

EXERCISE 13.1

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1. Evaluate the following:

(i) i^{457}

(ii) i^{528}

(iii) $1/i^{58}$

(iv) $i^{37} + 1/i^{67}$

(v) $[i^{41} + 1/i^{257}]$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(vii) $i^{30} + i^{40} + i^{60}$

(viii) $i^{49} + i^{68} + i^{89} + i^{110}$

Solution:

(i) i^{457}

Let us simplify we get,

$$\begin{aligned} i^{457} &= i^{4(114) + 1} \\ &= i^{4(114)} \times i \\ &= (1)^{114} \times i \\ &= i \text{ [since } i^4 = 1] \end{aligned}$$

(ii) i^{528}

Let us simplify we get,

$$\begin{aligned} i^{528} &= i^{4(132)} \\ &= (1)^{132} \\ &= 1 \text{ [since } i^4 = 1] \end{aligned}$$

(iii) $1/i^{58}$

Let us simplify we get,

$$\begin{aligned} 1/i^{58} &= 1/i^{56+2} \\ &= 1/i^{56} \times i^2 \\ &= 1/(i^4)^{14} \times i^2 \\ &= 1/i^2 \text{ [since, } i^4 = 1] \\ &= 1/-1 \text{ [since, } i^2 = -1] \\ &= -1 \end{aligned}$$

(iv) $i^{37} + 1/i^{67}$

Let us simplify we get,

$$\begin{aligned} i^{37} + 1/i^{67} &= i^{36+1} + 1/i^{64+3} \\ &= i + 1/i^3 \text{ [since, } i^4 = 1] \\ &= i + i/i^4 \end{aligned}$$

$$= i + i$$

$$= 2i$$

(v) $[i^{41} + 1/i^{257}]$

Let us simplify we get,

$$\begin{aligned} [i^{41} + 1/i^{257}] &= [i^{40+1} + 1/i^{256+1}] \\ &= [i + 1/i]^9 \text{ [since, } 1/i = -1] \\ &= [i - i] \\ &= 0 \end{aligned}$$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

Let us simplify we get,

$$\begin{aligned} (i^{77} + i^{70} + i^{87} + i^{414})^3 &= (i^{(76+1)} + i^{(68+2)} + i^{(84+3)} + i^{(412+2)})^3 \\ &= (i + i^2 + i^3 + i^2)^3 \text{ [since } i^3 = -i, i^2 = -1] \\ &= (i + (-1) + (-i) + (-1))^3 \\ &= (-2)^3 \\ &= -8 \end{aligned}$$

(vii) $i^{30} + i^{40} + i^{60}$

Let us simplify we get,

$$\begin{aligned} i^{30} + i^{40} + i^{60} &= i^{(28+2)} + i^{40} + i^{60} \\ &= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{15} \\ &= i^2 + 1^{10} + 1^{15} \\ &= -1 + 1 + 1 \\ &= 1 \end{aligned}$$

(viii) $i^{49} + i^{68} + i^{89} + i^{110}$

Let us simplify we get,

$$\begin{aligned} i^{49} + i^{68} + i^{89} + i^{110} &= i^{(48+1)} + i^{68} + i^{(88+1)} + i^{(116+2)} \\ &= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{11} \times i + (i^4)^{29} \times i^2 \\ &= i + 1 + i - 1 \\ &= 2i \end{aligned}$$

2. Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number?

Solution:

Given:

$$\begin{aligned} 1 + i^{10} + i^{20} + i^{30} &= 1 + i^{(8+2)} + i^{20} + i^{(28+2)} \\ &= 1 + (i^4)^2 \times i^2 + (i^4)^5 + (i^4)^7 \times i^2 \\ &= 1 - 1 + 1 - 1 \text{ [since, } i^4 = 1, i^2 = -1] \\ &= 0 \end{aligned}$$

Hence, $1 + i^{10} + i^{20} + i^{30}$ is a real number.

