EXERCISE 16.2 **PAGE NO: 16.14**

1. In a class, there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class in a function. In how many ways can the teacher make this selection?
Solution:
Given:
27 boys and 14 girls.
Here the teacher has to
(i) Select a boy among 27 boys, and
(ii) Select a girl among 14 girls.
Number of ways to select one boy is $^{27}C_1$ and similarly the number of ways to select one girl is $^{14}C_1$.
Hence, the number of ways to select 1 boy and 1 girl to represent the class in a function is $^{14}C_1 \times {}^{27}C_1 = 14 \times 27 = 378$ ways.

2. A person wants to buy one fountain pen, one ball pen, and one pencil from a stationery shop. If there are 10 fountain pen varieties, 12 ball pen varieties and 5 pencil varieties, in how many ways can he select these articles?
Solution:
Given:
10 fountain pens, 12 ball pens, and 5 pencil
Here the person has to
(i) Select a ball pen from 12 ball pens.
(ii) Select a fountain pen from 10 fountain pens, and
(iii) Select a pencil from 5 pencils.
The number of ways to select one fountain pen is $^{10}C_1$ and similarly the number of ways to select one ball pen is $^{12}C_1$ and number of ways to select one pencil from 5 pencils is $^5C_1$
Hence, the number of ways to select one fountain pen, one ball pen and one pencil from a stationery shop is $^{10}C_1 \times {}^{12}C_1 \times {}^5C_{1 =} 10 \times 12 \times 5 = 600$ ways.

3. From Goa to Bombay there are two roots; air, and sea. From Bombay to Delhi there are three routes; air, rail, and road. From Goa to Delhi via Bombay, how many kinds of routes are there?
Solution:
Given:
The number of roots from Goa to Bombay is air and sea.
So, the number of ways to go from Goa to Bombay is $^2C_1$

Given: The number of roots from Bombay to Delhi are: air, rail, and road.

So, the number of ways to go from Bombay to Delhi is $^3C_1$

Hence, the number of ways to go from Goa to Delhi via Bombay is $^2C_1 \times {}^3C_1 = 2 \times 3 = 6$ ways.

4. A mint prepares metallic calendars specifying months, dates and days in the form of monthly sheets (one plate for each month). How many types of calendars should it prepare to serve for all the possibilities in future years?

Solution:

The mint has to perform

(i) Select the number of days in the month of February (there can be 28 or 29 days), and

(ii) Select the first day of February.

Now,

In 2 ways mint can select the number of days in February and for selecting first day of February, it can start from any of one of the seven days of the week, so there are 7 possibilities.

Hence, the number of types calendars should it prepare to serve for all the possibilities in future years is $7 \times 2 = 14$.

5. There are four parcels and five post-offices. In how many different ways can the parcels be sent by registered post?

Solution:

Given:

Total number of parcels = 4

Total number of post-offices = 5

One parcel can be posted in 5 ways that is in either of the one post offices. So, $^5C_1$.

Similarly, for other parcels also it can be posted in $^5C_1$ ways.

Hence the number of ways the parcels be sent by registered post is

$^5C_1 \times {}^5C_1 \times {}^5C_1 \times {}^5C_1 = 5 \times 5 \times 5 \times 5 = 625$ ways.

6. A coin is tossed five times, and outcomes are recorded. How many possible outcomes are there?

Solution:

Given:

A coin is tossed 5 times, so each time the outcome is either heads or tails, so two possibilities are possible.

The total possible outcomes are:

$^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ outcomes.

7. In how many ways can an examinee answer a set of ten true/false type questions?
Solution:
Given:
An examinee can answer a question either true or false, so there are two possibilities.
The number of ways for an examinee to answer a set of ten true/false type questions are:
$^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = 1024$ ways

8. A letter lock consists of three rings each marked with 10 different letters. In how many ways it is possible to make an unsuccessful attempt to open the lock?
Solution:
The total number of ways to make an attempt to open the lock is $= 10 \times 10 \times 10 = 1000$
The number of successful attempts to open the lock $= 1$
The number of unsuccessful attempts to open the lock $= 1000 - 1 = 999$
Hence, required number of possible ways to make an unsuccessful attempt to open the lock is 999.

