

EXERCISE 15.4

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1. Find all pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11.

Solution:

Let 'x' be the smaller of the two consecutive odd positive integers. Then the other odd integer is $x + 2$.

Given:

Both the integers are smaller than 10 and their sum is more than 11.

So,

$$x + 2 < 10 \text{ and } x + (x + 2) > 11$$

$$x < 10 - 2 \text{ and } 2x + 2 > 11$$

$$x < 8 \text{ and } 2x > 11 - 2$$

$$x < 8 \text{ and } 2x > 9$$

$$x < 8 \text{ and } x > 9/2$$

$$9/2 < x < 8$$

Note the odd positive integers lying between 4.5 and 8.

$$x = 5, 7 \text{ [Since, } x \text{ is an odd integer]}$$

$$x < 10 \text{ [From the given statement]}$$

$$9/2 < x < 10$$

Note that, the upper limit here has shifted from 8 to 10. Now, x is odd integer from 4.5 to 10.

So, the odd integers from 4.5 to 10 are 5, 7 and 9.

Now, let us find pairs of consecutive odd integers.

$$\text{Let } x = 5, \text{ then } (x + 2) = (5 + 2) = 7.$$

$$\text{Let } x = 7, \text{ then } (x + 2) = (7 + 2) = 9.$$

$$\text{Let } x = 9, \text{ then } (x + 2) = (9 + 2) = 11. \text{ But, 11 is greater than 10.}$$

\therefore The required pairs of odd integers are (5, 7) and (7, 9)

2. Find all pairs of consecutive odd natural number, both of which are larger than 10, such that their sum is less than 40.

Solution:

Let 'x' be the smaller of the two consecutive odd natural numbers. Then the other odd number is $x + 2$.

Given:

Both the natural numbers are greater than 10 and their sum is less than 40.

So,

$$x > 10 \text{ and } x + x + 2 < 40$$

$$x > 10 \text{ and } 2x < 38$$

$$x > 10 \text{ and } x < 38/2$$

$$x > 10 \text{ and } x < 19$$

$$10 < x < 19$$

From this inequality, we can say that x lies between 10 and 19.

So, the odd natural numbers lying between 10 and 19 are 11, 13, 15 and 17. (Excluding 19 as $x < 19$)

Now, let us find pairs of consecutive odd natural numbers.

$$\text{Let } x = 11, \text{ then } (x + 2) = (11 + 2) = 13$$

$$\text{Let } x = 13, \text{ then } (x + 2) = (13 + 2) = 15$$

$$\text{Let } x = 15, \text{ then } (x + 2) = (15 + 2) = 17$$

$$\text{Let } x = 17, \text{ then } (x + 2) = (17 + 2) = 19.$$

$$x = 11, 13, 15, 17 \text{ [Since, } x \text{ is an odd number]}$$

\therefore The required pairs of odd natural numbers are (11, 13), (13, 15), (15, 17) and (17, 19)

3. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.

Solution:

Let ' x ' be the smaller of the two consecutive even positive integers. Then the other even integer is $x + 2$.

Given:

Both the even integers are greater than 5 and their sum is less than 23.

So,

$$x > 5 \text{ and } x + x + 2 < 23$$

$$x > 5 \text{ and } 2x < 21$$

$$x > 5 \text{ and } x < 21/2$$

$$5 < x < 21/2$$

$$5 < x < 10.5$$

From this inequality, we can say that x lies between 5 and 10.5.

So, the even positive integers lying between 5 and 10.5 are 6, 8 and 10.

Now, let us find pairs of consecutive even positive integers.

$$\text{Let } x = 6, \text{ then } (x + 2) = (6 + 2) = 8$$

$$\text{Let } x = 8, \text{ then } (x + 2) = (8 + 2) = 10$$

$$\text{Let } x = 10, \text{ then } (x + 2) = (10 + 2) = 12.$$

$$x = 6, 8, 10 \text{ [Since, } x \text{ is even integer]}$$

\therefore The required pairs of even positive integer are (6, 8), (8, 10) and (10, 12)

4. The marks scored by Rohit in two tests were 65 and 70. Find the minimum marks he should score in the third test to have an average of at least 65 marks.

Solution:

Given:

Marks scored by Rohit in two tests are 65 and 70.

Let marks in the third test be x .

So let us find minimum x for which the average of all three papers would be at least 65 marks.

That is,

Average marks in three papers $\geq 65 \dots(i)$

Average is given by:

$$\begin{aligned}\text{Average} &= (\text{sum of all numbers})/(\text{Total number of items}) \\ &= (\text{marks in 1}^{\text{st}} \text{ two papers} + \text{marks in third test})/3 \\ &= (65 + 70 + x)/3 \\ &= (135 + x)/3\end{aligned}$$

Substituting this value of average in the inequality (i), we get

$$(135 + x)/3 \geq 65$$

$$(135 + x) \geq 65 \times 3$$

$$(135 + x) \geq 195$$

$$x \geq 195 - 135$$

$$x \geq 60$$

This inequality means that Rohit should score at least 60 marks in his third test to have an average of at least 65 marks.

So, the minimum marks to get an average of 65 marks is 60.

\therefore The minimum marks required in the third test is 60.

5. A solution is to be kept between 86° and 95°F . What is the range of temperature in degree Celsius, if the Celsius (C)/Fahrenheit (F) conversion formula is given by $F = 9/5C + 32$.

Solution:

Let us consider $F_1 = 86^\circ\text{F}$

And $F_2 = 95^\circ$

We know, $F = 9/5C + 32$

$$F_1 = 9/5 C_1 + 32$$

$$F_1 - 32 = 9/5 C_1$$

$$C_1 = 5/9 (F_1 - 32)$$

$$= 5/9 (86 - 32)$$

$$= 5/9 (54)$$

$$= 5 \times 6$$

$$= 30^\circ\text{C}$$

Now,

$$F_2 = 9/5 C_2 + 32$$

$$\begin{aligned}F_2 - 32 &= 9/5 C_2 \\C_2 &= 5/9 (F_2 - 32) \\&= 5/9 (95 - 32) \\&= 5/9 (63) \\&= 5 \times 7 \\&= 35^\circ \text{C}\end{aligned}$$

\therefore The range of temperature of the solution in degree Celsius is 30°C and 35°C .

6. A solution is to be kept between 30°C and 35°C . What is the range of temperature in degree Fahrenheit?

Solution:

Let us consider $C_1 = 30^\circ \text{C}$

And $C_2 = 35^\circ$

We know, $F = 9/5C + 32$

$$\begin{aligned}F_1 &= 9/5 C_1 + 32 \\&= 9/5 \times 30 + 32 \\&= 9 \times 6 + 32 \\&= 54 + 32 \\&= 86^\circ \text{F}\end{aligned}$$

Now,

$$\begin{aligned}F_2 &= 9/5 C_2 + 32 \\&= 9/5 \times 35 + 32 \\&= 9 \times 7 + 32 \\&= 63 + 32 \\&= 95^\circ \text{F}\end{aligned}$$

\therefore The range of temperature of the solution in degree Fahrenheit is 86°F and 95°F .