3. Find the values of the following expressions:

(i) $i^{49} + i^{68} + i^{89} + i^{110}$

(ii) $i^{30} + i^{80} + i^{120}$

(iii) $i + i^2 + i^3 + i^4$

(iv) $i^5 + i^{10} + i^{15}$

(v) $[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

(vi) $1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20}$

(vii) $(1 + i)^6 + (1 - i)^3$

Solution:

(i) $i^{49} + i^{68} + i^{89} + i^{110}$

Let us simplify we get,

$$\begin{aligned} i^{49} + i^{68} + i^{89} + i^{110} &= i^{(48+1)} + i^{68} + i^{(88+1)} + i^{(108+2)} \\ &= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{11} \times i + (i^4)^{27} \times i^2 \\ &= i + 1 + i - 1 \text{ [since } i^4 = 1, i^2 = -1] \\ &= 2i \end{aligned}$$

$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$

(ii) $i^{30} + i^{80} + i^{120}$

Let us simplify we get,

$$\begin{aligned} i^{30} + i^{80} + i^{120} &= i^{(28+2)} + i^{80} + i^{120} \\ &= (i^4)^7 \times i^2 + (i^4)^{20} + (i^4)^{30} \\ &= -1 + 1 + 1 \text{ [since } i^4 = 1, i^2 = -1] \\ &= 1 \end{aligned}$$

$\therefore i^{30} + i^{80} + i^{120} = 1$

(iii) $i + i^2 + i^3 + i^4$

Let us simplify we get,

$$\begin{aligned} i + i^2 + i^3 + i^4 &= i + i^2 + i^2 \times i + i^4 \\ &= i - 1 + (-1) \times i + 1 \text{ [since } i^4 = 1, i^2 = -1] \\ &= i - 1 - i + 1 \\ &= 0 \end{aligned}$$

$\therefore i + i^2 + i^3 + i^4 = 0$

(iv) $i^5 + i^{10} + i^{15}$

Let us simplify we get,

$$\begin{aligned} i^5 + i^{10} + i^{15} &= i^{(4+1)} + i^{(8+2)} + i^{(12+3)} \\ &= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^3 \end{aligned}$$

$$\begin{aligned}
 &= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^2 \times i \\
 &= 1 \times i + 1 \times (-1) + 1 \times (-1) \times i \\
 &= i - 1 - i \\
 &= -1 \\
 \therefore i^5 + i^{10} + i^{15} &= -1
 \end{aligned}$$

(v) $[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

Let us simplify we get,

$$\begin{aligned}
 &[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}] \\
 &= [i^{10} (i^{582} + i^{580} + i^{578} + i^{576} + i^{574})] / (i^{582} + i^{580} + i^{578} + i^{576} + i^{574}) \\
 &= i^{10} \\
 &= i^8 i^2 \\
 &= (i^4)^2 i^2 \\
 &= (1)^2 (-1) \text{ [since } i^4 = 1, i^2 = -1] \\
 &= -1 \\
 \therefore [i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}] &= -1
 \end{aligned}$$

(vi) $1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20}$

Let us simplify we get,

$$\begin{aligned}
 1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20} &= 1 + (-1) + 1 + (-1) + 1 + \dots + 1 \\
 &= 1 \\
 \therefore 1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20} &= 1
 \end{aligned}$$

(vii) $(1 + i)^6 + (1 - i)^3$

Let us simplify we get,

$$\begin{aligned}
 (1 + i)^6 + (1 - i)^3 &= \{(1 + i)^2\}^3 + (1 - i)^2 (1 - i) \\
 &= \{1 + i^2 + 2i\}^3 + (1 + i^2 - 2i)(1 - i) \\
 &= \{1 - 1 + 2i\}^3 + (1 - 1 - 2i)(1 - i) \\
 &= (2i)^3 + (-2i)(1 - i) \\
 &= 8i^3 + (-2i) + 2i^2 \\
 &= -8i - 2i - 2 \text{ [since } i^3 = -i, i^2 = -1] \\
 &= -10i - 2 \\
 &= -2(1 + 5i) \\
 &= -2 - 10i \\
 \therefore (1 + i)^6 + (1 - i)^3 &= -2 - 10i
 \end{aligned}$$