9. There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 2 each?
Solution:
Given: Multiple choice question, only one answer is correct of the given options.
For the first three questions only one answer is correct out of four. So it can be answered in 4 ways.
Total number of ways to answer the first 3 questions $= {}^4C_1 \times {}^4C_1 \times {}^4C_1 = 4 \times 4 \times 4 = 64$

Each of the next 3 questions can be answered in 2 ways.
Total number of ways to answer the next 3 questions $= {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 2 \times 2 \times 2 = 8$
Hence, total possible outcomes possible are $64 \times 8 = 512$

10. There are 5 books on Mathematics and 6 books on Physics in a book shop. In how many ways can a student buy:
(i) a Mathematics book and a Physics book
(ii) either a Mathematics book or a Physics book?
Solution:
(i) Given: there are 5 books of mathematics and 6 books of physics.
In order to buy one mathematics book, number of ways is $^5C_1$ similarly to buy one physics book number of ways is $^6C_1$

Hence, the number of ways a student buy a Mathematics book and a Physics book
is $^5C_1 \times {}^6C_1 = 5 \times 6 = 30$

(ii) Given: there is a total of 11 books.
So in order to buy either a Mathematics book or a Physics book it means that only one
book out of eleven is bought.
Hence, the number of ways in which a student can either buy either a Mathematics book
or a Physics book is $^{11}C_1 = 11$

**11. Given 7 flags of different colours, how many different signals can be generated if
a signal requires the use of two flags, one below the other?**
Solution:
Given: seven flags are available and out of which two are needed to make a signal.
From this, we can say, that we have to select two flags out of seven and arrange these two
flags to get one signal.
Seven flags of different colours are available, so first flag can be selected in 7 ways.
Now, the second flag can be selected from any one of the remaining flags in 6 ways.
Hence, the required number of signals are $7 \times 6 = 42$.

**12. A team consists of 6 boys and 4 girls, and other has 5 boys and 3 girls. How
many single matches can be arranged between the two teams when a boy plays
against a boy, and a girl plays against a girl?**
Solution:
Given:
A team consists of 6 boys and 4 girls, and other team has 5 boys and 3 girls.
Let team 1 be = 6 boys and 4 girls
Team 2 be = 5 boys and 3 girls
Singles matches are to be played, either a boy plays against a boy, and a girl plays against
a girl.
So, number of ways to select a boy from team 1 is $^6C_1$. Similarly, Number of ways to
select a boy from team 2 is $^5C_1$
Hence number of singles matches between boys is $^6C_1 \times {}^5C_1 = 6 \times 5 = 30$

A number of ways to select a girl from team 1 is $^4C_1$. Similarly, Number of ways to select
a girl from team 2 is $^3C_1$
Hence number of singles matches between girls is $^3C_1 \times {}^4C_1 = 4 \times 3 = 12$
∴ The total number of single matches are = 30 + 12 = 42

13. Twelve students compete in a race. In how many ways first three prizes be

given?

Solution:

Given:

Twelve students compete in a race.

Number of ways to select the winner of the first prize is $^{12}C_1$

Number of ways to select the winner of the second prize is $^{11}C_1$ (11, since one student is already given a prize)

Number of ways to select the winner of the third prize is $^{10}C_1$ (10, since two students are already given a prize)

Hence, total number of ways is $^{12}C_1 \times {}^{11}C_1 \times {}^{10}C_1 = 12 \times 11 \times 10 = 1320$.

14. How many A.P.'s with 10 terms are there whose first term is in the set {1, 2, 3} and whose common difference is in the set {1, 2, 3, 4, 5}?

Solution:

We know that, each AP consists of a unique first term and a common difference.

So, number of ways to select the first term of a given set is $^3C_1 = 3$

And, number of ways to select a common difference of given set is $^5C_1 = 5$

Hence, total number of AP's possible are $^3C_1 \times {}^5C_1 = 3 \times 5 = 15$

15. From among the 36 teachers in a college, one principal, one vice-principal and the teacher-in-charge are to be appointed. In how many ways can this be done?

Solution:

Number of ways to appoint one principal, vice-principal and the teacher incharge is equal to the number of ways to select the three teachers from 36 members.

So, a total of three positions are to be appointed.

