

EXERCISE 18.2
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1. Find the 11th term from the beginning and the 11th term from the end in the expansion of $(2x - 1/x^2)^{25}$.

Solution:

Given:

$$(2x - 1/x^2)^{25}$$

The given expression contains 26 terms.

So, the 11th term from the end is the $(26 - 11 + 1)$ th term from the beginning.

In other words, the 11th term from the end is the 16th term from the beginning.

Then,

$$\begin{aligned} T_{16} = T_{15+1} &= {}^{25}C_{15} (2x)^{25-15} (-1/x^2)^{15} \\ &= {}^{25}C_{15} (2^{10}) (x)^{10} (-1/x^{30}) \\ &= - {}^{25}C_{15} (2^{10} / x^{20}) \end{aligned}$$

Now we shall find the 11th term from the beginning.

$$\begin{aligned} T_{11} = T_{10+1} &= {}^{25}C_{10} (2x)^{25-10} (-1/x^2)^{10} \\ &= {}^{25}C_{10} (2^{15}) (x)^{15} (1/x^{20}) \\ &= {}^{25}C_{10} (2^{15} / x^5) \end{aligned}$$

2. Find the 7th term in the expansion of $(3x^2 - 1/x^3)^{10}$.

Solution:

Given:

$$(3x^2 - 1/x^3)^{10}$$

Let us consider the 7th term as T_7

So,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^{10}C_6 (3x^2)^{10-6} (-1/x^3)^6 \\ &= {}^{10}C_6 (3)^4 (x)^8 (1/x^{18}) \\ &= [10 \times 9 \times 8 \times 7 \times 81] / [4 \times 3 \times 2 \times x^{10}] \\ &= 17010 / x^{10} \end{aligned}$$

∴ The 7th term of the expression $(3x^2 - 1/x^3)^{10}$ is $17010 / x^{10}$.

3. Find the 5th term in the expansion of $(3x - 1/x^2)^{10}$.

Solution:

Given:

$$(3x - 1/x^2)^{10}$$

The 5th term from the end is the $(11 - 5 + 1)$ th, is., 7th term from the beginning.

So,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^{10}C_6 (3x)^{10-6} (-1/x^2)^6 \\
 &= {}^{10}C_6 (3)^4 (x)^4 (1/x^{12}) \\
 &= [10 \times 9 \times 8 \times 7 \times 81] / [4 \times 3 \times 2 \times x^8] \\
 &= 17010 / x^8
 \end{aligned}$$

∴ The 5th term of the expression $(3x - 1/x^2)^{10}$ is $17010 / x^8$.

4. Find the 8th term in the expansion of $(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$.

Solution:

Given:

$$(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$$

Let us consider the 8th term as T_8

So,

$$\begin{aligned}
 T_8 &= T_{7+1} \\
 &= {}^{10}C_7 (x^{3/2} y^{1/2})^{10-7} (-x^{1/2} y^{3/2})^7 \\
 &= -[10 \times 9 \times 8] / [3 \times 2] x^{9/2} y^{3/2} (x^{7/2} y^{21/2}) \\
 &= -120 x^8 y^{12}
 \end{aligned}$$

∴ The 8th term of the expression $(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$ is $-120 x^8 y^{12}$.

5. Find the 7th term in the expansion of $(4x/5 + 5/2x)^8$.

Solution:

Given:

$$(4x/5 + 5/2x)^8$$

Let us consider the 7th term as T_7

So,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^8C_6 \left(\frac{4x}{5}\right)^{8-6} \left(\frac{5}{2x}\right)^6 \\
 &= \frac{8 \times 7 \times 4 \times 4 \times 125 \times 125}{2 \times 1 \times 25 \times 64} x^2 \left(\frac{1}{x^6}\right) \\
 &= \frac{4375}{x^4}
 \end{aligned}$$

∴ The 7th term of the expression $(4x/5 + 5/2x)^8$ is $4375/x^4$.

6. Find the 4th term from the beginning and 4th term from the end in the expansion of $(x + 2/x)^9$.

Solution:

Given:

$$(x + 2/x)^9$$

Let T_{r+1} be the 4th term from the end.

Then, T_{r+1} is $(10 - 4 + 1)$ th, i.e., 7th, term from the beginning.

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 (x^{9-6}) \left(\frac{2}{x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} (x^3) \left(\frac{64}{x^6}\right) \\ &= \frac{5376}{x^3} \end{aligned}$$

4th term from the beginning = $T_4 = T_{3+1}$

$$\begin{aligned} T_4 &= {}^9C_3 (x^{9-3}) \left(\frac{2}{x}\right)^3 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} (x^6) \left(\frac{8}{x^3}\right) \\ &= 672 x^3 \end{aligned}$$

7. Find the 4th term from the end in the expansion of $(4x/5 - 5/2x)^8$.

Solution:

Given:

$$(4x/5 - 5/2x)^8$$

Let T_{r+1} be the 4th term from the end of the given expression.

Then, T_{r+1} is $(10 - 4 + 1)$ th term, i.e., 7th term, from the beginning.

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 \left(\frac{4x}{5}\right)^{9-6} \left(\frac{5}{2x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} \left(\frac{64}{125} x^3\right) \left(\frac{125 \times 125}{64x^6}\right) \\ &= \frac{10500}{x^3} \end{aligned}$$

\therefore The 4th term from the end is $10500/x^3$.

