

**EXERCISE 18.1**
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**1. Using binomial theorem, write down the expressions of the following:**

**(i)  $(2x + 3y)^5$**

**(ii)  $(2x - 3y)^4$**

**(iii)  $\left(x - \frac{1}{x}\right)^6$**

**(iv)  $(1 - 3x)^7$**

**(v)  $\left(ax - \frac{b}{x}\right)^6$**

**(vi)  $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$**

**(vii)  $\left(\sqrt[3]{x} - \sqrt[3]{a}\right)^6$**

**(viii)  $(1 + 2x - 3x^2)^5$**

**(ix)  $\left(x + 1 - \frac{1}{x}\right)^3$**

**(x)  $(1 - 2x + 3x^2)^3$**

**Solution:**

**(i)  $(2x + 3y)^5$**

Let us solve the given expression:

$$(2x + 3y)^5 = {}^5C_0 (2x)^5 (3y)^0 + {}^5C_1 (2x)^4 (3y)^1 + {}^5C_2 (2x)^3 (3y)^2 + {}^5C_3 (2x)^2 (3y)^3 + {}^5C_4$$

$$(2x)^1 (3y)^4 + {}^5C_5 (2x)^0 (3y)^5$$

$$= 32x^5 + 5 (16x^4) (3y) + 10 (8x^3) (9y)^2 + 10 (4x)^2 (27y)^3 + 5 (2x) (81y^4) + 243y^5$$

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$$

**(ii)  $(2x - 3y)^4$**

Let us solve the given expression:

$$(2x - 3y)^4 = {}^4C_0 (2x)^4 (3y)^0 - {}^4C_1 (2x)^3 (3y)^1 + {}^4C_2 (2x)^2 (3y)^2 - {}^4C_3 (2x)^1 (3y)^3 + {}^4C_4 (2x)^0$$

$$(3y)^4$$

$$= 16x^4 - 4 (8x^3) (3y) + 6 (4x^2) (9y^2) - 4 (2x) (27y^3) + 81y^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

**(iii)**  $\left(x - \frac{1}{x}\right)^6$

Let us solve the given expression:

$$\begin{aligned} & \left(x - \frac{1}{x}\right)^6 \\ &= {}^6C_0 x^6 \left(\frac{1}{x}\right)^0 - {}^6C_1 x^5 \left(\frac{1}{x}\right)^1 + {}^6C_2 x^4 \left(\frac{1}{x}\right)^2 - {}^6C_3 x^3 \left(\frac{1}{x}\right)^3 \\ &+ {}^6C_4 x^2 \left(\frac{1}{x}\right)^4 - {}^6C_5 x^1 \left(\frac{1}{x}\right)^5 + {}^6C_6 x^0 \left(\frac{1}{x}\right)^6 \\ &= x^6 - 6x^5 \times \frac{1}{x} + 15x^4 \times \frac{1}{x^2} - 20x^3 \times \frac{1}{x^3} + 15x^2 \times \frac{1}{x^4} - 6x \times \frac{1}{x^5} + \frac{1}{x^6} \\ &= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6} \end{aligned}$$

**(iv)**  $(1 - 3x)^7$

Let us solve the given expression:

$$\begin{aligned} (1 - 3x)^7 &= {}^7C_0 (3x)^0 - {}^7C_1 (3x)^1 + {}^7C_2 (3x)^2 - {}^7C_3 (3x)^3 + {}^7C_4 (3x)^4 - {}^7C_5 (3x)^5 + {}^7C_6 (3x)^6 - {}^7C_7 (3x)^7 \\ &= 1 - 7(3x) + 21(9x^2) - 35(27x^3) + 35(81x^4) - 21(243x^5) + 7(729x^6) - 2187(x^7) \\ &= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7 \end{aligned}$$

