

EXERCISE 14.1

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Solve the following quadratic equations by factorization method only:

1. $x^2 + 1 = 0$

Solution:

Given: $x^2 + 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$x^2 - i^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(x + i)(x - i) = 0$$

$$x + i = 0 \text{ or } x - i = 0$$

$$x = -i \text{ or } x = i$$

\therefore The roots of the given equation are $i, -i$

2. $9x^2 + 4 = 0$

Solution:

Given: $9x^2 + 4 = 0$

$$9x^2 + 4 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

So,

$$9x^2 + 4(-i^2) = 0$$

$$9x^2 - 4i^2 = 0$$

$$(3x)^2 - (2i)^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(3x + 2i)(3x - 2i) = 0$$

$$3x + 2i = 0 \text{ or } 3x - 2i = 0$$

$$3x = -2i \text{ or } 3x = 2i$$

$$x = -2i/3 \text{ or } x = 2i/3$$

\therefore The roots of the given equation are $2i/3, -2i/3$

3. $x^2 + 2x + 5 = 0$

Solution:

Given: $x^2 + 2x + 5 = 0$

$$x^2 + 2x + 1 + 4 = 0$$

$$x^2 + 2(x)(1) + 1^2 + 4 = 0$$

$$(x + 1)^2 + 4 = 0 \text{ [since, } (a + b)^2 = a^2 + 2ab + b^2]$$

$$(x + 1)^2 + 4 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$(x + 1)^2 + 4(-i^2) = 0$$

$$(x + 1)^2 - 4i^2 = 0$$

$$(x + 1)^2 - (2i)^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(x + 1 + 2i)(x + 1 - 2i) = 0$$

$$x + 1 + 2i = 0 \text{ or } x + 1 - 2i = 0$$

$$x = -1 - 2i \text{ or } x = -1 + 2i$$

\therefore The roots of the given equation are $-1+2i, -1-2i$

4. $4x^2 - 12x + 25 = 0$

Solution:

Given: $4x^2 - 12x + 25 = 0$

$$4x^2 - 12x + 9 + 16 = 0$$

$$(2x)^2 - 2(2x)(3) + 3^2 + 16 = 0$$

$$(2x - 3)^2 + 16 = 0 \text{ [Since, } (a + b)^2 = a^2 + 2ab + b^2]$$

$$(2x - 3)^2 + 16 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$(2x - 3)^2 + 16(-i^2) = 0$$

$$(2x - 3)^2 - 16i^2 = 0$$

$$(2x - 3)^2 - (4i)^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(2x - 3 + 4i)(2x - 3 - 4i) = 0$$

$$2x - 3 + 4i = 0 \text{ or } 2x - 3 - 4i = 0$$

$$2x = 3 - 4i \text{ or } 2x = 3 + 4i$$

$$x = 3/2 - 2i \text{ or } x = 3/2 + 2i$$

\therefore The roots of the given equation are $3/2 + 2i, 3/2 - 2i$

5. $x^2 + x + 1 = 0$

Solution:

Given: $x^2 + x + 1 = 0$

$$x^2 + x + 1/4 + 3/4 = 0$$

$$x^2 + 2(x)(1/2) + (1/2)^2 + 3/4 = 0$$

$$(x + 1/2)^2 + 3/4 = 0 \text{ [Since, } (a + b)^2 = a^2 + 2ab + b^2]$$

$$(x + 1/2)^2 + 3/4 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$(x + \frac{1}{2})^2 + \frac{3}{4}(-1)^2 = 0$$

$$(x + \frac{1}{2})^2 + \frac{3}{4}i^2 = 0$$

$$(x + \frac{1}{2})^2 - (\frac{\sqrt{3}i}{2})^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(x + \frac{1}{2} + \frac{\sqrt{3}i}{2})(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}) = 0$$

$$(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}) = 0 \text{ or } (x + \frac{1}{2} - \frac{\sqrt{3}i}{2}) = 0$$

$$x = -\frac{1}{2} - \frac{\sqrt{3}i}{2} \text{ or } x = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

\therefore The roots of the given equation are $-\frac{1}{2} + \frac{\sqrt{3}i}{2}$, $-\frac{1}{2} - \frac{\sqrt{3}i}{2}$

6. $4x^2 + 1 = 0$

Solution:

Given: $4x^2 + 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$4x^2 - i^2 = 0$$

$$(2x)^2 - i^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(2x + i)(2x - i) = 0$$

$$2x + i = 0 \text{ or } 2x - i = 0$$

$$2x = -i \text{ or } 2x = i$$

$$x = -i/2 \text{ or } x = i/2$$

\therefore The roots of the given equation are $i/2$, $-i/2$

7. $x^2 - 4x + 7 = 0$

Solution:

