

EXERCISE 17.1

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1. Evaluate the following:

(i) ${}^{14}C_3$

(ii) ${}^{12}C_{10}$

(iii) ${}^{35}C_{35}$

(iv) ${}^{n+1}C_n$

(v) $\sum_{r=1}^5 {}^5C_r$

Solution:

(i) ${}^{14}C_3$

Let us use the formula,

$${}^nC_r = n! / r!(n - r)!$$

So now, value of $n = 14$ and $r = 3$

$${}^nC_r = n! / r!(n - r)!$$

$$\begin{aligned} {}^{14}C_3 &= 14! / 3!(14 - 3)! \\ &= 14! / (3! 11!) \\ &= [14 \times 13 \times 12 \times 11!] / (3! 11!) \\ &= [14 \times 13 \times 12] / (3 \times 2) \\ &= 14 \times 13 \times 2 \\ &= 364 \end{aligned}$$

(ii) ${}^{12}C_{10}$

Let us use the formula,

$${}^nC_r = n! / r!(n - r)!$$

So now, value of $n = 12$ and $r = 10$

$${}^nC_r = n! / r!(n - r)!$$

$$\begin{aligned} {}^{12}C_{10} &= 12! / 10!(12 - 10)! \\ &= 12! / (10! 2!) \\ &= [12 \times 11 \times 10!] / (10! 2!) \\ &= [12 \times 11] / (2) \\ &= 6 \times 11 \\ &= 66 \end{aligned}$$

(iii) ${}^{35}C_{35}$

Let us use the formula,

$${}^nC_r = n! / r!(n - r)!$$

So now, value of $n = 35$ and $r = 35$

$${}^n C_r = n! / r!(n - r)!$$

$$\begin{aligned} {}^{35} C_{35} &= 35! / 35!(35 - 35)! \\ &= 35! / (35! 0!) \text{ [Since, } 0! = 1 \text{]} \\ &= 1 \end{aligned}$$

(iv) ${}^{n+1} C_n$

Let us use the formula,

$${}^n C_r = n! / r!(n - r)!$$

So now, value of $n = n+1$ and $r = n$

$${}^n C_r = n! / r!(n - r)!$$

$$\begin{aligned} {}^{n+1} C_n &= (n+1)! / n!(n+1 - n)! \\ &= (n+1)! / n!(1!) \\ &= (n + 1) / 1 \\ &= n + 1 \end{aligned}$$

$$\sum_{r=1}^5 {}^5 C_r$$

(v)

Let us use the formula,

$${}^n C_r = n! / r!(n - r)!$$

$$\begin{aligned} \sum_{r=1}^5 {}^5 C_r &= {}^5 C_1 + {}^5 C_2 + {}^5 C_3 + {}^5 C_4 + {}^5 C_5 \\ &= \frac{5!}{(5-1)!1!} + \frac{5!}{(5-2)!2!} + \frac{5!}{(5-3)!3!} + \frac{5!}{(5-4)!4!} + \frac{5!}{(5-5)!5!} \\ &= \frac{5!}{4!1!} + \frac{5!}{3!2!} + \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!} \\ &= \frac{5}{1} + \frac{5 \times 4}{2 \times 1} + \frac{5 \times 4}{2 \times 1} + \frac{5}{1} + \frac{1}{1} \\ &= 5 + 10 + 10 + 5 + 1 \\ &= 31 \end{aligned}$$

2. If ${}^n C_{12} = {}^n C_5$, find the value of n .

Solution:

We know that if ${}^n C_p = {}^n C_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^n C_{12} = {}^n C_5$, we can say that

$$12 \neq 5$$

So, the condition (ii) must be satisfied,

$$n = 12 + 5$$

$$n = 17$$

∴ The value of n is 17.

3. If ${}^n C_4 = {}^n C_6$, find ${}^{12} C_n$.

Solution:

We know that if ${}^n C_p = {}^n C_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^n C_4 = {}^n C_6$, we can say that

$$4 \neq 6$$

So, the condition (ii) must be satisfied,

$$n = 4 + 6$$

$$n = 10$$

Now, we need to find ${}^{12} C_n$,

We know the value of n so, ${}^{12} C_n = {}^{12} C_{10}$

Let us use the formula,

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

So now, value of $n = 12$ and $r = 10$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

$${}^{12} C_{10} = \frac{12!}{10!(12-10)!}$$

$$= \frac{12!}{(10! 2!)}$$

$$= \frac{[12 \times 11 \times 10!]}{(10! 2!)}$$

$$= \frac{[12 \times 11]}{(2)}$$

$$= 6 \times 11$$

$$= 66$$

∴ The value of ${}^{12} C_{10} = 66$.