**(v)**  $\left(ax - \frac{b}{x}\right)^6$

Let us solve the given expression:

$$\begin{aligned} & \left(ax - \frac{b}{x}\right)^6 \\ &= {}^6C_0 (ax)^6 \left(\frac{b}{x}\right)^0 - {}^6C_1 (ax)^5 \left(\frac{b}{x}\right)^1 + {}^6C_2 (ax)^4 \left(\frac{b}{x}\right)^2 - {}^6C_3 (ax)^3 \left(\frac{b}{x}\right)^3 \\ &+ {}^6C_4 (ax)^2 \left(\frac{b}{x}\right)^4 - {}^6C_5 (ax)^1 \left(\frac{b}{x}\right)^5 + {}^6C_6 (ax)^0 \left(\frac{b}{x}\right)^6 \\ &= a^6 x^6 - 6a^5 x^5 \times \frac{b}{x} + 15a^4 x^4 \times \frac{b^2}{x^2} - 20a^3 b^3 \times \frac{b^3}{x^3} + 15a^2 x^2 \times \frac{b^4}{x^4} - 6ax \times \frac{b^5}{x^5} + \frac{b^6}{x^6} \\ &= a^6 x^6 - 6a^5 x^4 b + 15a^4 x^2 b^2 - 20a^3 b^3 + 15 \frac{a^2 b^4}{x^2} - 6 \frac{ab^5}{x^4} + \frac{b^6}{x^6} \end{aligned}$$

$$(vi) \left( \sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 \left( \sqrt{\frac{x}{a}} \right)^6 \left( \sqrt{\frac{a}{x}} \right)^0 - {}^6C_1 \left( \sqrt{\frac{x}{a}} \right)^5 \left( \sqrt{\frac{a}{x}} \right)^1 + {}^6C_2 \left( \sqrt{\frac{x}{a}} \right)^4 \left( \sqrt{\frac{a}{x}} \right)^2 - {}^6C_3 \left( \sqrt{\frac{x}{a}} \right)^3 \left( \sqrt{\frac{a}{x}} \right)^3 \\ &+ {}^6C_4 \left( \sqrt{\frac{x}{a}} \right)^2 \left( \sqrt{\frac{a}{x}} \right)^4 - {}^6C_5 \left( \sqrt{\frac{x}{a}} \right)^1 \left( \sqrt{\frac{a}{x}} \right)^5 + {}^6C_6 \left( \sqrt{\frac{x}{a}} \right)^0 \left( \sqrt{\frac{a}{x}} \right)^6 \\ &= \frac{x^3}{a^3} - 6 \frac{x^2}{a^2} + 15 \frac{x}{a} - 20 + 15 \frac{a}{x} - 6 \frac{a^2}{x^2} + \frac{a^3}{x^3} \end{aligned}$$

$$(vii) \left( \sqrt[3]{x} - \sqrt[3]{a} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 (\sqrt[3]{x})^6 (\sqrt[3]{a})^0 - {}^6C_1 (\sqrt[3]{x})^5 (\sqrt[3]{a})^1 + {}^6C_2 (\sqrt[3]{x})^4 (\sqrt[3]{a})^2 - {}^6C_3 (\sqrt[3]{x})^3 (\sqrt[3]{a})^3 \\ &+ {}^6C_4 (\sqrt[3]{x})^2 (\sqrt[3]{a})^4 - {}^6C_5 (\sqrt[3]{x})^1 (\sqrt[3]{a})^5 + {}^6C_6 (\sqrt[3]{x})^0 (\sqrt[3]{a})^6 \\ &= x^2 - 6x^{5/3}a^{1/3} + 15x^{4/3}a^{2/3} - 20xa + 15x^{2/3}a^{4/3} - 6x^{1/3}a^{5/3} + a^2 \end{aligned}$$

$$(viii) (1 + 2x - 3x^2)^5$$

Let us solve the given expression:

Let us consider  $(1 + 2x)$  and  $3x^2$  as two different entities and apply the binomial theorem.