Given: $x^2 - 4x + 7 = 0$

$$x^2 - 4x + 4 + 3 = 0$$

$$x^2 - 2(x)(2) + 2^2 + 3 = 0$$

$$(x - 2)^2 + 3 = 0 \text{ [Since, } (a - b)^2 = a^2 - 2ab + b^2]$$

$$(x - 2)^2 + 3 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$(x - 2)^2 + 3(-i^2) = 0$$

$$(x - 2)^2 - 3i^2 = 0$$

$$(x - 2)^2 - (\sqrt{3}i)^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(x - 2 + \sqrt{3}i)(x - 2 - \sqrt{3}i) = 0$$

$$(x - 2 + \sqrt{3}i) = 0 \text{ or } (x - 2 - \sqrt{3}i) = 0$$

$$x = 2 - \sqrt{3}i \text{ or } x = 2 + \sqrt{3}i$$

$$x = 2 \pm \sqrt{3}i$$

∴ The roots of the given equation are $2 \pm \sqrt{3}i$

8. $x^2 + 2x + 2 = 0$

Solution:

Given: $x^2 + 2x + 2 = 0$

$$x^2 + 2x + 1 + 1 = 0$$

$$x^2 + 2(x)(1) + 1^2 + 1 = 0$$

$$(x + 1)^2 + 1 = 0 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$(x + 1)^2 + (-i^2) = 0$$

$$(x + 1)^2 - i^2 = 0$$

$$(x + 1)^2 - (i)^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(x + 1 + i)(x + 1 - i) = 0$$

$$x + 1 + i = 0 \text{ or } x + 1 - i = 0$$

$$x = -1 - i \text{ or } x = -1 + i$$

$$x = -1 \pm i$$

∴ The roots of the given equation are $-1 \pm i$

9. $5x^2 - 6x + 2 = 0$

Solution:

Given: $5x^2 - 6x + 2 = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 5$, $b = -6$, $c = 2$

So,

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(2)}}{2(5)}$$

$$= \frac{6 \pm \sqrt{36-40}}{10}$$

$$= \frac{6 \pm \sqrt{-4}}{10}$$

$$= \frac{6 \pm \sqrt{4(-1)}}{10}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = \frac{6 \pm \sqrt{4i^2}}{10}$$

$$= \frac{6 \pm 2i}{10}$$

$$= \frac{2(3 \pm i)}{10}$$

$$= \frac{(3 \pm i)}{5}$$

$$x = 3/5 \pm i/5$$

∴ The roots of the given equation are $3/5 \pm i/5$

10. $21x^2 + 9x + 1 = 0$

Solution:

Given: $21x^2 + 9x + 1 = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Here, $a = 21$, $b = 9$, $c = 1$

So,

$$x = (-9 \pm \sqrt{9^2 - 4(21)(1)})/2(21)$$

$$= (-9 \pm \sqrt{81-84})/42$$

$$= (-9 \pm \sqrt{-3})/42$$

$$= (-9 \pm \sqrt{3}(-1))/42$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$x = (-9 \pm \sqrt{3i^2})/42$$

$$= (-9 \pm \sqrt{(\sqrt{3}i)^2})/42$$

$$= (-9 \pm \sqrt{3}i)/42$$

$$= -9/42 \pm \sqrt{3}i/42$$

$$= -3/14 \pm \sqrt{3}i/42$$

∴ The roots of the given equation are $-3/14 \pm \sqrt{3}i/42$

11. $x^2 - x + 1 = 0$

Solution:

Given: $x^2 - x + 1 = 0$

$$x^2 - x + 1/4 + 3/4 = 0$$

$$x^2 - 2(x)(1/2) + (1/2)^2 + 3/4 = 0$$

$$(x - 1/2)^2 + 3/4 = 0 \text{ [Since, } (a + b)^2 = a^2 + 2ab + b^2]$$

$$(x - 1/2)^2 + 3/4 \times 1 = 0$$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$$(x - 1/2)^2 + 3/4 (-1)^2 = 0$$

$$(x - 1/2)^2 + 3/4 (-i)^2 = 0$$

$$(x - 1/2)^2 - (\sqrt{3}i/2)^2 = 0$$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$$(x - 1/2 + \sqrt{3}i/2)(x - 1/2 - \sqrt{3}i/2) = 0$$

$$(x - 1/2 + \sqrt{3}i/2) = 0 \text{ or } (x - 1/2 - \sqrt{3}i/2) = 0$$

$$x = 1/2 - \sqrt{3}i/2 \text{ or } x = 1/2 + \sqrt{3}i/2$$

∴ The roots of the given equation are $1/2 + \sqrt{3}i/2$, $1/2 - \sqrt{3}i/2$

12. $x^2 + x + 1 = 0$

Solution:

Given: $x^2 + x + 1 = 0$

$x^2 + x + 1/4 + 3/4 = 0$

$x^2 + 2(x)(1/2) + (1/2)^2 + 3/4 = 0$

$(x + 1/2)^2 + 3/4 = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x + 1/2)^2 + 3/4 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x + 1/2)^2 + 3/4 (-1)^2 = 0$

$(x + 1/2)^2 + 3/4 i^2 = 0$

$(x + 1/2)^2 - (\sqrt{3}i/2)^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]

$(x + 1/2 + \sqrt{3}i/2)(x + 1/2 - \sqrt{3}i/2) = 0$

$(x + 1/2 + \sqrt{3}i/2) = 0$ or $(x + 1/2 - \sqrt{3}i/2) = 0$

$x = -1/2 - \sqrt{3}i/2$ or $x = -1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $-1/2 + \sqrt{3}i/2$, $-1/2 - \sqrt{3}i/2$

13. $17x^2 - 8x + 1 = 0$

Solution:

Given: $17x^2 - 8x + 1 = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Here, $a = 17$, $b = -8$, $c = 1$

So,

$x = (-(-8) \pm \sqrt{(-8)^2 - 4(17)(1)})/2(17)$

$= (8 \pm \sqrt{64-68})/34$

$= (8 \pm \sqrt{-4})/34$

$= (8 \pm \sqrt{4(-1)})/34$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$x = (8 \pm \sqrt{(2i)^2})/34$

$= (8 \pm 2i)/34$

$= 2(4 \pm i)/34$

$= (4 \pm i)/17$

$x = 4/17 \pm i/17$

∴ The roots of the given equation are $4/17 \pm i/17$

EXERCISE 14.2

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1. Solving the following quadratic equations by factorization method:

(i) $x^2 + 10ix - 21 = 0$

(ii) $x^2 + (1 - 2i)x - 2i = 0$

(iii) $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

(iv) $6x^2 - 17ix - 12 = 0$

Solution:

(i) $x^2 + 10ix - 21 = 0$

Given: $x^2 + 10ix - 21 = 0$

$x^2 + 10ix - 21 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$x^2 + 10ix - 21(-i^2) = 0$

$x^2 + 10ix + 21i^2 = 0$

$x^2 + 3ix + 7ix + 21i^2 = 0$

$x(x + 3i) + 7i(x + 3i) = 0$

$(x + 3i)(x + 7i) = 0$

$x + 3i = 0$ or $x + 7i = 0$

$x = -3i$ or $-7i$

\therefore The roots of the given equation are $-3i, -7i$

(ii) $x^2 + (1 - 2i)x - 2i = 0$

Given: $x^2 + (1 - 2i)x - 2i = 0$

$x^2 + x - 2ix - 2i = 0$

$x(x + 1) - 2i(x + 1) = 0$

$(x + 1)(x - 2i) = 0$

$x + 1 = 0$ or $x - 2i = 0$

$x = -1$ or $2i$

\therefore The roots of the given equation are $-1, 2i$

(iii) $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

Given: $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

$x^2 - (2\sqrt{3}x + 3ix) + 6\sqrt{3}i = 0$

$x^2 - 2\sqrt{3}x - 3ix + 6\sqrt{3}i = 0$

$x(x - 2\sqrt{3}) - 3i(x - 2\sqrt{3}) = 0$

$(x - 2\sqrt{3})(x - 3i) = 0$

$(x - 2\sqrt{3}) = 0$ or $(x - 3i) = 0$

$x = 2\sqrt{3}$ or $x = 3i$

∴ The roots of the given equation are $2\sqrt{3}$, $3i$

(iv) $6x^2 - 17ix - 12 = 0$

Given: $6x^2 - 17ix - 12 = 0$

$6x^2 - 17ix - 12 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$6x^2 - 17ix - 12(-i^2) = 0$