Number of ways to select principal is $^{36}C_1 = 36$

Number of ways to select vice-principal is $^{35}C_1 = 35$ (35, since one position is already given)

Number of ways to select teacher in charge is $^{34}C_1$ (34, since two positions are already given)

Hence, number of ways to appoint three teachers is $^{36}C_1 \times {}^{35}C_1 \times {}^{34}C_1 = 36 \times 35 \times 34 = 42840$.

16. How many three-digit numbers are there with no digit repeated?

Solution:

Let us assume we have three boxes.

The first box can be filled with any one of the nine digits (0 not allowed at first place).

So, the possibilities are $^9C_1$

The second box can be filled with any one of the nine digits

So the available possibilities are $^9C_1$
The third box can be filled with any one of the eight digits
So the available possibilities are $^8C_1$
Hence, the total number of possible outcomes are $^9C_1 \times ^9C_1 \times ^8C_1 = 9 \times 9 \times 8 = 648$.

17. How many three-digit numbers are there?
Solution:
Let us assume we have three boxes.
The first box can be filled with any one of the nine digits (zero not allowed at first position)
So, possibilities are $^9C_1$
The second box can be filled with any one of the ten digits
So the available possibilities are $^{10}C_1$
Third box can be filled with any one of the ten digits
So the available possibilities are $^{10}C_1$
Hence, the total number of possible outcomes are $^9C_1 \times ^{10}C_1 \times ^{10}C_1 = 9 \times 10 \times 10 = 900$.

18. How many three-digit odd numbers are there?
Solution:
We know that in odd numbers, the last digit consists of (1, 3, 5, 7, 9).
Let us assume we have three boxes.
The first box can be filled with any one of the nine digits (zero not allowed at first position)
So the possibilities are $^9C_1$
The second box can be filled with any one of the ten digits
So the available possibilities are $^{10}C_1$
The third box can be filled with any one of the five digits (1,3,5,7,9)
So the available possibilities are $^5C_1$
Hence, the total number of possible outcomes are $^9C_1 \times ^{10}C_1 \times ^5C_1 = 9 \times 10 \times 5 = 450$

19. How many different five-digit number license plates can be made if
(i) the first digit cannot be zero, and the repetition of digits is not allowed,
(ii) the first-digit cannot be zero, but the repetition of digits is allowed?
Solution:
(i) We know that zero cannot be the first digit of the license plates. And the repetition of digits is not allowed.
Let us assume five boxes, now the first box can be filled with one of the nine available digits, so the possibility is $^9C_1$
Similarly, the second box can be filled with one of the nine available digits, so the

possibility is $^9C_1$

The third box can be filled with one of the eight available digits, so the possibility is $^8C_1$
The fourth box can be filled with one of the seven available digits, so the possibility is $^7C_1$
The fifth box can be filled with one of the six available digits, so the possibility is $^6C_1$
Hence, the total number of possible outcomes is $^9C_1 \times {}^9C_1 \times {}^8C_1 \times {}^7C_1 \times {}^6C_1 = 9 \times 9 \times 8 \times 7 \times 6 = 27,216$

(ii) We know that zero cannot be the first digit of the license plates. And the repetition of digits is allowed.
Let us assume five boxes, now the first box can be filled with one of the nine available digits, so the possibility is $^9C_1$
Similarly, the second box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$
The third box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$
The fourth box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$
The fifth box can be filled with one of the ten available digits, so the possibility is $^{10}C_1$
Hence, the total number of possible outcomes is $^9C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 9 \times 10 \times 10 \times 10 \times 10 = 90,000$

20. How many four-digit numbers can be formed with the digits 3, 5, 7, 8, 9 which are greater than 7000, if repetition of digits is not allowed?
Solution:
The required numbers are greater than 7000.
So, the thousand's place can be filled with any of the 3 digits: 7, 8, 9.
Let us assume four boxes, now in the first box can either be one of the three numbers 7, 8 or 9, so there are three possibilities which are $^3C_1$
In the second box, the numbers can be any of the four digits left, so the possibility is $^4C_1$
In the third box, the numbers can be any of the three digits left, so the possibility is $^3C_1$
In the fourth box, the numbers can be any of the two digits left, so the possibility is $^2C_1$
Hence, the total number of possible outcomes is $^3C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 = 3 \times 4 \times 3 \times 2 = 72$.

