EXERCISE 7.7

Integrate the functions in exercise 1 to 9

1. $\sqrt{4-x^2}$

Solution:

Given:

$\sqrt{4-x^2}$

Upon integration we get,

$$\int\sqrt{4-x^2}\,dx = \int\sqrt{(2)^2-(x)^2}\,dx$$

By using the formula,

$$\int\sqrt{a^2-x^2}\,dx = \frac{\pi}{2}\sqrt{a^2-x^2}\,\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

So,

$$\int\sqrt{4-x^2}\,dx = \frac{\pi}{2}\sqrt{4-x^2}\,\frac{4}{2}\sin^{-1}\frac{x}{a}+C$$

$$= \frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{a}+C$$

2. $\sqrt{1-4x^2}$

Solution:

Given:

$\sqrt{1-4x^2}$

Upon integration we get,

$$\sqrt{1-4x^2}\,dx = \int\sqrt{(1)^2-(2x)^2\,dx}$$

Let $2x = t$

So,

$2dx = dt$

$dx = dt/2$

Then,

$$I = \frac{1}{2}\int\sqrt{(1)^2-(t)^2\,dt}$$

By using the formula,

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2}\, \frac{a^2}{2} \sin^{-1}\frac{x}{a} + C$$

So,

$$I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1 - t^2} + \frac{1}{2}\sin^{-1} t\right] + C$$

$$= \frac{t}{4}\sqrt{1 - t^2} + \frac{1}{4}\sin^{-1} t + C$$

$$= \frac{2x}{4}\sqrt{1 - 4x^2} + \frac{1}{4}\sin^{-1} 2x + C$$

$$= \frac{x}{2}\sqrt{1 - 4x^2} + \frac{1}{4}\sin^{-1} 2x + C$$

3. $\sqrt{x^2 + 4x + 6}$

Solution:

Given:

$\sqrt{x^2 + 4x + 6}$

Upon integration we get,

$$I = \int \sqrt{x^2 + 4x + 6}\, dx$$

$$= \int \sqrt{x^2 + 4x + 4 + 2}\, dx$$

$$= \int \sqrt{(x + 2)^2 + \left(\sqrt{2}\right)^2}\, dx$$

By using the formula,

$$\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$$

So,

$$I = \frac{(x + 2)}{2}\sqrt{x^2 + 4x + 6} + \frac{2}{2}\log\left|(x + 2) + \sqrt{x^2 + 4x + 6}\right| + C$$

$$= \frac{(x + 2)}{2}\sqrt{x^2 + 4x + 6} + \log\left|(x + 2) + \sqrt{x^2 + 4x + 6}\right| + C$$

4. $\sqrt{x^2+4x+1}$

Solution:

Given:

$\sqrt{x^2+4x+1}$

Upon integration we get,

$$I = \int \sqrt{x^2+4x+1}\,dx$$
$$= \int \sqrt{(x^2+4x+4)-3}\,dx$$
$$= \int \sqrt{(x+2)^2-(\sqrt{3})^2}\,dx$$

By using the formula,

$$\int \sqrt{(x+2)^2-(\sqrt{3})^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$$

So,

$$I = \frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C$$

5. $\sqrt{1-4x-x^2}$

Solution:

Given:

$\sqrt{1-4x-x^2}$

Upon integration we get,

$$I = \int \sqrt{1-4x-x^2}\,dx$$
$$= \int \sqrt{1-(x^2+4x+4-4)}\,dx$$
$$= \int \sqrt{1+4-(x+2)^2}\,dx$$
$$= \int \sqrt{(\sqrt{5})^2-(x+2)^2}\,dx$$

By using the formula,

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

So,

$$I = \frac{(x+2)}{2}\sqrt{1 - 4x - x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$$

6. $\sqrt{x^2 + 4x - 5}$

Solution:

Given:

$$\sqrt{x^2 + 4x - 5}$$

Upon integration we get,

$$I = \sqrt{x^2 + 4x - 5}\, dx$$

$$= \int \sqrt{(x^2 + 4x + 4) - 9}\, dx$$

$$= \int \sqrt{(x+2)^2 - (3)^2}\, dx$$

By using the formula,

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$$

So,

$$I = \frac{(x+2)}{2}\sqrt{x^2 + 4x - 5} - \frac{9}{2}\log\left|(x+2) + \sqrt{x^2 + 4x - 5}\right| + C$$

7. $\sqrt{1 + 3x - x^2}$

Solution:

Given:

$$\sqrt{1 + 3x - x^2}$$

Upon integration we get,

$$I = \int \sqrt{1 + 3x - x^2}\, dx$$

$$= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)}\, dx$$

$$= \int \sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx$$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

By using the formula,

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

So,

$$I = \frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$$

$$= \frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

8. $\sqrt{x^2+3x}$

Solution:

Given:

$\sqrt{x^2+3x}$

Upon integration we get,

$$I = \int \sqrt{x^2+3x}\,dx$$

$$= \int \sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$

$$= \int \sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

By using the formula,

$$\int \sqrt{x^2-a^2x}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

So,

$$I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2-3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2-3x}\right|+C$$

$$=\frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

9. $\sqrt{1+\frac{x^2}{9}}$

Solution:

Given:

$\sqrt{1+\frac{x^2}{9}}$

Upon integration we get,

$$I=\int\sqrt{1+\frac{x^2}{9}}\,dx$$

$$=\frac{1}{3}\int\sqrt{9+x^2}\,dx$$

$$=\frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$$

By using the formula,

$$\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x^2+a^2\right|+C$$

So,

$$I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

Choose the correct answer in Exercises 10 to 11

10. $\int \sqrt{1+x^2}\, dx$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C}$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

D. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C$

Solution:

Given:

$$\int \sqrt{1+x^2}\, dx$$

By using the formula,

$$\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

So,

$$\int \sqrt{1+x^2}dx = \frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

Hence the correct option is A.

11. $\int \sqrt{x^2-8x+7}\, dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-2x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Solution:

Given:

$$\int\sqrt{x^2-8x+7}\,dx$$

Upon integration we get,

$$I=\int\sqrt{x^2-8x+7}\,dx$$
$$=\int\sqrt{(x^2-8x+16)-9}\,dx$$
$$=\int\sqrt{(x-4)^2-(3)^2}\,dx$$

By using the formula,

$$\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

So,

$$I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$$

Hence the correct option is D.