$6x^2 - 17ix + 12i^2 = 0$

$6x^2 - 9ix - 8ix + 12i^2 = 0$

$3x(2x - 3i) - 4i(2x - 3i) = 0$

$(2x - 3i)(3x - 4i) = 0$

$2x - 3i = 0$ or $3x - 4i = 0$

$2x = 3i$ or $3x = 4i$

$x = 3i/2$ or $x = 4i/3$

∴ The roots of the given equation are $3i/2$, $4i/3$

2. Solve the following quadratic equations:

(i) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

(ii) $x^2 - (5 - i)x + (18 + i) = 0$

(iii) $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

(iv) $x^2 - (2 + i)x - (1 - 7i) = 0$

(v) $ix^2 - 4x - 4i = 0$

(vi) $x^2 + 4ix - 4 = 0$

(vii) $2x^2 + \sqrt{15}ix - i = 0$

(viii) $x^2 - x + (1 + i) = 0$

(ix) $ix^2 - x + 12i = 0$

(x) $x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$

(xi) $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

(xii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Solution:

(i) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Given: $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

$x^2 - (3\sqrt{2}x + 2ix) + 6\sqrt{2}i = 0$

$x^2 - 3\sqrt{2}x - 2ix + 6\sqrt{2}i = 0$

$x(x - 3\sqrt{2}) - 2i(x - 3\sqrt{2}) = 0$

$(x - 3\sqrt{2})(x - 2i) = 0$

$(x - 3\sqrt{2}) = 0$ or $(x - 2i) = 0$

$x = 3\sqrt{2}$ or $x = 2i$

∴ The roots of the given equation are $3\sqrt{2}$, $2i$

(ii) $x^2 - (5 - i)x + (18 + i) = 0$

Given: $x^2 - (5 - i)x + (18 + i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 1$, $b = -(5 - i)$, $c = (18 + i)$

So,

$$\begin{aligned} x &= \frac{-(-(5 - i)) \pm \sqrt{(-(5 - i))^2 - 4(1)(18 + i)}}{2(1)} \\ &= \frac{(5 - i) \pm \sqrt{(5 - i)^2 - 4(18 + i)}}{2} \\ &= \frac{(5 - i) \pm \sqrt{25 - 10i + i^2 - 72 - 4i}}{2} \\ &= \frac{(5 - i) \pm \sqrt{-47 - 14i + i^2}}{2} \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned} &= \frac{(5 - i) \pm \sqrt{-47 - 14i + (-1)}}{2} \\ &= \frac{(5 - i) \pm \sqrt{-48 - 14i}}{2} \\ &= \frac{(5 - i) \pm \sqrt{(-1)(48 + 14i)}}{2} \\ &= \frac{(5 - i) \pm \sqrt{i^2(48 + 14i)}}{2} \\ &= \frac{(5 - i) \pm i\sqrt{48 + 14i}}{2} \end{aligned}$$

We can write $48 + 14i = 49 - 1 + 14i$

So,

$$\begin{aligned} 48 + 14i &= 49 + i^2 + 14i \quad [\because i^2 = -1] \\ &= 7^2 + i^2 + 2(7)(i) \\ &= (7 + i)^2 \quad [\text{Since, } (a + b)^2 = a^2 + b^2 + 2ab] \end{aligned}$$

By using the result $48 + 14i = (7 + i)^2$, we get

$$\begin{aligned}
 &= \frac{(5-i) \pm i\sqrt{(7+i)^2}}{2} \\
 &= \frac{(5-i) \pm i(7+i)}{2} \\
 &= \frac{(5-i) + i(7+i)}{2} \text{ or } \frac{(5-i) - i(7+i)}{2} \\
 &= \frac{5-i+7i+i^2}{2} \text{ or } \frac{5-i-7i-i^2}{2} \\
 &= \frac{5+6i+(-1)}{2} \text{ or } \frac{5-8i-(-1)}{2} \\
 &= \frac{5+6i-1}{2} \text{ or } \frac{5-8i+1}{2} \\
 &= \frac{4+6i}{2} \text{ or } \frac{6-8i}{2} \\
 &= \frac{2(2+3i)}{2} \text{ or } \frac{2(3-4i)}{2}
 \end{aligned}$$

$$x = 2 + 3i \text{ or } 3 - 4i$$

∴ The roots of the given equation are $3 - 4i$, $2 + 3i$

(iii) $(2+i)x^2 - (5-i)x + 2(1-i) = 0$

Given: $(2+i)x^2 - (5-i)x + 2(1-i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = (2+i)$, $b = -(5-i)$, $c = 2(1-i)$