$$(1 + 2x - 3x^2)^5 = {}^5C_0 (1 + 2x)^5 (3x^2)^0 - {}^5C_1 (1 + 2x)^4 (3x^2)^1 + {}^5C_2 (1 + 2x)^3 (3x^2)^2 - {}^5C_3 (1 + 2x)^2 (3x^2)^3 + {}^5C_4 (1 + 2x)^1 (3x^2)^4 - {}^5C_5 (1 + 2x)^0 (3x^2)^5$$

$$= (1 + 2x)^5 - 5(1 + 2x)^4 (3x^2) + 10(1 + 2x)^3 (9x^4) - 10(1 + 2x)^2 (27x^6) + 5(1 + 2x)(81x^8) - 243x^{10}$$

$$= {}^5C_0 (2x)^0 + {}^5C_1 (2x)^1 + {}^5C_2 (2x)^2 + {}^5C_3 (2x)^3 + {}^5C_4 (2x)^4 + {}^5C_5 (2x)^5 - 15x^2 [{}^4C_0 (2x)^0 + {}^4C_1 (2x)^1 + {}^4C_2 (2x)^2 + {}^4C_3 (2x)^3 + {}^4C_4 (2x)^4] + 90x^4 [1 + 8x^3 + 6x + 12x^2] - 270x^6(1 + 4x^2 + 4x) + 405x^8 + 810x^9 - 243x^{10}$$

$$= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5 - 15x^2 - 120x^3 - 360x^4 - 480x^5 - 240x^6 + 90x^4 + 720x^7 + 540x^5 + 1080x^6 - 270x^6 - 1080x^8 - 1080x^7 + 405x^8 + 810x^9 - 243x^{10}$$

$$= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10}$$

$$(ix) \left(x + 1 - \frac{1}{x}\right)^3$$

Let us solve the given expression:

$$\begin{aligned} &= {}^3C_0(x+1)^3\left(\frac{1}{x}\right)^0 - {}^3C_1(x+1)^2\left(\frac{1}{x}\right)^1 + {}^3C_2(x+1)^1\left(\frac{1}{x}\right)^2 - {}^3C_3(x+1)^0\left(\frac{1}{x}\right)^3 \\ &= (x+1)^3 - 3(x+1)^2 \times \frac{1}{x} + 3 \frac{x+1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x + 3x^2 - \frac{3x^2 + 3 + 6x}{x} + 3 \frac{x+1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x + 3x^2 - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3} \\ &= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3} \end{aligned}$$

$$(x) (1 - 2x + 3x^2)^3$$

Let us solve the given expression:

$$\begin{aligned} &= {}^3C_0(1-2x)^3 + {}^3C_1(1-2x)^2(3x^2) + {}^3C_2(1-2x)(3x^2)^2 + {}^3C_3(3x^2)^3 \\ &= (1-2x)^3 + 9x^2(1-2x)^2 + 27x^4(1-2x) + 27x^6 \\ &= 1 - 8x^3 + 12x^2 - 6x + 9x^2(1 + 4x^2 - 4x) + 27x^4 - 54x^5 + 27x^6 \\ &= 1 - 8x^3 + 12x^2 - 6x + 9x^2 + 36x^4 - 36x^3 + 27x^4 - 54x^5 + 27x^6 \\ &= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6 \end{aligned}$$

**2. Evaluate the following:**

$$(i) (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6$$

$$(ii) \left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$$

$$(iii) (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$$

$$(iv) (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$



$$(v) (3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$$

$$(vi) (2 + \sqrt{3})^7 + (2 - \sqrt{3})^7$$

$$(vii) (\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$$

$$(viii) (0.99)^5 + (1.01)^5$$

$$(ix) (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

$$(x) \left\{ a^2 + \sqrt{a^2 - 1} \right\}^4 + \left\{ a^2 - \sqrt{a^2 - 1} \right\}^4$$

