EXERCISE 7.3

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1. $\sin^2(2x + 5)$

Solution:-

We have,

By standard trigonometric identity, $\sin^2 x = (1 - \cos 4x)/2$

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

Taking integrals on both sides, we get,

$$= \int \sin^2(2x+5)dx = \int \frac{1-\cos(4x+10)}{2}dx$$

Splitting the integrals,

$$= \frac{1}{2}\int 1.dx - \frac{1}{2}\int \cos(4x+10)dx$$

$$= \frac{1}{2}x - \frac{1}{2}\int \cos(4x+10)dx$$

On integrating, we get,

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

2. sin 3x cos 4x

Solution:-

By standard trigonometric identity sinA cosB = ½ {sin(A + B) + cos(A - B)}

$$\int \sin 3x \cos 4x dx = \frac{1}{2}\int \{\sin(3x+4x) + \sin(3x-4x)\}dx$$

On simplifying,

$$= \frac{1}{2}\int\left\{\sin 7x + \sin(-x)\right\}dx$$

$$= \frac{1}{2}\int\left\{\sin 7x - \sin x\right\}dx$$

Splitting the integrals, we have,

$$= \frac{1}{2}\int \sin 7x dx - \frac{1}{2}\int \sin x dx$$

On integrating, we get,

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

3. cos 2x cos 4x cos 6x

Solution:-

By standard trigonometric identity cosA cosB = ½ {cos(A + B) + cos(A - B)}

$$\int \cos 2x \cos 4x \cos 6x dx = \int \cos 2x\left[\frac{1}{2}\left\{\cos(4x+6x) + \cos(4x-6x)\right\}\right]dx$$

$$= \frac{1}{2}\int\left\{\cos 2x \cos 10x + \cos 2x \cos(-2x)\right\}dx$$

We know that, cos(-x) = cos x,

$$= \frac{1}{2}\int\left\{\cos 2x \cos 10x + \cos^2 2x\right\}dx$$

Again by, standard trigonometric identity cosA cosB = ½ {cos(A + B) + cos(A - B)} and $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x) + \cos(2x-10x)\right\} + \left(\frac{1+\cos 4x}{2}\right)\right]dx$$

On simplifying, we get,

$$= \frac{1}{4}\int(\cos 12x + \cos 8x + 1 + \cos 4x)dx$$

By integrating,

$$= \frac{1}{4}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4}\right] + C$$

4. $\sin^3(2x + 1)$

Solution:-

Given, $\sin^3(2x+1)$

By splitting,

$$= \int \sin^3(2x+1)dx = \int \sin^2(2x+1).\sin(2x+1)dx$$

We know that, $\sin^2 x = 1 - \cos^2 x$

$$= \int\left(1-\cos^2(2x+1)\right)\sin(2x+1)dx$$

Let us assume cos (2x+1) = t

Then,

=> $-2\sin(2x+1)dx = dt$

=> $\sin(2x+1)dx = \frac{-dt}{2}$

$\sin^3(2x+1) = \frac{-1}{2}\int(1-t^2)dt$

$$= \frac{-1}{2}\left\{t - \frac{t^3}{3}\right\}$$

Now substitute the value 't' in equation,

$$= \frac{-1}{2}\left\{\cos(2x+1) - \frac{\cos^3(2x+1)}{3}\right\}$$

$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C$$

5. $\sin^3 x \cos^3 x$
Solution:-

Given, $\int \sin^3 x \cos^3 x.dx$

By splitting the given function,

$$= \int \cos^3 x.\sin^2 x.\sin x.dx$$

We know that, $\sin^2 x = 1 - \cos^2 x$

$$= \int \cos^3 x\left(1 - \cos^2 x\right)\sin x.dx$$

So, let us assume $\cos x = t$

Then,

$$\Rightarrow -\sin x \times dx = dt$$

$\sin^3 x \cos^3 x \quad = -\int t^3\left(1 - t^2\right)dt$

$$= -\int\left(t^3 - t^5\right)dt$$

On integrating, we get,

$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$

Now substitute the value ‘t’ in equation,

$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

6. $\sin x \sin 2x \sin 3x$
Solution:-

By standard trigonometric identity sinA sinB = -½ {cos (A + B) - cos (A - B)}

$$\int \sin x \sin 2x \sin 3x\,dx = \int \sin x.\frac{1}{2}\left[\left\{\cos\left(2x-3x\right)-\cos\left(2x+3x\right)\right\}\right]dx$$