So,

$$\begin{aligned}
 x &= \frac{-(-(5-i)) \pm \sqrt{(-(5-i))^2 - 4(2+i)(2(1-i))}}{2(2+i)} \\
 &= \frac{(5-i) \pm \sqrt{(5-i)^2 - 8(2+i)(1-i)}}{2(2+i)} \\
 &= \frac{(5-i) \pm \sqrt{25 - 10i + i^2 - 8(2 - 2i + i - i^2)}}{2(2+i)}
 \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned}
 &= \frac{(5-i) \pm \sqrt{25-10i+(-1)-8(2-i-(-1))}}{2(2+i)} \\
 &= \frac{(5-i) \pm \sqrt{24-10i-8(3-i)}}{2(2+i)} \\
 &= \frac{(5-i) \pm \sqrt{24-10i-24+8i}}{2(2+i)} \\
 &= \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)}
 \end{aligned}$$

We can write $-2i = -2i + 1 - 1$

$-2i = -2i + 1 + i^2$ [Since, $i^2 = -1$]

$$= 1 - 2i + i^2$$

$$= 1^2 - 2(1)(i) + i^2$$

$$= (1-i)^2 \text{ [By using the formula, } (a-b)^2 = a^2 - 2ab + b^2]$$

By using the result $-2i = (1-i)^2$, we get

$$\begin{aligned}
 x &= \frac{(5-i) \pm \sqrt{(1-i)^2}}{2(2+i)} \\
 &= \frac{(5-i) \pm (1-i)}{2(2+i)} \\
 &= \frac{(5-i) + (1-i)}{2(2+i)} \text{ or } \frac{(5-i) - (1-i)}{2(2+i)} \\
 &= \frac{5-i+1-i}{2(2+i)} \text{ or } \frac{5-i-1+i}{2(2+i)} \\
 &= \frac{6-2i}{2(2+i)} \text{ or } \frac{4}{2(2+i)} \\
 &= \frac{3-i}{2+i} \text{ or } \frac{2}{2+i}
 \end{aligned}$$

Let us multiply and divide by $(2-i)$, we get

$$\begin{aligned}
 &= \frac{3-i}{2+i} \times \frac{2-i}{2-i} \text{ or } \frac{2}{2+i} \times \frac{2-i}{2-i} \\
 &= \frac{(3-i)(2-i)}{(2+i)(2-i)} \text{ or } \frac{2(2-i)}{(2+i)(2-i)} \\
 &= \frac{6-3i-2i+i^2}{2^2-i^2} \text{ or } \frac{4-2i}{2^2-i^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{6-5i+(-1)}{4-(-1)} \text{ or } \frac{4-2i}{4-(-1)} \\
 &= \frac{5-5i}{4+1} \text{ or } \frac{4-2i}{4+1} \\
 &= \frac{5(1-i)}{5} \text{ or } \frac{4-2i}{5}
 \end{aligned}$$

$$x = (1-i) \text{ or } 4/5 - 2i/5$$

∴ The roots of the given equation are $(1-i)$, $4/5 - 2i/5$

$$\text{(iv) } x^2 - (2+i)x - (1-7i) = 0$$

$$\text{Given: } x^2 - (2+i)x - (1-7i) = 0$$

We shall apply discriminant rule,

$$\text{Where, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Here, } a = 1, b = -(2+i), c = -(1-7i)$$

So,

$$\begin{aligned}
 x &= \frac{-(-(2+i)) \pm \sqrt{(-(2+i))^2 - 4(1)(-(1-7i))}}{2(1)} \\
 &= \frac{(2+i) \pm \sqrt{(2+i)^2 + 4(1-7i)}}{2} \\
 &= \frac{(2+i) \pm \sqrt{4+4i+i^2+4-28i}}{2} \\
 &= \frac{(2+i) \pm \sqrt{8-24i+i^2}}{2}
 \end{aligned}$$

$$\text{We have } i^2 = -1$$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned}
 &= \frac{(2+i) \pm \sqrt{8-24i+(-1)}}{2} \\
 &= \frac{(2+i) \pm \sqrt{7-24i}}{2}
 \end{aligned}$$