**Solution:**

$$(i) (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[ {}^6C_0 (\sqrt{x+1})^6 (\sqrt{x-1})^0 + {}^6C_2 (\sqrt{x+1})^4 (\sqrt{x-1})^2 \right. \\ &+ {}^6C_4 (\sqrt{x+1})^2 (\sqrt{x-1})^4 + {}^6C_6 (\sqrt{x+1})^0 (\sqrt{x-1})^6 \left. \right] \\ &= 2 \left[ (x+1)^3 + 15(x+1)^2(x-1) + 15(x+1)(x-1)^2 + (x-1)^3 \right] \\ &= 2 \left[ x^3 + 1 + 3x + 3x^2 + 15(x^2 + 2x + 1)(x-1) + 15(x+1)(x^2 + 1 - 2x) + x^3 - 1 + 3x - 3x^2 \right] \\ &= 2 \left[ 2x^3 + 6x + 15x^3 - 15x^2 + 30x^2 - 30x + 15x - 15 + 15x^3 + 15x^2 - 30x^2 - 30x + 15x + 15 \right] \\ &= 2 \left[ 32x^3 - 24x \right] \\ &= 16x \left[ 4x^2 - 3 \right] \end{aligned}$$

$$(ii) \left( x + \sqrt{x^2 - 1} \right)^6 + \left( x - \sqrt{x^2 - 1} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[ {}^6C_0 x^6 (\sqrt{x^2 - 1})^0 + {}^6C_2 x^4 (\sqrt{x^2 - 1})^2 + {}^6C_4 x^2 (\sqrt{x^2 - 1})^4 + \right. \\ &{}^6C_6 x^0 (\sqrt{x^2 - 1})^6 \left. \right] \\ &= 2 \left[ x^6 + 15x^4(x^2 - 1) + 15x^2(x^2 - 1)^2 + (x^2 - 1)^3 \right] \end{aligned}$$

$$\begin{aligned}
 &= 2 \left[ x^6 + 15x^6 - 15x^4 + 15x^2 (x^4 - 2x^2 + 1) + (x^6 - 1 + 3x^2 - 3x^4) \right] \\
 &= 2 \left[ x^6 + 15x^6 - 15x^4 + 15x^6 - 30x^4 + 15x^2 + x^6 - 1 + 3x^2 - 3x^4 \right] \\
 &= 64x^6 - 96x^4 + 36x^2 - 2
 \end{aligned}$$

$$(iii) (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^5C_0 (2\sqrt{x})^0 + {}^5C_2 (2\sqrt{x})^2 + {}^5C_4 (2\sqrt{x})^4 \right] \\
 &= 2 \left[ 1 + 10(4x) + 5(16x^2) \right] \\
 &= 2 \left[ 1 + 40x + 80x^2 \right]
 \end{aligned}$$

$$(iv) (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^6C_0 (\sqrt{2})^6 + {}^6C_2 (\sqrt{2})^4 + {}^6C_4 (\sqrt{2})^2 + {}^6C_6 (\sqrt{2})^0 \right] \\
 &= 2 \left[ 8 + 15(4) + 15(2) + 1 \right] \\
 &= 2 \left[ 99 \right] \\
 &= 198
 \end{aligned}$$

$$(v) (3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^5C_1 (3^4) (\sqrt{2})^1 + {}^5C_3 (3^2) (\sqrt{2})^3 + {}^5C_5 (3^0) (\sqrt{2})^5 \right] \\
 &= 2 \left[ 5(81)(\sqrt{2}) + 10(9)(2\sqrt{2}) + 4\sqrt{2} \right] \\
 &= 2\sqrt{2} (405 + 180 + 4) \\
 &= 1178\sqrt{2}
 \end{aligned}$$