8. Find the 7th term from the end in the expansion of $(2x^2 - 3/2x)^8$.

Solution:

Given:

$$(2x^2 - 3/2x)^8$$

Let T_{r+1} be the 4th term from the end of the given expression.

Then, T_{r+1} is $(9 - 7 + 1)$ th term, i.e., 3rd term, from the beginning.

$$\begin{aligned} T_3 &= T_{2+1} \\ &= {}^8C_2 (2x^2)^{8-2} \left(-\frac{3}{2x}\right)^2 \\ &= \frac{8 \times 7}{2 \times 1} (64x^{12}) \frac{9}{4x^2} \\ &= 4032 x^{10} \end{aligned}$$

\therefore The 7th term from the end is $4032 x^{10}$.

9. Find the coefficient of:

- (i) x^{10} in the expansion of $(2x^2 - 1/x)^{20}$
- (ii) x^7 in the expansion of $(x - 1/x^2)^{40}$
- (iii) x^{-15} in the expansion of $(3x^2 - a/3x^3)^{10}$
- (iv) x^9 in the expansion of $(x^2 - 1/3x)^9$
- (v) x^m in the expansion of $(x + 1/x)^n$
- (vi) x in the expansion of $(1 - 2x^3 + 3x^5)(1 + 1/x)^8$
- (vii) a^5b^7 in the expansion of $(a - 2b)^{12}$
- (viii) x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$

Solution:

(i) x^{10} in the expansion of $(2x^2 - 1/x)^{20}$

Given:

$$(2x^2 - 1/x)^{20}$$

If x^{10} occurs in the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{20}C_r (2x^2)^{20-r} \left(\frac{-1}{x}\right)^r \\ &= (-1)^r {}^{20}C_r (2^{20-r}) (x^{40-2r-r}) \end{aligned}$$

For this term to contain x^{10} , we must have:

$$40 - 3r = 10$$

$$3r = 30$$

$$r = 10$$

$$\therefore \text{Coefficient of } x^{10} = (-1)^{10} {}^{20}C_{10} (2^{20-10}) = {}^{20}C_{10} (2^{10})$$

(ii) x^7 in the expansion of $(x - 1/x^2)^{40}$

Given:

$$(x - 1/x^2)^{40}$$

If x^7 occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{40}C_r x^{40-r} \left(\frac{-1}{x^2}\right)^r \\ &= (-1)^r {}^{40}C_r x^{40-r-2r} \end{aligned}$$

For this term to contain x^7 , we must have:

$$40 - 3r = 7$$

$$3r = 40 - 7$$

$$3r = 33$$

$$r = 33/3$$

$$= 11$$

$$\therefore \text{Coefficient of } x^7 = (-1)^{11} {}^{40}C_{11} = -{}^{40}C_{11}$$

(iii) x^{-15} in the expansion of $(3x^2 - a/3x^3)^{10}$

Given:

$$(3x^2 - a/3x^3)^{10}$$

If x^{-15} occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{10}C_r (3x^2)^{10-r} \left(\frac{-a}{3x^3}\right)^r \\ &= (-1)^r {}^{10}C_r (3^{10-r-r}) (x^{20-2r-3r}) (a^r) \end{aligned}$$

For this term to contain x^{-15} , we must have:

$$20 - 5r = -15$$

$$5r = 20 + 15$$

$$5r = 35$$

$$r = 35/5$$

$$= 7$$

$$\therefore \text{Coefficient of } x^{-15} = (-1)^7 {}^{10}C_7 3^{10-14} (a^7) = -\frac{10 \times 9 \times 8}{3 \times 2 \times 9 \times 9} a^7 = -\frac{40}{27} a^7$$

(iv) x^9 in the expansion of $(x^2 - 1/3x)^9$

Given:

$$(x^2 - 1/3x)^9$$

If x^9 occurs at the $(r + 1)$ th term in the above expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^9C_r (x^2)^{9-r} \left(\frac{-1}{3x}\right)^r \\ &= (-1)^r {}^9C_r (x^{18-2r-r}) \left(\frac{1}{3^r}\right) \end{aligned}$$

For this term to contain x^9 , we must have:

$$18 - 3r = 9$$

$$3r = 18 - 9$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

$$\therefore \text{Coefficient of } x^9 = (-1)^3 {}^9C_3 \frac{1}{3^3} = -\frac{9 \times 8 \times 7}{2 \times 9 \times 9} = -\frac{28}{9}$$

(v) x^m in the expansion of $(x + 1/x)^n$

Given:

$$(x + 1/x)^n$$

If x^m occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$T_{r+1} = {}^nC_r x^{n-r} \frac{1}{x^r}$$

$$= {}^nC_r x^{n-2r}$$

For this term to contain x^m , we must have:

$$n - 2r = m$$

$$2r = n - m$$

$$r = (n - m)/2$$

$$\therefore \text{Coefficient of } x^m = {}^nC_{(n-m)/2} = \frac{n!}{\left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!}$$

(vi) x in the expansion of $(1 - 2x^3 + 3x^5)(1 + 1/x)^8$

Given:

$$(1 - 2x^3 + 3x^5)(1 + 1/x)^8$$

If x occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$(1 - 2x^3 + 3x^5)(1 + 1/x)^8 = (1 - 2x^3 + 3x^5)({}^8C_0 + {}^8C_1(1/x) + {}^8C_2(1/x)^2 + {}^8C_3(1/x)^3 + {}^8C_4(1/x)^4 + {}^8C_5(1/x)^5 + {}^8C_6(1/x)^6 + {}^8C_7(1/x)^7 + {}^8C_8(1/x)^8)$$