On simplifying, we get,

$$=\frac{1}{2}\int\left\{\sin x\cos\left(-x\right)-\sin x\cos 5x\right\}dx$$

We know that, cos(-x) = cos x,

$$=\frac{1}{2}\int\left\{\sin x\cos x-\sin x\cos 5x\right\}dx$$

Splitting the integrals, by using sin 2x = 2sinx cosx, we get,

$$=\frac{1}{2}\int\frac{\sin 2x}{2}dx-\frac{1}{2}\int\sin x\cos 5x\,dx$$

On integrating the first term, and substituting sinA cosB = ½ {sin(A + B) + sin (A - B)}

$$=\frac{1}{4}\left[\frac{-\cos 2x}{2}\right]-\frac{1}{2}\int\left\{\frac{1}{2}\sin\left(x+5x\right)+\sin\left(x-5x\right)\right\}dx$$

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\int\left(\sin 6x+\sin\left(-4x\right)\right)dx$$

Computing and simplifying, we get,

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{4}\right]+C$$

$$=\frac{-\cos 2x}{8}-\frac{1}{8}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{2}\right]+C$$

$$=\frac{1}{8}\left[\frac{\cos 6x}{3}-\frac{\cos 4x}{2}-\cos 2x\right]+C$$

7. sin 4x sin 8x
Solution:-

By standard trigonometric identity sinA sinB = -½ {cos (A + B) - cos (A - B)}

Then,

$$\int \sin 4x \sin 8x dx = \int \left\{\frac{1}{2}\cos\left(4x-8x\right)-\cos\left(4x+8x\right)\right\}dx$$

$$=\frac{1}{2}\int(\cos\left(-4x\right)-\cos 12x dx$$

We know that, cos (-x) = cos x,

$$=\frac{1}{2}\int\left\{\cos 4x-\cos 12x\right\}dx$$

On integrating we get,

$$=\frac{1}{2}\left[\frac{\sin 4x}{4}-\frac{\sin 12x}{12}\right]+C$$

8. $\frac{1-\cos x}{1+\cos x}$

Solution:-

By standard trigonometric identity, we have,

$$\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

We know that, (Sin x/cos x) = tan x

$$=2\tan^2\frac{x}{2}$$

Also, we know that, $\tan^{-1}$ x = sec x

$$=\left(\sec^2\frac{x}{2}-1\right)$$

Integrating both the sides, we get,

$$\therefore \int \frac{1-\cos x}{1+\cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2\tan \frac{x}{2} - x + C$$

9. $\dfrac{\cos x}{1+\cos x}$

Solution:-

By standard trigonometric identity, we have,

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$$

We know that, (Sin x/cos x) = tan x and takeout ½ as common, we get

$$= \frac{1}{2}\left[1 - \tan^2 \frac{x}{2} \right]$$

Integrating both the sides, we get,

$$\int \frac{\cos x}{1+\cos x} dx = \int \frac{1}{2}\left[1 - \tan^2 \frac{x}{2} \right] dx$$