4. If ${}^n C_{10} = {}^n C_{12}$, find ${}^{23} C_n$.

Solution:

We know that if ${}^n C_p = {}^n C_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^n C_{10} = {}^n C_{12}$, we can say that

$$10 \neq 12$$

So, the condition (ii) must be satisfied,

$$n = 10 + 12$$

$$n = 22$$

Now, we need to find ${}^{23} C_n$,

We know the value of n so, ${}^{23} C_n = {}^{23} C_{22}$

Let us use the formula,

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

So now, value of $n = 23$ and $r = 22$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

$$\begin{aligned} {}^{23} C_{22} &= \frac{23!}{22!(23-22)!} \\ &= \frac{23!}{(22! 1!)} \\ &= \frac{[23 \times 22!]}{(22!)} \\ &= 23 \end{aligned}$$

\therefore The value of ${}^{23} C_{22} = 23$.

5. If ${}^{24} C_x = {}^{24} C_{2x+3}$, find x .

Solution:

We know that if ${}^n C_p = {}^n C_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^{24} C_x = {}^{24} C_{2x+3}$, we can say that

Let us check for condition (i)

$$x = 2x + 3$$

$$2x - x = -3$$

$$x = -3$$

We know that for a combination ${}^n C_r$, $r \geq 0$, r should be a positive integer which is not satisfied here,

So, the condition (ii) must be satisfied,

$$24 = x + 2x + 3$$

$$3x = 21$$

$$x = \frac{21}{3}$$

$$x = 7$$

\therefore The value of x is 7.

6. If ${}^{18} C_x = {}^{18} C_{x+2}$, find x .

Solution:

We know that if ${}^n C_p = {}^n C_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^{18} C_x = {}^{18} C_{x+2}$, we can say that

$$x \neq x + 2$$

So, the condition (ii) must be satisfied,

$$18 = x + x + 2$$

$$18 = 2x + 2$$

$$2x = 18 - 2$$

$$2x = 16$$

$$x = 16/2$$

$$= 8$$

∴ The value of x is 8.

7. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

Solution:

We know that if ${}^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^{15}C_{3r} = {}^{15}C_{r+3}$, we can say that

Let us check for condition (i)

$$3r = r + 3$$

$$3r - r = 3$$

$$2r = 3$$

$$r = 3/2$$

We know that for a combination nC_r , $r \geq 0$, r should be a positive integer which is not satisfied here,

So, the condition (ii) must be satisfied,

$$15 = 3r + r + 3$$

$$15 - 3 = 4r$$

$$4r = 12$$

$$r = 12/4$$

$$= 3$$

∴ The value of r is 3.

8. If ${}^8C_r - {}^7C_3 = {}^7C_2$, find r.

Solution:

To find r, let us consider the given expression,

$${}^8C_r - {}^7C_3 = {}^7C_2$$

$${}^8C_r = {}^7C_2 + {}^7C_3$$

We know that ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

$${}^8C_r = {}^{7+1}C_{2+1}$$

$${}^8C_r = {}^8C_3$$

Now, we know that if ${}^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:

(i) $p = q$

(ii) $n = p + q$

So from the question ${}^8C_r = {}^8C_3$, we can say that

Let us check for condition (i)

$$r = 3$$

Let us also check for condition (ii)

$$8 = 3 + r$$

$$r = 5$$

∴ The values of 'r' are 3 and 5.

9. If ${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$, find r.

Solution:

Given:

$${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$$

$${}^{15}C_r / {}^{15}C_{r-1} = 11 / 5$$

Let us use the formula,

$${}^nC_r = n! / r!(n - r)!$$

$$\frac{\frac{15!}{(15-r)!r!}}{\frac{15!}{(15-(r-1))!(r-1)!}} = \frac{11}{5}$$

$$\frac{(16-r)!}{(15-r)!r} = \frac{11}{5}$$

$$\frac{16-r}{r} = \frac{11}{5}$$

$$5(16 - r) = 11r$$

$$80 - 5r = 11r$$

$$80 = 11r + 5r$$

$$16r = 80$$

$$r = 80/16$$

$$= 5$$

∴ The value of r is 5.

10. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, find n.

Solution:

Given:

$${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$$

$${}^{n+2}C_8 / {}^{n-2}P_4 = 57 / 16$$

Let us use the formula,

$${}^nC_r = n! / r!(n - r)!$$

$$\frac{\frac{(n+2)!}{(n+2-8)!8!}}{\frac{(n-2)!}{(n-2-4)!}} = \frac{57}{16}$$

$$[(n+2)! (n-6)!] / [(n-6)! (n-2)! 8!] = 57/16$$

$$(n+2)(n+1)(n)(n-1) / 8! = 57/16$$

$$(n+2)(n+1)(n)(n-1) = (57 \times 8!) / 16$$

$$(n+2)(n+1)(n)(n-1) = [19 \times 3 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1] / 16$$

$$(n+2)(n+1)(n)(n-1) = 21 \times 20 \times 19 \times 18$$

Equating the corresponding terms on both sides we get,

$$n = 19$$

∴ The value of n is 19.