So, 'x' occurs in the above expression at $-2x^3 \cdot {}^8C_2(1/x^2) + 3x^5 \cdot {}^8C_4(1/x^4)$

$$\begin{aligned} \therefore \text{Coefficient of } x &= -2(8!/(2!6!)) + 3(8!/(4!4!)) \\ &= -56 + 210 \\ &= 154 \end{aligned}$$

(vii) a^5b^7 in the expansion of $(a - 2b)^{12}$

Given:

$$(a - 2b)^{12}$$

If a^5b^7 occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{12}C_r a^{12-r} (-2b)^r \\ &= (-1)^r {}^{12}C_r (a^{12-r}) (b^r) (2^r) \end{aligned}$$

For this term to contain a^5b^7 , we must have:

$$12 - r = 5$$

$$r = 12 - 5$$

$$= 7$$

$$\begin{aligned} \therefore \text{Required coefficient} &= (-1)^7 {}^{12}C_7 (2^7) \\ &= -\frac{12 \times 11 \times 10 \times 9 \times 8 \times 128}{5 \times 4 \times 3 \times 2} \\ &= -101376 \end{aligned}$$

(viii) x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$

Given:

$$(1 - 3x + 7x^2)(1 - x)^{16}$$

If x occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$\begin{aligned} (1 - 3x + 7x^2)(1 - x)^{16} &= (1 - 3x + 7x^2) ({}^{16}C_0 + {}^{16}C_1(-x) + {}^{16}C_2(-x)^2 + {}^{16}C_3(-x)^3 + {}^{16}C_4(-x)^4 \\ &+ {}^{16}C_5(-x)^5 + {}^{16}C_6(-x)^6 + {}^{16}C_7(-x)^7 + {}^{16}C_8(-x)^8 + {}^{16}C_9(-x)^9 + {}^{16}C_{10}(-x)^{10} + {}^{16}C_{11}(-x)^{11} \\ &+ {}^{16}C_{12}(-x)^{12} + {}^{16}C_{13}(-x)^{13} + {}^{16}C_{14}(-x)^{14} + {}^{16}C_{15}(-x)^{15} + {}^{16}C_{16}(-x)^{16}) \end{aligned}$$

So, ' x ' occurs in the above expression at ${}^{16}C_1(-x) - 3x{}^{16}C_0$

$$\begin{aligned} \therefore \text{Coefficient of } x &= -(16!/(1! 15!)) - 3(16!/(0! 16!)) \\ &= -16 - 3 \\ &= -19 \end{aligned}$$

10. Which term in the expansion of $\left\{ \left(\frac{x}{\sqrt{y}} \right)^{1/3} + \left(\frac{y}{x^{1/3}} \right)^{1/2} \right\}^{21}$ contains x and y to one and the same power.

Solution:

Let us consider T_{r+1} th term in the given expansion contains x and y to one and the same power.

Then we have,

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{21}C_r \left[\left(\frac{x}{\sqrt{y}} \right)^{1/3} \right]^{21-r} \left[\left(\frac{y}{x^{1/3}} \right)^{1/2} \right]^r \\ &= {}^{21}C_r \left(\frac{x^{(21-r)/3}}{x^{r/6}} \right) \left(\frac{y^{r/2}}{y^{(21-r)/6}} \right) \end{aligned}$$

$$= {}^{21}C_r (x)^{7-r/2} (y)^{2r/3-7/2}$$

If x and y have the same power, then

$$7 - r/2 = 2r/3 - 7/2$$

$$2r/3 + r/2 = 7 + 7/2$$

$$(4r + 3r)/6 = (14+7)/2$$

$$7r/6 = 21/2$$

$$r = (21 \times 6)/(2 \times 7)$$

$$= 3 \times 3$$

$$= 9$$

Hence, the required term is the 10th term.

11. Does the expansion of $(2x^2 - 1/x)$ contain any term involving x^9 ?

Solution:

Given:

$$(2x^2 - 1/x)$$

If x^9 occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{20}C_r (2x^2)^{20-r} \left(\frac{-1}{x}\right)^r \\ &= (-1)^r {}^{20}C_r (2)^{20-r} (x)^{40-2r-r} \end{aligned}$$

For this term to contain x^9 , we must have

$$40 - 3r = 9$$

$$3r = 40 - 9$$

$$3r = 31$$

$$r = 31/3$$

It is not possible, since r is not an integer.

Hence, there is no term with x^9 in the given expansion.

12. Show that the expansion of $(x^2 + 1/x)^{12}$ does not contain any term involving x^{-1} .

Solution:

Given:

$$(x^2 + 1/x)^{12}$$

If x^{-1} occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{12}C_r (x^2)^{12-r} \left(\frac{1}{x}\right)^r \\ &= {}^{12}C_r x^{24-2r-r} \end{aligned}$$

For this term to contain x^{-1} , we must have

$$24 - 3r = -1$$

$$3r = 24 + 1$$

$$3r = 25$$

$$r = 25/3$$

It is not possible, since r is not an integer.

Hence, there is no term with x^{-1} in the given expansion.