Using standard trigonometric identity $\tan^2 x + 1 = \sec^2 (x)$

$$= \frac{1}{2} \int \left[2 - \sec^2 \frac{x}{2} \right] dx$$

On integrating, we get,

$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$

$$= x - \tan\frac{x}{2} + C$$

10. $\sin^4 x$

Solution:-

By splitting the given function, we get,

$\sin^4 x = \sin^2 x \sin^2 x$

By standard trigonometric identity, we have, $\sin^2 x = (1 - \cos 2x)/2$

$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{4}(1-\cos 2x)^2$$

By using the formula $(a - b)^2 = a^2 - 2ab + b^2$, we get,

$$= \frac{1}{4}\left[1 + \cos^2 2x - 2\cos 2x\right]$$

From the standard trigonometric identity, $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{4}\left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

On simplifying, we get,

$$= \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

Integrating on both the sides,

$$\int \sin^4 x dx = \frac{1}{4}\int\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]dx$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C$$

By simplifying,

$$= \frac{1}{8}\left[3x + \left(\frac{\sin 4x}{4}\right) - 2\sin 2x\right] + C$$

$$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

11. $\cos^4 2x$

Solution:-

By splitting the given function,

$\cos^4 2x = (\cos^2 2x)^2$

By standard trigonometric identity, we have, $\cos^2 2x = (1 + \cos 4x)/2$

$$= \left(\frac{1 + \cos 4x}{2}\right)^2$$

On simplifying, we get,

$$= \frac{1}{4}\left[1 + \cos^2 4x - 2\cos 4x\right]$$

By standard trigonometric identity, we have, $\cos^2 2x = (1 + \cos 4x)/2$

$$= \frac{1}{4}\left[1 + \left(\frac{1 + \cos 8x}{2}\right) + 2\cos 4x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2}\cos 8x + 2\cos 4x\right]$$

By simplifying,

$$= \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\cos 8x + 2\cos 4x\right]$$

Integrating both side,

$$\int \cos^4 2x dx = \int\left[\frac{3}{8} + \frac{1}{8}\cos 8x + \frac{1}{2}\cos 4x\right]dx$$

$$= \frac{3x}{8} + \frac{1}{64}\sin 8x + \frac{1}{8}\sin 4x + C$$

12. $\dfrac{\sin^2 x}{1 + \cos x}$

Solution:-

By standard trigonometric identity, we have,

$$\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

On simplifying, we get,

$$= 2\sin^2\frac{x}{2}$$

From the standard trigonometric identity, we have, $1 - \cos x = 2\sin^2\frac{x}{2}$

= 1- cosx

On integrating both the sides, we get,

$$\int \frac{\sin^2 x}{1+\cos x}\,dx = \int (1- \cos x)\,dx$$

$$= x - \sin x + C$$

13. $\dfrac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$

Solution:-

By using the trigonometry identity i.e.,

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

So, we have,

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} = \frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\sin\frac{x+\alpha}{2}\sin\sin\frac{x-\alpha}{2}}$$

By simplifying, we get,

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

Then,

From the identity sin 2x = 2 sin x cos x, we have

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

On simplifying, we get,

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

By using the trigonometry identity 2 cos A cos B = cos (A + B) + cos (A - B), we have

$$= 2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$= 2[\cos(x) + \cos\alpha]$

$= 2\cos x + 2\cos\alpha$

Then,

Integrating on both the sides,

$$\int \therefore \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx = \int(2\cos x + 2\cos\alpha)dx$$

We have,

$= 2[\sin x + x\cos\alpha] + C$

14. $\dfrac{\cos x - \sin x}{1+\sin 2x}$

Solution:-

Given = $\dfrac{\cos x - \sin x}{1+\sin 2x}$

By using the standard trigonometric identity, $(1 + \sin 2x) = \sin^2 x + \cos^2 x + 2\sin x\cos x$.