$$\text{We can write } 7-24i = 16-9-24i$$

$$7-24i = 16+9(-1)-24i$$

$$= 16+9i^2-24i \quad [\because i^2 = -1]$$

$$= 4^2 + (3i)^2 - 2(4)(3i)$$

$$= (4-3i)^2 \quad [\because (a-b)^2 = a^2 - b^2 + 2ab]$$

By using the result $7 - 24i = (4 - 3i)^2$, we get

$$\begin{aligned} x &= \frac{(2 + i) \pm \sqrt{(4 - 3i)^2}}{2} \\ &= \frac{(2 + i) \pm (4 - 3i)}{2} \\ &= \frac{(2 + i) + (4 - 3i)}{2} \text{ or } \frac{(2 + i) - (4 - 3i)}{2} \\ &= \frac{2 + i + 4 - 3i}{2} \text{ or } \frac{2 + i - 4 + 3i}{2} \\ &= \frac{6 - 2i}{2} \text{ or } \frac{-2 + 4i}{2} \\ &= \frac{2(3 - i)}{2} \text{ or } \frac{2(-1 + 2i)}{2} \end{aligned}$$

$$x = 3 - i \text{ or } -1 + 2i$$

\therefore The roots of the given equation are $(-1 + 2i)$, $(3 - i)$

(v) $ix^2 - 4x - 4i = 0$

Given: $ix^2 - 4x - 4i = 0$

$ix^2 + 4x(-1) - 4i = 0$ [We know, $i^2 = -1$]

So by substituting $-1 = i^2$ in the above equation, we get

$$ix^2 + 4xi^2 - 4i = 0$$

$$i(x^2 + 4ix - 4) = 0$$

$$x^2 + 4ix - 4 = 0$$

$$x^2 + 4ix + 4(-1) = 0$$

$$x^2 + 4ix + 4i^2 = 0 \text{ [Since, } i^2 = -1]$$

$$x^2 + 2ix + 2ix + 4i^2 = 0$$

$$x(x + 2i) + 2i(x + 2i) = 0$$

$$(x + 2i)(x + 2i) = 0$$

$$(x + 2i)^2 = 0$$

$$x + 2i = 0$$

$$x = -2i, -2i$$

\therefore The roots of the given equation are $-2i, -2i$

(vi) $x^2 + 4ix - 4 = 0$

Given: $x^2 + 4ix - 4 = 0$

$$x^2 + 4ix + 4(-1) = 0 \text{ [We know, } i^2 = -1]$$

So by substituting $-1 = i^2$ in the above equation, we get

$$x^2 + 4ix + 4i^2 = 0$$

$$x^2 + 2ix + 2ix + 4i^2 = 0$$

$$x(x + 2i) + 2i(x + 2i) = 0$$

$$(x + 2i)(x + 2i) = 0$$

$$(x + 2i)^2 = 0$$

$$x + 2i = 0$$

$$x = -2i, -2i$$

\therefore The roots of the given equation are $-2i, -2i$

(vii) $2x^2 + \sqrt{15}ix - i = 0$

Given: $2x^2 + \sqrt{15}ix - i = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = 2$, $b = \sqrt{15}i$, $c = -i$

So,

$$\begin{aligned} x &= \frac{-(\sqrt{15}i) \pm \sqrt{(\sqrt{15}i)^2 - 4(2)(-i)}}{2(2)} \\ &= \frac{-\sqrt{15}i \pm \sqrt{15i^2 + 8i}}{4} \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned} &= \frac{-\sqrt{15}i \pm \sqrt{15(-1) + 8i}}{4} \\ &= \frac{-\sqrt{15}i \pm \sqrt{8i - 15}}{4} \\ &= \frac{-\sqrt{15}i \pm \sqrt{(-1)(15 - 8i)}}{4} \\ &= \frac{-\sqrt{15}i \pm \sqrt{i^2(15 - 8i)}}{4} \\ &= \frac{-\sqrt{15}i \pm i\sqrt{15 - 8i}}{4} \end{aligned}$$

We can write $15 - 8i = 16 - 1 - 8i$

$$\begin{aligned}
 15 - 8i &= 16 + (-1) - 8i \\
 &= 16 + i^2 - 8i \quad [\because i^2 = -1] \\
 &= 4^2 + (i)^2 - 2(4)(i) \\
 &= (4 - i)^2 \quad [\text{Since, } (a - b)^2 = a^2 - b^2 + 2ab]
 \end{aligned}$$