$$(vi) (2 + \sqrt{3})^7 + (2 - \sqrt{3})^7$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^7C_0 (2^7) (\sqrt{3})^0 + {}^7C_2 (2^5) (\sqrt{3})^2 + {}^7C_4 (2^3) (\sqrt{3})^4 + {}^7C_6 (2^1) (\sqrt{3})^6 \right] \\
 &= 2 \left[ 128 + 21(32)(3) + 35(8)(9) + 7(2)(27) \right] \\
 &= 2 \left[ 128 + 2016 + 2520 + 378 \right] \\
 &= 2 \left[ 5042 \right] \\
 &= 10084
 \end{aligned}$$

$$(vii) (\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 [{}^5C_1 (\sqrt{3})^4 + {}^5C_3 (\sqrt{3})^2 + {}^5C_5 (\sqrt{3})^0] \\
 &= 2 [5(9) + 10(3) + 1] \\
 &= 2 [76] \\
 &= 152
 \end{aligned}$$

$$(viii) (0.99)^5 + (1.01)^5$$

Let us solve the given expression:

$$\begin{aligned}
 &= (1 - 0.01)^5 + (1 + 0.01)^5 \\
 &= 2 [{}^5C_0 (0.01)^0 + {}^5C_2 (0.01)^2 + {}^5C_4 (0.01)^4] \\
 &= 2 [1 + 10(0.0001) + 5(0.00000001)] \\
 &= 2 [1.00100005] \\
 &= 2.0020001
 \end{aligned}$$

$$(ix) (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 [{}^6C_1 (\sqrt{3})^5 (\sqrt{2})^1 + {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_5 (\sqrt{3})^1 (\sqrt{2})^5] \\
 &= 2 [6(9\sqrt{3})(\sqrt{2}) + 20(3\sqrt{3})(2\sqrt{2}) + 6(\sqrt{3})(4\sqrt{2})] \\
 &= 2 [\sqrt{6}(54 + 120 + 24)] \\
 &= 396\sqrt{6}
 \end{aligned}$$

$$(x) \left\{ a^2 + \sqrt{a^2 - 1} \right\}^4 + \left\{ a^2 - \sqrt{a^2 - 1} \right\}^4$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^4C_0 (a^2)^4 (\sqrt{a^2 - 1})^0 + {}^4C_2 (a^2)^2 (\sqrt{a^2 - 1})^2 + {}^4C_4 (a^2)^0 (\sqrt{a^2 - 1})^4 \right] \\
 &= 2 \left[ a^8 + 6a^4 (a^2 - 1) + (a^2 - 1)^2 \right] \\
 &= 2 [a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2] \\
 &= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2
 \end{aligned}$$

**3. Find  $(a + b)^4 - (a - b)^4$ . Hence, evaluate  $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$ .**

**Solution:**

Firstly, let us solve the given expression:

$$(a + b)^4 - (a - b)^4$$

The above expression can be expressed as,  
 $(a + b)^4 - (a - b)^4 = 2 [{}^4C_1 a^3b^1 + {}^4C_3 a^1b^3]$   
 $= 2 [4a^3b + 4ab^3]$   
 $= 8 (a^3b + ab^3)$

Now,

Let us evaluate the expression:

$$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$$

So consider,  $a = \sqrt{3}$  and  $b = \sqrt{2}$  we get,

$$\begin{aligned} (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8 (a^3b + ab^3) \\ &= 8 [(\sqrt{3})^3 (\sqrt{2}) + (\sqrt{3}) (\sqrt{2})^3] \\ &= 8 [(3\sqrt{6}) + (2\sqrt{6})] \\ &= 8 (5\sqrt{6}) \\ &= 40\sqrt{6} \end{aligned}$$

**4. Find  $(x + 1)^6 + (x - 1)^6$ . Hence, or otherwise evaluate  $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$ .**

**Solution:**

Firstly, let us solve the given expression:

$$(x + 1)^6 + (x - 1)^6$$

The above expression can be expressed as,

$$\begin{aligned} (x + 1)^6 + (x - 1)^6 &= 2 [{}^6C_0 x^6 + {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6 x^0] \\ &= 2 [x^6 + 15x^4 + 15x^2 + 1] \end{aligned}$$