Then,

$$= \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2\sin x\cos x}$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Now,

Let us assume that, sinx + cosx = t

And also, (cosx-sinx)dx = dt

Integrating on both the sides and substitute the value of (cosx – sinx) dx and (sinx + cosx) we get,

$$= \int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

$$= \int \frac{dt}{t^2}$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

15. $\tan^3 2x \sec 2x$

Solution:-

By splitting the given function, we have,

$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$

From the standard trigonometric identity, $\tan^2 2x = \sec^2 2x – 1$,

$= (\sec^2 2x - 1) \tan 2x \sec 2x$

By multiplying, we get,

$= (\sec^2 2x \times \tan 2x \sec 2x) – (\tan 2x \sec 2x)$

Integrating both sides,

$$\int \tan^3 2x \sec 2x dx = \int \sec^2 2x \tan 2x \sec 2x dx - \int \tan 2x \sec 2x dx$$

$$= \int \sec^2 2x \tan 2x \sec 2x dx - \frac{\sec 2x}{2} + C$$

Then,

Let us assume sec2x = t

And also assume 2sec2x tan2x dx = dt

$$\int \tan^3 2x \sec 2x dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$

On simplifying, we get,

$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$

$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

16. $\tan^4 x$

Solution:-

By splitting the given function, we have,

$\tan^4 x = \tan^2 x \times \tan^2 x$

Then,

From trigonometric identity, $\tan^2 x = \sec^2 x - 1$

$= (\sec^2 x - 1) \tan^2 x$

By multiplying, we get,

$= \sec^2 x \tan^2 x - \tan^2 x$

Again by using trigonometric identity, $\tan^2 x = \sec^2 x - 1$

$= \sec^2 x \tan^2 x - (\sec^2 x - 1)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

Now, integrating on both sides we get,

$$\int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx - \int 1.dx$$

$$= \int \sec^2 x \tan^2 x dx - \tan x + x + C$$

Then, let us assume tanx = t

And also assume $\sec^2 x$ dx =dt

$$\int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

$$\int \tan^4 x dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Solution:-

By splitting up the given function,

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x\cos^2 x} = \frac{\sin^3 x}{\sin^2 x\cos^2 x} + \frac{\cos^3 x}{\sin^2 x\cos^2 x}$$

By simplifying, we get,

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

We know that, (sinx/cosx) = tanx and (1/cosx) = secx.

Again, we have (cosx/sinx) = cotx and (1/sinx) = cosecx

= tanx secx + cotx cosecx

Integrating on both the sides, we get

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x\cos^2 x}\, dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x)\, dx$$

= secx – cosecx + C

18. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Solution:-

By using the standard trigonometric identity, $2\sin^2 x = (1 - \cos 2x)$

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x} = \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x}$$

By simplification, we get,

$$= \frac{1}{\cos^2 x}$$

We know that, $(1/\cos^2 x) = \sec^2 x$

$= \sec^2 x$

Integrating on both sides, we get,

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx = \int \sec^2 x\, dx$$

= tanx + C

19. $\dfrac{1}{\sin x\cos^3 x}$

Solution:-

For further simplification, the given function can be written as,

$$\frac{1}{\sin x\cos^3 x}=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$

Divide both numerator and denominator by $\cos^2 x$

$$=\tan x\sec^2 x+\frac{\frac{1}{\cos^2 x}}{\frac{\sin x\cos x}{\cos^2 x}}$$

On simplification, we get,

$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$

By applying the integrals, we get,

$$\int\frac{1}{\sin x\cos^3 x}dx=\int\tan x\sec^2 x\,dx+\int\frac{\sec^2 x}{\tan x}dx$$

Let us assume that, $\tan x = t$

Then, $\sec^2 x\, dx = dt$

By substituting above values, we get,

$$\int\frac{1}{\sin x\cos^3 x}dx=\int t\,dt+\int\frac{1}{t}dt$$

On integrating,

$$=\frac{t^2}{2}+\log|t|+C$$

Now, by substituting the value of 't' we get,

$$=\frac{1}{2}\tan^2 x+\log|\tan x|+C$$

20. $\dfrac{\cos 2x}{(\cos x+\sin x)^2}$

Solution:-

We know that, $(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x + 2\sin x\cos x$

$$\frac{\cos 2x}{(\cos x+\sin x)^2} = \frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}$$