By using the result $15 - 8i = (4 - i)^2$, we get

$$\begin{aligned}
 x &= \frac{-\sqrt{15}i \pm i\sqrt{(4 - i)^2}}{4} \\
 &= \frac{-\sqrt{15}i \pm i(4 - i)}{4} \\
 &= \frac{-\sqrt{15}i + i(4 - i)}{4} \quad \text{or} \quad \frac{-\sqrt{15}i - i(4 - i)}{4} \\
 &= \frac{-\sqrt{15}i + 4i - i^2}{4} \quad \text{or} \quad \frac{-\sqrt{15}i - 4i + i^2}{4} \\
 &= \frac{-\sqrt{15}i + 4i - (-1)}{4} \quad \text{or} \quad \frac{-\sqrt{15}i - 4i + (-1)}{4} \\
 &= \frac{-\sqrt{15}i + 4i + 1}{4} \quad \text{or} \quad \frac{-\sqrt{15}i - 4i - 1}{4} \\
 &= \frac{1 + (4 - \sqrt{15})i}{4} \quad \text{or} \quad \frac{-1 - (4 + \sqrt{15})i}{4}
 \end{aligned}$$

\therefore The roots of the given equation are $[1 + (4 - \sqrt{15})i/4]$, $[-1 - (4 + \sqrt{15})i/4]$

(viii) $x^2 - x + (1 + i) = 0$

Given: $x^2 - x + (1 + i) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Here, $a = 1$, $b = -1$, $c = (1 + i)$

So,

$$\begin{aligned}
 x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1 + i)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1 - 4(1 + i)}}{2}
 \end{aligned}$$

$$= \frac{1 \pm \sqrt{1 - 4 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{-3 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{(-1)(3 + 4i)}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{1 \pm \sqrt{i^2(3 + 4i)}}{2}$$

$$= \frac{1 \pm i\sqrt{3 + 4i}}{2}$$

We can write $3 + 4i = 4 - 1 + 4i$

$$3 + 4i = 4 + i^2 + 4i [\because i^2 = -1]$$

$$= 2^2 + i^2 + 2(2)(i)$$

$$= (2 + i)^2 \text{ [Since, } (a + b)^2 = a^2 + b^2 + 2ab]$$

By using the result $3 + 4i = (2 + i)^2$, we get

$$x = \frac{1 \pm i\sqrt{(2 + i)^2}}{2}$$

$$= \frac{1 \pm i(2 + i)}{2}$$

$$= \frac{1 + i(2 + i)}{2} \text{ or } \frac{1 - i(2 + i)}{2}$$

$$= \frac{1 + 2i + i^2}{2} \text{ or } \frac{1 - 2i - i^2}{2}$$

$$= \frac{1 + 2i + (-1)}{2} \text{ or } \frac{1 - 2i - (-1)}{2}$$

$$= \frac{1 + 2i - 1}{2} \text{ or } \frac{1 - 2i + 1}{2}$$

$$x = 2i/2 \text{ or } (2 - 2i)/2$$

$$x = i \text{ or } 2(1-i)/2$$

$$x = i \text{ or } (1 - i)$$

\therefore The roots of the given equation are $(1-i)$, i

(ix) $ix^2 - x + 12i = 0$

Given: $ix^2 - x + 12i = 0$

$$ix^2 + x(-1) + 12i = 0 \text{ [We know, } i^2 = -1]$$

so by substituting $-1 = i^2$ in the above equation, we get

$$ix^2 + xi^2 + 12i = 0$$

$$i(x^2 + ix + 12) = 0$$

$$x^2 + ix + 12 = 0$$

$$x^2 + ix - 12(-1) = 0$$

$$x^2 + ix - 12i^2 = 0 \text{ [Since, } i^2 = -1]$$

$$x^2 - 3ix + 4ix - 12i^2 = 0$$

$$x(x - 3i) + 4i(x - 3i) = 0$$

$$(x - 3i)(x + 4i) = 0$$

$$x - 3i = 0 \text{ or } x + 4i = 0$$

$$x = 3i \text{ or } -4i$$

∴ The roots of the given equation are $-4i, 3i$

$$(x) x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$$

$$\text{Given: } x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$$

We shall apply discriminant rule,

$$\text{Where, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Here, } a = 1, b = -(3\sqrt{2} - 2i), c = -\sqrt{2}i$$

So,

$$\begin{aligned} x &= \frac{-(-(3\sqrt{2} - 2i)) \pm \sqrt{(-(3\sqrt{2} - 2i))^2 - 4(1)(-\sqrt{2}i)}}{2(1)} \\ &= \frac{(3\sqrt{2} - 2i) \pm \sqrt{(3\sqrt{2} - 2i)^2 + 4\sqrt{2}i}}{2} \\ &= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 12\sqrt{2}i + 4i^2 + 4\sqrt{2}i}}{2} \\ &= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i + 4i^2}}{2} \end{aligned}$$

$$\text{We have } i^2 = -1$$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i + 4(-1)}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i - 4}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{14 - 8\sqrt{2}i}}{2}$$