13. Find the middle term in the expansion of:

(i) $(2/3x - 3/2x)^{20}$

(ii) $(a/x + bx)^{12}$

(iii) $(x^2 - 2/x)^{10}$

(iv) $(x/a - a/x)^{10}$

Solution:

(i) $(2/3x - 3/2x)^{20}$

We have,

$(2/3x - 3/2x)^{20}$ where, $n = 20$ (even number)

So the middle term is $(n/2 + 1) = (20/2 + 1) = (10 + 1) = 11$. ie., 11th term

Now,

$$\begin{aligned} T_{11} &= T_{10+1} \\ &= {}^{20}C_{10} (2/3x)^{20-10} (3/2x)^{10} \\ &= {}^{20}C_{10} 2^{10}/3^{10} \times 3^{10}/2^{10} x^{10-10} \\ &= {}^{20}C_{10} \end{aligned}$$

Hence, the middle term is ${}^{20}C_{10}$.

(ii) $(a/x + bx)^{12}$

We have,

$(a/x + bx)^{12}$ where, $n = 12$ (even number)

So the middle term is $(n/2 + 1) = (12/2 + 1) = (6 + 1) = 7$. ie., 7th term

Now,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^{12}C_6 \left(\frac{a}{x}\right)^{12-6} (bx)^6 \\ &= {}^{12}C_6 a^6 b^6 \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2} a^6 b^6 \end{aligned}$$

$$= 924 a^6 b^6$$

Hence, the middle term is $924 a^6 b^6$.

(iii) $(x^2 - 2/x)^{10}$

We have,

$(x^2 - 2/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. i.e., 6th term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned} &= {}^{10}C_5 \left(x^2\right)^{10-5} \left(\frac{-2}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 32x^5 \\ &= -8064x^5 \end{aligned}$$

Hence, the middle term is $-8064x^5$.

(iv) $(x/a - a/x)^{10}$

We have,

$(x/a - a/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. i.e., 6th term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned} &= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\ &= -252 \end{aligned}$$

Hence, the middle term is -252 .

14. Find the middle terms in the expansion of:

(i) $(3x - x^3/6)^9$

(ii) $(2x^2 - 1/x)^7$

(iii) $(3x - 2/x^2)^{15}$

(iv) $(x^4 - 1/x^3)^{11}$

Solution:

(i) $(3x - x^3/6)^9$

We have,

$(3x - x^3/6)^9$ where, $n = 9$ (odd number)

So the middle terms are $((n+1)/2) = ((9+1)/2) = 10/2 = 5$ and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5th and 6th.

Now,

$$\begin{aligned} T_5 &= T_{4+1} \\ &= {}^9C_4 (3x)^{9-4} \left(\frac{-x^3}{6}\right)^4 \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 27 \times 9 \times \frac{1}{36 \times 36} x^{17} \\ &= \frac{189}{8} x^{17} \end{aligned}$$

And,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^9C_5 (3x)^{9-5} \left(\frac{-x^3}{6}\right)^5 \\ &= -\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 81 \times \frac{1}{216 \times 36} x^{19} \\ &= -\frac{21}{16} x^{19} \end{aligned}$$

Hence, the middle term are $\frac{189}{8} x^{17}$ and $-\frac{21}{16} x^{19}$.

(ii) $(2x^2 - 1/x)^7$

We have,

$(2x^2 - 1/x)^7$ where, $n = 7$ (odd number)

So the middle terms are $((n+1)/2) = ((7+1)/2) = 8/2 = 4$ and

$((n+1)/2 + 1) = ((7+1)/2 + 1) = (8/2 + 1) = (4 + 1) = 5$

The terms are 4th and 5th.

Now,

$$\begin{aligned} T_4 &= T_{3+1} \\ &= {}^7C_3 (2x^2)^{7-3} \left(\frac{-1}{x}\right)^3 \\ &= -\frac{7 \times 6 \times 5}{3 \times 2} \times 16 x^8 \times \frac{1}{x^3} \\ &= -560 x^5 \end{aligned}$$

And,

$$\begin{aligned} T_5 &= T_{4+1} \\ &= {}^7C_4 (2x^2)^{7-4} \left(\frac{-1}{x}\right)^4 \\ &= 35 \times 8 \times x^6 \times \frac{1}{x^4} \\ &= 280 x^2 \end{aligned}$$

Hence, the middle term are $-560x^5$ and $280x^2$.

(iii) $(3x - 2/x^2)^{15}$

We have,

$(3x - 2/x^2)^{15}$ where, $n = 15$ (odd number)

So the middle terms are $((n+1)/2) = ((15+1)/2) = 16/2 = 8$ and

$((n+1)/2 + 1) = ((15+1)/2 + 1) = (16/2 + 1) = (8 + 1) = 9$

The terms are 8th and 9th.

Now,

$T_8 = T_{7+1}$

$$\begin{aligned} &= {}^{15}C_7 (3x)^{15-7} \left(\frac{-2}{x^2}\right)^7 \\ &= -\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \times 3^8 \times 2^7 x^{8-14} \\ &= \frac{-6435 \times 3^8 \times 2^7}{x^6} \end{aligned}$$

And,

$T_9 = T_{8+1}$

$$\begin{aligned} &= {}^{15}C_8 (3x)^{15-8} \left(\frac{-2}{x^2}\right)^8 \\ &= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \times 3^7 \times 2^8 \times x^{7-16} \\ &= \frac{6435 \times 3^7 \times 2^8}{x^9} \end{aligned}$$

Hence, the middle term are $(-6435 \times 3^8 \times 2^7)/x^6$ and $(6435 \times 3^7 \times 2^8)/x^9$.