Now,

Let us evaluate the expression:

$$(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$

So consider,  $x = \sqrt{2}$  then we get,

$$\begin{aligned} (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 &= 2 [x^6 + 15x^4 + 15x^2 + 1] \\ &= 2 [(\sqrt{2})^6 + 15 (\sqrt{2})^4 + 15 (\sqrt{2})^2 + 1] \\ &= 2 [8 + 15 (4) + 15 (2) + 1] \\ &= 2 [8 + 60 + 30 + 1] \\ &= 198 \end{aligned}$$

**5. Using binomial theorem evaluate each of the following:**

(i)  $(96)^3$

(ii)  $(102)^5$

(iii)  $(101)^4$

(iv)  $(98)^5$

**Solution:**

(i)  $(96)^3$

We have,



$$(96)^3$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned} (96)^3 &= (100 - 4)^3 \\ &= {}^3C_0 (100)^3 (4)^0 - {}^3C_1 (100)^2 (4)^1 + {}^3C_2 (100)^1 (4)^2 - {}^3C_3 (100)^0 (4)^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 884736 \end{aligned}$$

$$\text{(ii) } (102)^5$$

We have,

$$(102)^5$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned} (102)^5 &= (100 + 2)^5 \\ &= {}^5C_0 (100)^5 (2)^0 + {}^5C_1 (100)^4 (2)^1 + {}^5C_2 (100)^3 (2)^2 + {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100)^1 (2)^4 + {}^5C_5 (100)^0 (2)^5 \\ &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 \\ &= 11040808032 \end{aligned}$$

$$\text{(iii) } (101)^4$$

We have,

$$(101)^4$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned} (101)^4 &= (100 + 1)^4 \\ &= {}^4C_0 (100)^4 + {}^4C_1 (100)^3 + {}^4C_2 (100)^2 + {}^4C_3 (100)^1 + {}^4C_4 (100)^0 \\ &= 100000000 + 4000000 + 60000 + 400 + 1 \\ &= 104060401 \end{aligned}$$

$$\text{(iv) } (98)^5$$

We have,

$$(98)^5$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned} (98)^5 &= (100 - 2)^5 \\ &= {}^5C_0 (100)^5 (2)^0 - {}^5C_1 (100)^4 (2)^1 + {}^5C_2 (100)^3 (2)^2 - {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100)^1 (2)^4 - {}^5C_5 (100)^0 (2)^5 \\ &= 10000000000 - 1000000000 + 40000000 - 800000 + 8000 - 32 \\ &= 9039207968 \end{aligned}$$

**6. Using binomial theorem, prove that  $2^{3n} - 7n - 1$  is divisible by 49, where  $n \in \mathbb{N}$ .**

**Solution:**

Given:

$$2^{3n} - 7n - 1$$

So,  $2^{3n} - 7n - 1 = 8^n - 7n - 1$

Now,

$$8^n - 7n - 1$$

$$8^n = 7n + 1$$

$$= (1 + 7)^n$$

$$= {}^n C_0 + {}^n C_1 (7)^1 + {}^n C_2 (7)^2 + {}^n C_3 (7)^3 + {}^n C_4 (7)^4 + {}^n C_5 (7)^5 + \dots + {}^n C_n (7)^n$$

$$8^n = 1 + 7n + 49 [{}^n C_2 + {}^n C_3 (7^1) + {}^n C_4 (7^2) + \dots + {}^n C_n (7)^{n-2}]$$

$$8^n - 1 - 7n = 49 (\text{integer})$$

So now,

$$8^n - 1 - 7n \text{ is divisible by } 49$$

Or

$$2^{3n} - 1 - 7n \text{ is divisible by } 49.$$

Hence proved.

**EXERCISE 18.2**
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**1. Find the 11<sup>th</sup> term from the beginning and the 11<sup>th</sup> term from the end in the expansion of  $(2x - 1/x^2)^{25}$ .**

**Solution:**

Given:

$$(2x - 1/x^2)^{25}$$

The given expression contains 26 terms.