We can write $14 - 8\sqrt{2}i = 16 - 2 - 8\sqrt{2}i$

$$14 - 8\sqrt{2}i = 16 + 2(-1) - 8\sqrt{2}i$$

$$= 16 + 2i^2 - 8\sqrt{2}i \text{ [Since, } i^2 = -1]$$

$$= 4^2 + (\sqrt{2}i)^2 - 2(4)(\sqrt{2}i)$$

$$= (4 - \sqrt{2}i)^2 \text{ [By using the formula, } (a - b)^2 = a^2 - 2ab + b^2]$$

By using the result $14 - 8\sqrt{2}i = (4 - \sqrt{2}i)^2$, we get

$$x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{(4 - \sqrt{2}i)^2}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm (4 - \sqrt{2}i)}{2}$$

$$= \frac{3\sqrt{2} - 2i}{2} \pm \frac{4 - \sqrt{2}i}{2}$$

$$\therefore \text{ The roots of the given equation are } \frac{3\sqrt{2} - 2i}{2} \pm \frac{4 - \sqrt{2}i}{2}$$

(xi) $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

Given: $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

$$x^2 - (\sqrt{2}x + ix) + \sqrt{2}i = 0$$

$$x^2 - \sqrt{2}x - ix + \sqrt{2}i = 0$$

$$x(x - \sqrt{2}) - i(x - \sqrt{2}) = 0$$

$$(x - \sqrt{2})(x - i) = 0$$

$$(x - \sqrt{2}) = 0 \text{ or } (x - i) = 0$$

$$x = \sqrt{2} \text{ or } x = i$$

\therefore The roots of the given equation are $i, \sqrt{2}$

(xii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Given: $2x^2 - (3 + 7i)x + (9i - 3) = 0$

We shall apply discriminant rule,

Where, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 2, b = -(3 + 7i), c = (9i - 3)$

So,

$$\begin{aligned}
 x &= \frac{-(-(3+7i)) \pm \sqrt{(-(3+7i))^2 - 4(2)(9i-3)}}{2(2)} \\
 &= \frac{(3+7i) \pm \sqrt{(3+7i)^2 - 8(9i-3)}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{9+42i+49i^2 - 72i+24}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{33-30i+49i^2}}{4}
 \end{aligned}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$\begin{aligned}
 &= \frac{(3+7i) \pm \sqrt{33-30i+49(-1)}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{33-30i-49}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{-16-30i}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{(-1)(16+30i)}}{4} \\
 &= \frac{(3+7i) \pm \sqrt{i^2(16+30i)}}{4} \\
 &= \frac{(3+7i) \pm i\sqrt{16+30i}}{4}
 \end{aligned}$$

We can write $16+30i = 25-9+30i$

$$\begin{aligned}
 16+30i &= 25+9(-1)+30i \\
 &= 25+9i^2+30i \quad [\because i^2 = -1] \\
 &= 5^2+(3i)^2+2(5)(3i) \\
 &= (5+3i)^2 \quad [\because (a+b)^2 = a^2+b^2+2ab]
 \end{aligned}$$

By using the result $16+30i = (5+3i)^2$, we get

$$\begin{aligned}
 x &= \frac{(3+7i) \pm i\sqrt{(5+3i)^2}}{4} \\
 &= \frac{(3+7i) \pm i(5+3i)}{4}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(3 + 7i) + i(5 + 3i)}{4} \text{ or } \frac{(3 + 7i) - i(5 + 3i)}{4} \\
 &= \frac{3 + 7i + 5i + 3i^2}{4} \text{ or } \frac{3 + 7i - 5i - 3i^2}{4} \\
 &= \frac{3 + 12i + 3i^2}{4} \text{ or } \frac{3 + 2i - 3i^2}{4} \\
 &= \frac{3 + 12i + 3(-1)}{4} \text{ or } \frac{3 + 2i - 3(-1)}{4} \\
 &= \frac{3 + 12i - 3}{4} \text{ or } \frac{3 + 2i + 3}{4} \\
 &= \frac{12}{4}i \text{ or } \frac{6 + 2i}{4} \\
 &= 3i \text{ or } \frac{6}{4} + \frac{2}{4}i \\
 x &= 3i \text{ or } \frac{3}{2} + \frac{1}{2}i \\
 &= 3i \text{ or } (3 + i)/2
 \end{aligned}$$

∴ The roots of the given equation are $(3 + i)/2$, $3i$