(iv) $(x^4 - 1/x^3)^{11}$

We have,

$(x^4 - 1/x^3)^{11}$

where, $n = 11$ (odd number)

So the middle terms are $((n+1)/2) = ((11+1)/2) = 12/2 = 6$ and

$((n+1)/2 + 1) = ((11+1)/2 + 1) = (12/2 + 1) = (6 + 1) = 7$

The terms are 6th and 7th.

Now,

$T_6 = T_{5+1}$

$$\begin{aligned} &= {}^{11}C_5 (x^4)^{11-5} \left(\frac{-1}{x^3}\right)^5 \\ &= -\frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} \times (x)^{24-15} \\ &= -462 x^9 \end{aligned}$$

And,

$T_7 = T_{6+1}$

$$\begin{aligned}
 &= {}^{11}C_6 (x^4)^{11-6} \left(\frac{-1}{x^3}\right)^6 \\
 &= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} (x)^{20-18} \\
 &= 462 x^2
 \end{aligned}$$

Hence, the middle term are $-462x^9$ and $462x^2$.

15. Find the middle terms in the expansion of:

(i) $(x - 1/x)^{10}$

(ii) $(1 - 2x + x^2)^n$

(iii) $(1 + 3x + 3x^2 + x^3)^{2n}$

(iv) $(2x - x^2/4)^9$

(v) $(x - 1/x)^{2n+1}$

(vi) $(x/3 + 9y)^{10}$

(vii) $(3 - x^3/6)^7$

(viii) $(2ax - b/x^2)^{12}$

(ix) $(p/x + x/p)^9$

(x) $(x/a - a/x)^{10}$

Solution:

(i) $(x - 1/x)^{10}$

We have,

$(x - 1/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. ie., 6th term

Now,

$T_6 = T_{5+1}$

$$= {}^{10}C_5 x^{10-5} \left(\frac{-1}{x}\right)^5$$

$$= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}$$

$$= -252$$

Hence, the middle term is -252 .

(ii) $(1 - 2x + x^2)^n$

We have,

$(1 - 2x + x^2)^n = (1 - x)^{2n}$ where, n is an even number.

So the middle term is $(2n/2 + 1) = (n + 1)$ th term.

Now,

$T_n = T_{n+1}$

$$= {}^{2n}C_n (-1)^n (x)^n$$

$$= (2n)!/(n!)^2 (-1)^n x^n$$

Hence, the middle term is $(2n)!/(n!)^2 (-1)^n x^n$.

(iii) $(1 + 3x + 3x^2 + x^3)^{2n}$

We have,

$(1 + 3x + 3x^2 + x^3)^{2n} = (1 + x)^{6n}$ where, n is an even number.

So the middle term is $(n/2 + 1) = (6n/2 + 1) = (3n + 1)$ th term.

Now,

$$\begin{aligned} T_{2n} &= T_{3n+1} \\ &= {}^{6n}C_{3n} x^{3n} \\ &= (6n)!/(3n!)^2 x^{3n} \end{aligned}$$

Hence, the middle term is $(6n)!/(3n!)^2 x^{3n}$.

(iv) $(2x - x^2/4)^9$

We have,

$(2x - x^2/4)^9$ where, $n = 9$ (odd number)

So the middle terms are $((n+1)/2) = ((9+1)/2) = 10/2 = 5$ and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5th and 6th.

Now,

$T_5 = T_{4+1}$

$$\begin{aligned} &= {}^9C_4 (2x)^{9-4} \left(\frac{-x^2}{4}\right)^4 \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 2^5 \frac{1}{4^4} x^{5+8} \\ &= \frac{63}{4} x^{13} \end{aligned}$$

And,

$T_6 = T_{5+1}$

$$\begin{aligned} &= {}^9C_5 (2x)^{9-5} \left(\frac{-x^2}{4}\right)^5 \\ &= -\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 2^4 \frac{1}{4^5} x^{4+10} \\ &= -\frac{63}{32} x^{14} \end{aligned}$$

Hence, the middle term is $63/4 x^{13}$ and $-63/32 x^{14}$.

(v) $(x - 1/x)^{2n+1}$

We have,

$(x - 1/x)^{2n+1}$ where, $n = (2n + 1)$ is an (odd number)

So the middle terms are $((n+1)/2) = ((2n+1+1)/2) = (2n+2)/2 = (n + 1)$ and

$$((n+1)/2 + 1) = ((2n+1+1)/2 + 1) = ((2n+2)/2 + 1) = (n + 1 + 1) = (n + 2)$$

The terms are $(n + 1)^{\text{th}}$ and $(n + 2)^{\text{th}}$.

Now,

$$\begin{aligned} T_n &= T_{n+1} \\ &= {}^{2n+1}C_n x^{2n+1-n} \times \frac{(-1)^n}{x^n} \\ &= (-1)^n {}^{2n+1}C_n x \end{aligned}$$

And,

$$\begin{aligned} T_{n+2} &= T_{n+1+1} \\ &= {}^{2n+1}C_n x^{2n+1-n-1} \frac{(-1)^{n+1}}{x^{n+1}} \\ &= (-1)^{n+1} {}^{2n+1}C_n \times \frac{1}{x} \end{aligned}$$

Hence, the middle term is $(-1)^n \cdot {}^{2n+1}C_n x$ and $(-1)^{n+1} \cdot {}^{2n+1}C_n (1/x)$.

(vi) $(x/3 + 9y)^{10}$

We have,

$(x/3 + 9y)^{10}$ where, $n = 10$ is an even number.

