EXERCISE 15.3 PAGE NO: 15.22

Solve each of the following system of equations in R.

1. $|x + 1/3| > 8/3$

Solution:

Let 'r' be a positive real number and 'a' be a fixed real number. Then,

$|x + a| > r \Leftrightarrow x > r - a$ or $x < -(a + r)$

Here, $a = 1/3$ and $r = 8/3$

$x > 8/3 - 1/3$ or $x < -(8/3 + 1/3)$

$x > (8-1)/3$ or $x < -(8+1)/3$

$x > 7/3$ or $x < -9/3$

$x > 7/3$ or $x < -3$

$x \in (7/3, \infty)$ or $x \in (-\infty, -3)$

$\therefore x \in (-\infty, -3) \cup (7/3, \infty)$

2. $|4 - x| + 1 < 3$

Solution:

$|4 - x| + 1 < 3$

Let us subtract 1 from both the sides, we get

$|4 - x| + 1 - 1 < 3 - 1$

$|4 - x| < 2$

Let 'r' be a positive real number and 'a' be a fixed real number. Then,

$|a - x| < r \Leftrightarrow a - r < x < a + r$

Here, a=4 and r=2

$4 - 2 < x < 4 + 2$

$2 < x < 6$

$\therefore x \in (2, 6)$

3. $|(3x - 4)/2| \leq 5/12$

Solution:

Given:

$|(3x - 4)/2| \leq 5/12$

We can rewrite it as

$|3x/2 - 4/2| \leq 5/12$

$|3x/2 - 2| \leq 5/12$

Let 'r' be a positive real number and 'a' be a fixed real number. Then,

$|x - a| \leq r \Leftrightarrow a - r \leq x \leq a + r$

Here, $a = 2$ and $r = 5/12$

$2 - 5/12 \leq 3x/2 \leq 2 + 5/12$
$(24-5)/12 \leq 3x/2 \leq (24+5)/12$
$19/12 \leq 3x/2 \leq 29/12$
Now, multiplying the whole inequality by 2 and dividing by 3, we get
$19/18 \leq x \leq 29/18$
$\therefore x \in [19/18, 29/18]$

4. $|x - 2| / (x - 2) > 0$
Solution:
Given:
$|x - 2| / (x - 2) > 0$
Clearly it states, $x \neq 2$ so two case arise:

Case1: $x – 2 > 0$
$x > 2$
In this case $|x–2| = x – 2$
$x \in (2, \infty)$….(1)

Case 2: $x–2<0$
$x<2$
In this case, $|x–2| = – (x–2)$
$- (x - 2) / (x - 2) > 0$
$- 1 > 0$
Inequality doesn't get satisfy
This case gets nullified.
$\therefore x \in (2, \infty)$ from (1)

5. $1 / (|x| - 3) < ½$
Solution:
We know that, if we take reciprocal of any inequality we need to change the inequality as well.
Also, $|x| – 3 \neq 0$
$|x| > 3$ or $|x| < 3$
For $|x| < 3$
$–3 < x < 3$
$x \in (–3, 3)$ …. (1)
The equation can be re–written as
$|x| - 3 > 2$
Let us add 3 on both the sides, we get

$|x| - 3 + 3 > 2 + 3$
$|x| > 5$
Let 'a' be a fixed real number. Then,
$|x| > a \Leftrightarrow x < -a$ or $x > a$
Here, $a = 5$
$x < -5$ or $x > 5$ …. (2)
From (1) and (2)
$x \in (-\infty, -5)$ or $x \in (5, \infty)$
$\therefore x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)$

6. $(|x + 2| - x) / x < 2$
Solution:
Given:
$(|x + 2| - x) / x < 2$
Let us rewrite the equation as
$|x + 2|/x - x/x < 2$
$|x + 2|/x - 1 < 2$
By adding 1 on both sides, we get
$|x + 2|/x - 1 + 1 < 2 + 1$
$|x + 2|/x < 3$
By subtracting 3 on both sides, we get
$|x + 2|/x - 3 < 3 - 3$
$|x + 2|/x - 3 < 0$
Clearly it states, $x \neq 2$ so two case arise:

Case1: $x + 2 > 0$
$x > -2$
In this case $|x+2| = x + 2$
$x + 2/x - 3 < 0$
$(x + 2 - 3x)/x < 0$
$-(2x - 2)/x < 0$
$(2x - 2)/x < 0$
Let us consider only the numerators, we get
$2x - 2 > 0$
$x>1$
$x \in (1, \infty)$ ….(1)

Case 2: $x + 2 < 0$
$x < -2$

In this case, $|x+2| = -(x + 2)$
$-(x+2)/x - 3 < 0$
$(-x - 2 - 3x)/x < 0$
$-(4x + 2)/x < 0$
$(4x + 2)/x < 0$
Let us consider only the numerators, we get
$4x + 2 > 0$
$x > -\frac{1}{2}$
But $x < -2$
From the denominator we have,
$x \in (-\infty, 0)$ …(2)
From (1) and (2)
$\therefore x \in (-\infty, 0) \cup (1, \infty)$