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1. Find the square root of the following complex numbers.
(i) – 5 + 12i
(ii) -7 - 24i
(iii) 1 - i
(iv) - 8 - 6i
(v) 8 – 15i
(vi) -11 - 60√-1
(vii) 1 + 4√-3
(viii) 4i
(ix) -i
Solution:

$$if\ b > 0, \sqrt{a+ib} = \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$if\ b < 0, \sqrt{a+ib} = \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right]$$

(i) – 5 + 12i
Given:
– 5 + 12i
We know, Z = a + ib
So, √(a + ib) = √(-5+12i)
Here, b > 0
Let us simplify now,

$$\sqrt{-5+12i} = \pm\left[\left(\frac{-5+\sqrt{(-5)^2+12^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{(-5)^2+12^2}}{2}\right)^{\frac{1}{2}}\right]$$ [Since, b > 0]

$$= \pm\left[\left(\frac{-5+\sqrt{25+144}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{25+144}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-5+\sqrt{169}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{169}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-5+13}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+13}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8}{2}\right)^{\frac{1}{2}} + i\left(\frac{18}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[4^{\frac{1}{2}} + i9^{\frac{1}{2}}\right]$$

$$= \pm[2 + 3i]$$

∴ Square root of (– 5 + 12i) is ±[2 + 3i]

(ii) -7 – 24i

Given:

-7 – 24i

We know, Z = -7 – 24i

So, √(a + ib) = √(-7 – 24i)

Here, b < 0

Let us simplify now,

$$\sqrt{-7-24i} = \pm\left[\left(\frac{-7+\sqrt{(-7)^2+(-24)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{(-7)^2+(-24)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{-7+\sqrt{49+576}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{49+576}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-7+\sqrt{625}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{625}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-7+25}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+25}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{18}{2}\right)^{\frac{1}{2}} - i\left(\frac{32}{2}\right)^{\frac{1}{2}}\right]$$

$= \pm [9^{1/2} – i16^{1/2}]$

$= \pm [3 – 4i]$

∴ Square root of (-7 – 24i) is ± [3 - 4i]

(iii) 1 - i

Given:

1 – i

We know, Z = (1 - i)

So, √(a + ib) = √(1 - i)

Here, b < 0
Let us simplify now,

$$\sqrt{1-i} = \pm\left[\left(\frac{1+\sqrt{(1)^2+(-1)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{(1)^2+(-1)^2}}{2}\right)^{\frac{1}{2}}\right] \text{ [Since, b < 0]}$$

$$= \pm\left[\left(\frac{1+\sqrt{1+1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{1+1}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1+\sqrt{2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{2}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\sqrt{\frac{\sqrt{2}+1}{2}}\right) - i\left(\sqrt{\frac{\sqrt{2}-1}{2}}\right)\right]$$

∴ Square root of (1 - i) is ± [(√(√2+1)/2) – i (√(√2-1)/2)]

(iv) -8 -6i
Given:
-8 -6i
We know, Z = -8 -6i
So, √(a + ib) = -8 -6i
Here, b < 0
Let us simplify now,

$$\sqrt{-8-6i} = \pm\left[\left(\frac{-8+\sqrt{(-8)^2+(-6)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{(-8)^2+(-6)^2}}{2}\right)^{\frac{1}{2}}\right] \text{ [Since, b < 0]}$$

$$= \pm\left[\left(\frac{-8+\sqrt{64+36}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{64+36}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-8+\sqrt{100}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{100}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-8+10}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+10}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{2}{2}\right)^{\frac{1}{2}} - i\left(\frac{18}{2}\right)^{\frac{1}{2}}\right]$$

$= [1^{1/2} – i\ 9^{1/2}]$

= ± [1 – 3i]
∴ Square root of (-8 -6i) is ± [1 – 3i]