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. i.e., 6th term.

Now,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5 \\ &= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times \frac{1}{3^5} \times 9^5 \times x^5 y^5 \\ &= 61236 x^5 y^5 \end{aligned}$$

Hence, the middle term is $61236x^5y^5$.

(vii) $(3 - x^3/6)^7$

We have,

$(3 - x^3/6)^7$ where, $n = 7$ (odd number).

So the middle terms are $((n+1)/2) = ((7+1)/2) = 8/2 = 4$ and

$((n+1)/2 + 1) = ((7+1)/2 + 1) = (8/2 + 1) = (4 + 1) = 5$

The terms are 4th and 5th.

Now,

$$\begin{aligned} T_4 &= T_{3+1} \\ &= {}^7C_3 (3)^{7-3} (-x^3/6)^3 \\ &= -105/8 x^9 \end{aligned}$$

And,

$$T_5 = T_{4+1}$$

$$\begin{aligned}
 &= {}^9C_4 (3)^{9-4} (-x^3/6)^4 \\
 &= \frac{7 \times 6 \times 5}{3 \times 2} \times 3^5 \times \frac{1}{6^4} x^{12} \\
 &= \frac{35}{48} x^{12}
 \end{aligned}$$

Hence, the middle terms are $-105/8 x^9$ and $35/48 x^{12}$.

(viii) $(2ax - b/x^2)^{12}$

We have,

$(2ax - b/x^2)^{12}$ where, $n = 12$ is an even number.

So the middle term is $(n/2 + 1) = (12/2 + 1) = (6 + 1) = 7$. i.e., 7th term.

Now,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^{12}C_6 (2ax)^{12-6} \left(\frac{-b}{x^2}\right)^6 \\
 &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \times \left(\frac{2ab}{x}\right)^6 \\
 &= \frac{59136 a^6 b^6}{x^6}
 \end{aligned}$$

Hence, the middle term is $(59136a^6b^6)/x^6$.

(ix) $(p/x + x/p)^9$

We have,

$(p/x + x/p)^9$ where, $n = 9$ (odd number).

So the middle terms are $((n+1)/2) = ((9+1)/2) = 10/2 = 5$ and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5th and 6th.

Now,

$$\begin{aligned}
 T_5 &= T_{4+1} \\
 &= {}^9C_4 \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \left(\frac{p}{x}\right) \\
 &= \frac{126 p}{x}
 \end{aligned}$$

And,

$$\begin{aligned}
 T_6 &= T_{5+1} \\
 &= {}^9C_5 (p/x)^{9-5} (x/p)^5 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \left(\frac{x}{p}\right) \\
 &= \frac{126 x}{p}
 \end{aligned}$$

Hence, the middle terms are $126p/x$ and $126x/p$.

$$(x) (x/a - a/x)^{10}$$

We have,

$$(x/a - a/x)^{10} \text{ where, } n = 10 \text{ (even number)}$$

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. ie., 6th term

Now,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\ &= -252 \end{aligned}$$

Hence, the middle term is -252.

16. Find the term independent of x in the expansion of the following expressions:

(i) $(3/2 x^2 - 1/3x)^9$

(ii) $(2x + 1/3x^2)^9$

(iii) $(2x^2 - 3/x^3)^{25}$

(iv) $(3x - 2/x^2)^{15}$

(v) $((\sqrt{x}/3) + \sqrt{3/2x^2})^{10}$

(vi) $(x - 1/x^2)^{3n}$

(vii) $(1/2 x^{1/3} + x^{-1/5})^8$

(viii) $(1 + x + 2x^3)(3/2x^2 - 3/3x)^9$

(ix) $(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}$, $x > 0$

(x) $(3/2x^2 - 1/3x)^6$

Solution:

(i) $(3/2 x^2 - 1/3x)^9$

Given:

$$(3/2 x^2 - 1/3x)^9$$

If $(r + 1)$ th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r \\ &= (-1)^r {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}} \times x^{18-2r-r} \end{aligned}$$

For this term to be independent of x, we must have

$$18 - 3r = 0$$

$$3r = 18$$

$$r = 18/3$$

$$= 6$$

So, the required term is 7th term.

We have,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 \times (3^{9-12})/(2^{9-6}) \\ &= (9 \times 8 \times 7)/(3 \times 2) \times 3^{-3} \times 2^{-3} \\ &= 7/18 \end{aligned}$$

Hence, the term independent of x is 7/18.

(ii) $(2x + 1/3x^2)^9$

Given:

$$(2x + 1/3x^2)^9$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^9C_r (2x)^{9-r} \left(\frac{1}{3x^2}\right)^r \\ &= {}^9C_r \cdot \frac{2^{9-r}}{3^r} x^{9-r-2r} \end{aligned}$$

For this term to be independent of x, we must have

$$9 - 3r = 0$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

So, the required term is 4th term.

We have,

$$\begin{aligned} T_4 &= T_{3+1} \\ &= {}^9C_3 \times (2^6)/(3^3) \\ &= {}^9C_3 \times 64/27 \end{aligned}$$

Hence, the term independent of x is ${}^9C_3 \times 64/27$.

(iii) $(2x^2 - 3/x^3)^{25}$

Given:

$$(2x^2 - 3/x^3)^{25}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^{25}C_r (2x^2)^{25-r} (-3/x^3)^r \\ &= (-1)^r {}^{25}C_r \times 2^{25-r} \times 3^r x^{50-2r-3r} \end{aligned}$$

For this term to be independent of x, we must have

$$50 - 5r = 0$$

$$5r = 50$$

$$r = 50/5$$

$$= 10$$

So, the required term is 11th term.