And also we know that, $\cos^2 x + \sin^2 x = 1$ and $2\sin x \cos x = \sin 2x$,

Then,

$$= \frac{\cos 2x}{1+\sin 2x}$$

By applying the integrals, we get,

$$\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx = \int \frac{\cos 2x}{1+\sin 2x}dx$$

Let us assume that, $1 + \sin 2x = t$

So, $2\cos 2x\, dx = dt$

By substituting above values, we get,

$$\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx = \frac{1}{2}\int \frac{1}{t}dt$$

On integrating,

$$= \frac{1}{2}\log|t| + C$$

Now, by substituting the value of 't' we get,

$$= \frac{1}{2}\log|1+\sin 2x| + C$$

$$= \frac{1}{2}\log\left|(\cos x+\sin x)^2\right| + C$$

$= \log|\sin x + \cos x| + C$

21. $\sin^{-1}(\cos x)$

Solution:-

Given, $\sin^{-1}(\cos x)$

Let us assume $\cos x = t$ … [equation (i)]

Then, substitute 't' in place of $\cos x$

$= \sin^{-1}(t)$

$\sin x = \sqrt{1-t^2}$

So, now differentiating both sides of (i), we get,

$(-\sin x)dx = dt$

$dx = \frac{-dt}{\sin x}$

$dx = \frac{-dt}{\sqrt{1-t^2}}$

By applying the integrals, we get,

$$\int \sin^{-1}(\cos x)\,dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$= \int \frac{\sin^{-1} t}{\sqrt{1-t^2}}\,dt$$

Let us assume that, $\sin^{-1} t = v$

$\frac{dt}{\sqrt{1-t^2}} = dv$

$\int \sin^{-1}(\cos x)\,dx = -\int v\,dv$

On integrating,

$= -\frac{v^2}{2} + C$

Now, by substituting the value of 'V' and 't', we get,

$= -\frac{(\sin^{-1} t)^2}{2} + C$

$= -\frac{(\sin^{-1}(\cos x))^2}{2} + C$ ... [equation (ii)]

As we know that,

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

22. $\dfrac{1}{\cos(x-a)\cos(x-b)}$

Solution:-

Multiplying and dividing by sin (a - b) to given function, we get,

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

For further simplification, the given function can be written as,

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$

Using sin (A - B) = sin A cos B – cos A sin B formula, we get,

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$$

We know that, sinx/cosx = tan x by applying this formula we get,

$$= \frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

Taking integrals,

$$\int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

On integrating,

$$= \frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$

We know that, log (a/b) = log a – log b, using in above equation, we get,

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

Choose the correct answer in Exercises 23 and 24.

23. $\displaystyle\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$ is equal to

(A) $\tan x + \cot x + C$ (B) $\tan x + \text{cosec } x + C$

(C) $-\tan x + \cot x + C$ (D) $\tan x + \sec x + C$

Solution:-

(A) $\tan x + \cot x + C$

By splitting the denominators of given equation,

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

On simplifying, we get,

$$= \int \left(\sec^2 x - \operatorname{cosec}^2 x \right) dx$$

As we know that,

$\int \sec^2 x \, dx = \tan x + c$

$\int \operatorname{cosec}^2 x \, dx = - \cot x + c$

$= \tan x + \cot x + C$

24. $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ equals

(A) $-\cot(ex^x) + C$

(B) $\tan(xe^x) + C$

(C) $\tan(e^x) + C$

(D) $\cot(e^x) + C$

Solution:-

(B) $\tan(xe^x) + C$

Let us assume that, $(xe^x) = t$

Differentiating both sides we get,

$((e^x \times x) + (e^x \times 1))\, dx = dt$

$e^x(x + 1) = dt$

Applying integrals,

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

We know that, $(1/\cos^2 t) = \sec^2 t$

$$= \int \sec^2 t . dt$$

$= \tan t + C$

Substituting the value of 't',

$= \tan(e^x x) + C$