(v) 8 – 15i
Given:
8 – 15i
We know, Z = 8 – 15i
So, √(a + ib) = 8 – 15i
Here, b < 0
Let us simplify now,

$$\sqrt{8-15i} = \pm\left[\left(\frac{8+\sqrt{8^2+(-15)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{8^2+(-15)^2}}{2}\right)^{\frac{1}{2}}\right]$$ [Since, b < 0]

$$= \pm\left[\left(\frac{8+\sqrt{64+225}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{64+225}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8+\sqrt{289}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{289}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8+17}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+17}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{25}{2}\right)^{\frac{1}{2}} - i\left(\frac{9}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\frac{5}{\sqrt{2}} - \frac{i3}{\sqrt{2}}\right]$$

= ± 1/√2 (5 – 3i)
∴ Square root of (8 – 15i) is ± 1/√2 (5 – 3i)

(vi) -11 - 60√-1
Given:
-11 - 60√-1
We know, Z = -11 - 60√-1
So, √(a + ib) = -11 - 60√-1
= -11 – 60i
Here, b < 0
Let us simplify now,

$$\sqrt{-11-60i} = \pm\left[\left(\frac{-11+\sqrt{(-11)^2+(-60)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{(-11)^2+(60)^2}}{2}\right)^{\frac{1}{2}}\right]$$ [Since, b < 0]

$$= \pm\left[\left(\frac{-11+\sqrt{121+3600}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{121+3600}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-11+\sqrt{3721}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{3721}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-11+61}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+61}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{50}{2}\right)^{\frac{1}{2}} - i\left(\frac{72}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[25^{\frac{1}{2}} - i36^{\frac{1}{2}}\right]$$

$$= \pm(5-6i)$$

∴ Square root of (-11 - 60√-1) is ± (5 – 6i)

(vii) 1 + 4√-3

Given:

1 + 4√-3

We know, Z = 1 + 4√-3

So, √(a + ib) = 1 + 4√-3

= 1 + 4(√3) (√-1)

= 1 + 4√3i

Here, b > 0

Let us simplify now,

$$\sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{1+\sqrt{1^2+(4\sqrt{3})^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{1^2+(4\sqrt{3})^2}}{2}\right)^{\frac{1}{2}}\right]$$ [Since, b > 0]

$$= \pm\left[\left(\frac{1+\sqrt{1+48}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{1+48}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1+\sqrt{49}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{49}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1+7}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+7}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8}{2}\right)^{\frac{1}{2}} + i\left(\frac{6}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[4^{\frac{1}{2}} + i3^{\frac{1}{2}}\right]$$

$$= \pm\left[2 + \sqrt{3}i\right]$$

∴ Square root of (1 + 4√-3) is ± (2 + √3i)

(viii) 4i

Given:

4i

We know, Z = 4i

So, √(a + ib) = 4i

Here, b > 0

Let us simplify now,

$$\sqrt{4i} = \pm\left[\left(\frac{0+\sqrt{0^2+4^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{0^2+4^2}}{2}\right)^{\frac{1}{2}}\right] \text{ [Since, b > 0]}$$

$$= \pm\left[\left(\frac{0+\sqrt{0+16}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{0+16}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+\sqrt{16}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{16}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+4}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+4}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{4}{2}\right)^{\frac{1}{2}} + i\left(\frac{4}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[2^{\frac{1}{2}} + i2^{\frac{1}{2}}\right]$$

$$= \pm\left[\sqrt{2} + \sqrt{2}i\right]$$

= ± √2 (1 + i)

∴ Square root of 4i is ± √2 (1 + i)

(ix) –i

Given:

-i

We know, Z = -i

So, √(a + ib) = -i

Here, b < 0

Let us simplify now,

$$\sqrt{-i} = \pm\left[\left(\frac{0+\sqrt{0^2+(-1)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{0^2+(-1)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{0+\sqrt{0+1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{0+1}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+\sqrt{1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{1}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+1}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+1}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1}{2}\right)^{\frac{1}{2}} - i\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right]$$

= ± 1/√2 (1 - i)

∴ Square root of –i is ± 1/√2 (1 - i)