We have,

$$\begin{aligned} T_{11} &= T_{10+1} \\ &= (-1)^{10} {}^{25}C_{10} \times 2^{25-10} \times 3^{10} \\ &= {}^{25}C_{10} (2^{15} \times 3^{10}) \end{aligned}$$

Hence, the term independent of x is ${}^{25}C_{10} (2^{15} \times 3^{10})$.

(iv) $(3x - 2/x^2)^{15}$

Given:

$$(3x - 2/x^2)^{15}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^{15}C_r (3x)^{15-r} (-2/x^2)^r \\ &= (-1)^r {}^{15}C_r \times 3^{15-r} \times 2^r x^{15-r-2r} \end{aligned}$$

For this term to be independent of x, we must have

$$15 - 3r = 0$$

$$3r = 15$$

$$r = 15/3$$

$$= 5$$

So, the required term is 6th term.

We have,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= (-1)^5 {}^{15}C_5 \times 3^{15-5} \times 2^5 \\ &= -3003 \times 3^{10} \times 2^5 \end{aligned}$$

Hence, the term independent of x is $-3003 \times 3^{10} \times 2^5$.

(v) $((\sqrt{x}/3) + \sqrt{3}/2x^2)^{10}$

Given:

$$((\sqrt{x}/3) + \sqrt{3}/2x^2)^{10}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned}
 &= {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r \\
 &= {}^{10}C_r \cdot \frac{3^{\frac{r}{2}-\frac{10-r}{2}}}{2^r} x^{\frac{10-r}{2}-2r}
 \end{aligned}$$

For this term to be independent of x , we must have

$$(10-r)/2 - 2r = 0$$

$$10 - 5r = 0$$

$$5r = 10$$

$$r = 10/5$$

$$= 2$$

So, the required term is 3rd term.

We have,

$$T_3 = T_{2+1}$$

$$\begin{aligned}
 &= {}^{10}C_2 \times \frac{3^{\frac{2}{2}-\frac{10-2}{2}}}{2^2} \\
 &= \frac{10 \times 9}{2 \times 4 \times 9} \\
 &= 90/72 \\
 &= 15/12 \\
 &= 5/4
 \end{aligned}$$

Hence, the term independent of x is $5/4$.

(vi) $(x - 1/x^2)^{3n}$

Given:

$$(x - 1/x^2)^{3n}$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^{3n}C_r x^{3n-r} (-1/x^2)^r \\
 &= {}^{3n}C_r x^{3n-r} (-1)^r x^{-2r} \\
 &= (-1)^r {}^{3n}C_r x^{3n-r-2r}
 \end{aligned}$$

For this term to be independent of x , we must have

$$3n - 3r = 0$$

$$r = n$$

So, the required term is $(n+1)$ th term.

We have,

$$(-1)^n {}^{3n}C_n$$

Hence, the term independent of x $(-1)^n {}^{3n}C_n$

(vii) $(1/2 x^{1/3} + x^{-1/5})^8$

Given:

$$\left(\frac{1}{2}x^{1/3} + x^{-1/5}\right)^8$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^n C_r x^{n-r} a^r \\ &= {}^8 C_r \left(\frac{1}{2}x^{1/3}\right)^{8-r} \left(x^{-1/5}\right)^r \\ &= {}^8 C_r \cdot \frac{1}{2^{8-r}} x^{\frac{8-r}{3} - \frac{r}{5}} \end{aligned}$$

For this term to be independent of x , we must have

$$(8-r)/3 - r/5 = 0$$

$$(40 - 5r - 3r)/15 = 0$$

$$40 - 5r - 3r = 0$$

$$40 - 8r = 0$$

$$8r = 40$$

$$r = 40/8$$

$$= 5$$

So, the required term is 6th term.

We have,

$$\begin{aligned} T_5 &= T_{5+1} \\ &= {}^8 C_5 \times 1/(2^{8-5}) \\ &= (8 \times 7 \times 6)/(3 \times 2 \times 8) \\ &= 7 \end{aligned}$$

Hence, the term independent of x is 7.

(viii) $(1 + x + 2x^3)(3/2x^2 - 3/3x)^9$

Given:

$$(1 + x + 2x^3)(3/2x^2 - 3/3x)^9$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned} (1 + x + 2x^3)(3/2x^2 - 3/3x)^9 &= \\ &= (1 + x + 2x^3) \left[\left(\frac{3}{2}x^2\right)^9 - {}^9 C_1 \left(\frac{3}{2}x^2\right)^8 \frac{1}{3x} \dots + {}^9 C_6 \left(\frac{3}{2}x^2\right)^3 \left(\frac{1}{3x}\right)^6 - {}^9 C_7 \left(\frac{3}{2}x^2\right)^2 \left(\frac{1}{3x}\right)^7 \right] \end{aligned}$$

By computing we get,

The term independent of x

$$\begin{aligned} &= 1 \left[{}^9 C_6 \frac{3^3}{2^3} \times \frac{1}{3^6} \right] - 2x^3 \left[{}^9 C_7 \frac{3^3}{2^3} \times \frac{1}{3^7} \times \frac{1}{x^3} \right] \\ &= \left[\frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{1}{8 \times 27} \right] - 2 \left[\frac{9 \times 8}{1 \times 2} - \frac{1}{4 \times 243} \right] \end{aligned}$$

$$\begin{aligned}
 &= 7/18 - 2/27 \\
 &= (189 - 36)/486 \\
 &= 153/486 \text{ (divide by 9)} \\
 &= 17/54
 \end{aligned}$$

Hence, the term independent of x is $17/54$.

