get

$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Take cos x - sin x = t

By differentiation

(-sin x – cos x) dx = dt

We get

$$I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

By integration

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

Substituting the value of t

$$= \frac{x}{2} - \frac{1}{2}\log\left|\cos x - \sin x\right| + C$$

34.
$$\frac{\sqrt{\tan x}}{\sin x \cos x}$$
Solution:

It is given that

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$

By multiplying cos x to both numerator and denominator

$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$

On further calculation

$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$

So we get

$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

Take tan x = t

We get $\sec^2$ x dx = dt

$$I = \int \frac{dt}{\sqrt{t}}$$

By integration we get

$$= 2\sqrt{t} + C$$

Substituting the value of t

$$= 2\sqrt{\tan x} + C$$

35.
$$\frac{(1+\log x)^2}{x}$$
Solution:

Consider

1 + log x = t

So we get

$$\frac{1}{x}dx = dt$$

Integrating both sides

$$\int\frac{(1+\log x)^2}{x}\,dx = \int t^2 dt$$

We get

$$=\frac{t^3}{3}+C$$

Substituting the value of t

$$=\frac{(1+\log x)^3}{3}+C$$

36.

$$\frac{(x+1)(x+\log x)^2}{x}$$

Solution:

It is given that

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2$$

We can write it as

$$=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Consider x + log x = t

By differentiation

$$\left(1+\frac{1}{x}\right)dx = dt$$

Integrating both sides

$$\int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx = \int t^2 dt$$

So we get

$$=\frac{t^3}{3}+C$$

Substituting the value of t

$$=\frac{1}{3}(x+\log x)^3+C$$

37.
$$\frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$$
Solution:

It is given that

$$\frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$$

Consider $x^4 = t$

We get $4x^3\, dx = dt$

$$\int\frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}dx=\frac{1}{4}\int\frac{\sin\left(\tan^{-1}t\right)}{1+t^2}dt \qquad \ldots(1)$$

Similarly take $\tan^{-1} t = u$

By differentiation we get

$$\frac{1}{1+t^2}dt=du$$

Using equation (1) we get

$$\int\frac{x^3\sin\left(\tan^{-1}x^4\right)dx}{1+x^8}=\frac{1}{4}\int\sin u\,du$$

By integration

$$=\frac{1}{4}(-\cos u)+C$$

Substituting the value of u

$$=\frac{-1}{4}\cos\left(\tan^{-1}t\right)+C$$

Now substituting the value of t
$$=\frac{-1}{4}\cos\left(\tan^{-1}x^4\right)+C$$

Choose the correct answer in Exercises 38 and 39.

38. $\int \frac{10x^9 + 10^x log_e 10dx}{x^{10} + 10^x}\ equals$

$(A) 10^x - x^{10} + C$

$(B) 10^x + x^{10} + C$

$(C) (10^x - x^{10})^{-1} + C$

$(D) log(10^x + x^{10}) + C$

Solution:

Take $x^{10} + 10^x = t$

Differentiating both sides

$$\left(10x^9 + 10^x \log_e 10\right)dx = dt$$

Integrating both sides we get

$$\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

So we get

= log t + C

Substituting the value of t

= log $(10^x + x^{10})$ + C

Therefore, D is the correct answer.

39. $\int \frac{dx}{sin^2 x cos^2 x}\ equals$

(A) tan x + cot x + C

(B) tan x – cot x + C

(C) tan x cot x + C

(D) tan x – cot 2x + C

Solution:

It is given that

$$I = \int \frac{dx}{\sin^2 x \cos^2 x}$$

We can write it as

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

Here we get

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

By separating the terms

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

We get

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

By integration

= tan x – cot x + C

Therefore, B is the correct answer.