So, the 11<sup>th</sup> term from the end is the  $(26 - 11 + 1)$ <sup>th</sup> term from the beginning.

In other words, the 11<sup>th</sup> term from the end is the 16<sup>th</sup> term from the beginning.

Then,

$$\begin{aligned} T_{16} = T_{15+1} &= {}^{25}C_{15} (2x)^{25-15} (-1/x^2)^{15} \\ &= {}^{25}C_{15} (2^{10}) (x)^{10} (-1/x^{30}) \\ &= - {}^{25}C_{15} (2^{10} / x^{20}) \end{aligned}$$

Now we shall find the 11<sup>th</sup> term from the beginning.

$$\begin{aligned} T_{11} = T_{10+1} &= {}^{25}C_{10} (2x)^{25-10} (-1/x^2)^{10} \\ &= {}^{25}C_{10} (2^{15}) (x)^{15} (1/x^{20}) \\ &= {}^{25}C_{10} (2^{15} / x^5) \end{aligned}$$

**2. Find the 7<sup>th</sup> term in the expansion of  $(3x^2 - 1/x^3)^{10}$ .**

**Solution:**

Given:

$$(3x^2 - 1/x^3)^{10}$$

Let us consider the 7<sup>th</sup> term as  $T_7$

So,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^{10}C_6 (3x^2)^{10-6} (-1/x^3)^6 \\ &= {}^{10}C_6 (3)^4 (x)^8 (1/x^{18}) \\ &= [10 \times 9 \times 8 \times 7 \times 81] / [4 \times 3 \times 2 \times x^{10}] \\ &= 17010 / x^{10} \end{aligned}$$

∴ The 7<sup>th</sup> term of the expression  $(3x^2 - 1/x^3)^{10}$  is  $17010 / x^{10}$ .

**3. Find the 5<sup>th</sup> term in the expansion of  $(3x - 1/x^2)^{10}$ .**

**Solution:**

Given:

$$(3x - 1/x^2)^{10}$$

The 5<sup>th</sup> term from the end is the  $(11 - 5 + 1)$ th, is., 7<sup>th</sup> term from the beginning.

So,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^{10}C_6 (3x)^{10-6} (-1/x^2)^6 \\
 &= {}^{10}C_6 (3)^4 (x)^4 (1/x^{12}) \\
 &= [10 \times 9 \times 8 \times 7 \times 81] / [4 \times 3 \times 2 \times x^8] \\
 &= 17010 / x^8
 \end{aligned}$$

∴ The 5<sup>th</sup> term of the expression  $(3x - 1/x^2)^{10}$  is  $17010 / x^8$ .

**4. Find the 8<sup>th</sup> term in the expansion of  $(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$ .**

**Solution:**

Given:

$$(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$$

Let us consider the 8<sup>th</sup> term as  $T_8$

So,

$$\begin{aligned}
 T_8 &= T_{7+1} \\
 &= {}^{10}C_7 (x^{3/2} y^{1/2})^{10-7} (-x^{1/2} y^{3/2})^7 \\
 &= -[10 \times 9 \times 8] / [3 \times 2] x^{9/2} y^{3/2} (x^{7/2} y^{21/2}) \\
 &= -120 x^8 y^{12}
 \end{aligned}$$

∴ The 8<sup>th</sup> term of the expression  $(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$  is  $-120 x^8 y^{12}$ .

**5. Find the 7<sup>th</sup> term in the expansion of  $(4x/5 + 5/2x)^8$ .**

**Solution:**

Given:

$$(4x/5 + 5/2x)^8$$

Let us consider the 7<sup>th</sup> term as  $T_7$

So,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^8C_6 \left(\frac{4x}{5}\right)^{8-6} \left(\frac{5}{2x}\right)^6 \\
 &= \frac{8 \times 7 \times 4 \times 4 \times 125 \times 125}{2 \times 1 \times 25 \times 64} x^2 \left(\frac{1}{x^6}\right) \\
 &= \frac{4375}{x^4}
 \end{aligned}$$

∴ The 7<sup>th</sup> term of the expression  $(4x/5 + 5/2x)^8$  is  $4375/x^4$ .