(ix) $(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}$, $x > 0$

Given:

$$(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}, x > 0$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^nC_r x^{n-r} a^r \\
 &= {}^{18}C_r \left(x^{1/3}\right)^{18-r} \left(\frac{1}{2x^{1/3}}\right)^r \\
 &= {}^{18}C_r \times \frac{1}{2^r} x^{\frac{18-r}{3} - \frac{r}{3}}
 \end{aligned}$$

For this term to be independent of r , we must have

$$(18-r)/3 - r/3 = 0$$

$$(18 - r - r)/3 = 0$$

$$18 - 2r = 0$$

$$2r = 18$$

$$r = 18/2$$

$$= 9$$

So, the required term is 10th term.

We have,

$$\begin{aligned}
 T_{10} &= T_{9+1} \\
 &= {}^{18}C_9 \times 1/2^9
 \end{aligned}$$

Hence, the term independent of x is ${}^{18}C_9 \times 1/2^9$.

(x) $(3/2x^2 - 1/3x)^6$

Given:

$$(3/2x^2 - 1/3x)^6$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^nC_r x^{n-r} a^r \\
 &= {}^6C_r \left(\frac{3}{2}x^2\right)^{6-r} \left(\frac{-1}{3x}\right)^r \\
 &= (-1)^r {}^6C_r \times \frac{3^{6-r-r}}{2^{6-r}} x^{12-2r-r}
 \end{aligned}$$

For this term to be independent of r , we must have

$$12 - 3r = 0$$

$$3r = 12$$

$$r = 12/3$$

$$= 4$$

So, the required term is 5th term.

We have,

$$T_5 = T_{4+1}$$

$$= {}^6C_4 \times \frac{3^{6-4-4}}{2^{6-4}}$$

$$= \frac{6 \times 5}{2 \times 1 \times 4 \times 9}$$

$$= \frac{5}{12}$$

Hence, the term independent of x is $5/12$.

17. If the coefficients of $(2r + 4)$ th and $(r - 2)$ th terms in the expansion of $(1 + x)^{18}$ are equal, find r .

Solution:

Given:

$$(1 + x)^{18}$$

We know, the coefficient of the r term in the expansion of $(1 + x)^n$ is ${}^nC_{r-1}$

So, the coefficients of the $(2r + 4)$ and $(r - 2)$ terms in the given expansion are ${}^{18}C_{2r+4-1}$ and ${}^{18}C_{r-2-1}$

For these coefficients to be equal, we must have

$${}^{18}C_{2r+4-1} = {}^{18}C_{r-2-1}$$

$${}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$$2r + 3 = r - 3 \text{ (or) } 2r + 3 + r - 3 = 18 \text{ [Since, } {}^nC_r = {}^nC_s \Rightarrow r = s \text{ (or) } r + s = n]$$

$$2r - r = -3 - 3 \text{ (or) } 3r = 18 - 3 + 3$$

$$r = -6 \text{ (or) } 3r = 18$$

$$r = -6 \text{ (or) } r = 18/3$$

$$r = -6 \text{ (or) } r = 6$$

$\therefore r = 6$ [since, r should be a positive integer.]

18. If the coefficients of $(2r + 1)$ th term and $(r + 2)$ th term in the expansion of $(1 + x)^{43}$ are equal, find r .

Solution:

Given:

$$(1 + x)^{43}$$

We know, the coefficient of the r term in the expansion of $(1 + x)^n$ is ${}^nC_{r-1}$

So, the coefficients of the $(2r + 1)$ and $(r + 2)$ terms in the given expansion are ${}^{43}C_{2r+1-1}$ and ${}^{43}C_{r+2-1}$

For these coefficients to be equal, we must have

$${}^{43}C_{2r+1-1} = {}^{43}C_{r+2-1}$$

$${}^{43}C_{2r} = {}^{43}C_{r+1}$$

$$2r = r + 1 \text{ (or) } 2r + r + 1 = 43 \text{ [Since, } {}^nC_r = {}^nC_s \Rightarrow r = s \text{ (or) } r + s = n]$$

$$2r - r = 1 \text{ (or) } 3r + 1 = 43$$

$$r = 1 \text{ (or) } 3r = 43 - 1$$

$$r = 1 \text{ (or) } 3r = 42$$

$$r = 1 \text{ (or) } r = 42/3$$

$$r = 1 \text{ (or) } r = 14$$

$\therefore r = 14$ [since, value '1' gives the same term]

19. Prove that the coefficient of $(r + 1)$ th term in the expansion of $(1 + x)^{n+1}$ is equal to the sum of the coefficients of r th and $(r + 1)$ th terms in the expansion of $(1 + x)^n$.

Solution:

We know, the coefficients of $(r + 1)$ th term in $(1 + x)^{n+1}$ is ${}^{n+1}C_r$

So, sum of the coefficients of the r th and $(r + 1)$ th terms in $(1 + x)^n$ is

$$\begin{aligned} (1 + x)^n &= {}^nC_{r-1} + {}^nC_r \\ &= {}^{n+1}C_r \text{ [since, } {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}] \end{aligned}$$

Hence proved.