21. How many four-digit numbers can be formed with the digits 3, 5, 7, 8, 9 which are greater than 8000, if repetition of digits is not allowed?
Solution:
The required numbers are greater than 8000.
So, the thousand's place can be filled with 2 digits: 8 or 9.
Let us assume four boxes, now in the first box can either be one of the two numbers 8 or

9, so there are two possibilities which is $^2C_1$

In the second box, the numbers can be any of the four digits left, so the possibility is $^4C_1$

In the third box, the numbers can be any of the three digits left, so the possibility is $^3C_1$

In the fourth box, the numbers can be any of the two digits left, so the possibility is $^2C_1$

Hence total number of possible outcomes is $^2C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 = 2 \times 4 \times 3 \times 2 = 48$.

22. In how many ways can six persons be seated in a row?
Solution:

Let us assume there are six seats.

In the first seat, any one of six members can be seated, so the total number of possibilities is $^6C_1$

In the second seat, any one of five members can be seated, so the total number of possibilities is $^5C_1$ ways.

In the third seat, any one of four members can be seated, so the total number of possibilities is $^4C_1$ ways.

In the fourth seat, any one of three members can be seated, so the total number of possibilities is $^3C_1$ ways.

In the fifth seat, any one of two members can be seated, so the total number of possibilities is $^2C_1$ ways.

In the sixth seat, only one remaining person can be seated, so the total number of possibilities is $^1C_1$ ways.

Hence, the total number of possible outcomes $= {}^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

23. How many 9-digit numbers of different digits can be formed?
Solution:

In a nine-digit number, 0 cannot appear in the first digit and the repetition of digits is not allowed. So, the number of ways of filling up the first digit is $^9C_1 = 9$

Now, 9 digits are left including 0. So, the second digit can be filled with any of the remaining 9 digits in 9 ways.

Similarly, the third box can be filled with one of the eight available digits, so the possibility is $^8C_1$

The fourth digit can be filled with one of the seven available digits, so the possibility is $^7C_1$

The fifth digit can be filled with one of the six available digits, so the possibility is $^6C_1$

The sixth digit can be filled with one of the six available digits, so the possibility is $^5C_1$

The seventh digit can be filled with one of the six available digits, so the possibility is $^4C_1$

The eighth digit can be filled with one of the six available digits, so the possibility is $^3C_1$

The ninth digit can be filled with one of the six available digits, so the possibility is $^2C_1$

Hence the number of total possible outcomes is $^9C_1 \times {}^9C_1 \times {}^8C_1 \times {}^7C_1 \times {}^6C_1 = 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 9\ (9!)$

24. How many odd numbers less than 1000 can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed?
Solution:
Any number less than 1000 may be any of a number from one-digit number, two-digit number and three-digit number.
Case 1: One-digit odd number
In order to make the number odd, the last digit has to be either of (3, 5, 7)
In the first box either of the three digits (3,5,7) can be placed, so the possibility is $^3C_1 = 3$ possible ways.

Case 2: two-digit odd number
Tens place can be filled up by 3 ways (using any of the digit among 3, 5 and 7) and then the ones place can be filled in any of the remaining 2 digits.
So, there are $3 \times 2 = 6$ such 2-digit numbers

Case 3: three-digit odd number
Ignore zero at one's place for some instance.
Hundreds place can be filled up in 3 ways (using any of any of the digit among 3, 5 and 7), then tens place in 3 ways by using remaining 3 digits (after using a digit, there will be three digits) and then the ones place in 2 ways.

So, there are a total of $3 \times 3 \times 2 = 18$ numbers of 3-digit numbers which includes both odd and even numbers (ones place digit are zero). In order to get the odd numbers, it is required to ignore the even numbers i.e. numbers ending with zero.

To obtain the even 3-digit numbers, one's place can be filled up in 1 way (only 0 to be filled), hundreds place in 3 ways (using any of the digit among 3, 5, 7) and then tens place in 2 ways (using remaining 2 digits after filling up hundreds place).

So, there are a total of $1 \times 3 \times 2 = 6$ even 3-digit numbers using the digits 0, 3, 5 and 7 (repetition not allowed)
Then, number of three-digit odd numbers using the digits 0, 3, 5 and 7 (repetition not allowed) = 18 - 6 = 12.

∴ The odd numbers less than 1000 that can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed are 3 + 6 + 12 = 21.