**6. Find the 4<sup>th</sup> term from the beginning and 4<sup>th</sup> term from the end in the expansion of  $(x + 2/x)^9$ .**

**Solution:**

Given:

$$(x + 2/x)^9$$

Let  $T_{r+1}$  be the 4th term from the end.

Then,  $T_{r+1}$  is  $(10 - 4 + 1)$ th, i.e., 7th, term from the beginning.

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 (x^{9-6}) \left(\frac{2}{x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} (x^3) \left(\frac{64}{x^6}\right) \\ &= \frac{5376}{x^3} \end{aligned}$$

4th term from the beginning =  $T_4 = T_{3+1}$

$$\begin{aligned} T_4 &= {}^9C_3 (x^{9-3}) \left(\frac{2}{x}\right)^3 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} (x^6) \left(\frac{8}{x^3}\right) \\ &= 672 x^3 \end{aligned}$$

**7. Find the 4<sup>th</sup> term from the end in the expansion of  $(4x/5 - 5/2x)^8$ .**

**Solution:**

Given:

$$(4x/5 - 5/2x)^8$$

Let  $T_{r+1}$  be the 4th term from the end of the given expression.

Then,  $T_{r+1}$  is  $(10 - 4 + 1)$ th term, i.e., 7th term, from the beginning.

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 \left(\frac{4x}{5}\right)^{9-6} \left(\frac{5}{2x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} \left(\frac{64}{125} x^3\right) \left(\frac{125 \times 125}{64x^6}\right) \\ &= \frac{10500}{x^3} \end{aligned}$$

$\therefore$  The 4<sup>th</sup> term from the end is  $10500/x^3$ .

**8. Find the 7th term from the end in the expansion of  $(2x^2 - 3/2x)^8$ .**

**Solution:**

Given:

$$(2x^2 - 3/2x)^8$$

Let  $T_{r+1}$  be the 4th term from the end of the given expression.

Then,  $T_{r+1}$  is  $(9 - 7 + 1)$ th term, i.e., 3rd term, from the beginning.

$$\begin{aligned} T_3 &= T_{2+1} \\ &= {}^8C_2 (2x^2)^{8-2} \left(-\frac{3}{2x}\right)^2 \\ &= \frac{8 \times 7}{2 \times 1} (64x^{12}) \frac{9}{4x^2} \\ &= 4032 x^{10} \end{aligned}$$



∴ The 7<sup>th</sup> term from the end is  $4032 x^{10}$ .

**9. Find the coefficient of:**

- (i)  $x^{10}$  in the expansion of  $(2x^2 - 1/x)^{20}$
- (ii)  $x^7$  in the expansion of  $(x - 1/x^2)^{40}$
- (iii)  $x^{-15}$  in the expansion of  $(3x^2 - a/3x^3)^{10}$
- (iv)  $x^9$  in the expansion of  $(x^2 - 1/3x)^9$
- (v)  $x^m$  in the expansion of  $(x + 1/x)^n$
- (vi)  $x$  in the expansion of  $(1 - 2x^3 + 3x^5)(1 + 1/x)^8$
- (vii)  $a^5b^7$  in the expansion of  $(a - 2b)^{12}$
- (viii)  $x$  in the expansion of  $(1 - 3x + 7x^2)(1 - x)^{16}$

**Solution:**

(i)  $x^{10}$  in the expansion of  $(2x^2 - 1/x)^{20}$

Given:

$$(2x^2 - 1/x)^{20}$$

If  $x^{10}$  occurs in the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{20}C_r (2x^2)^{20-r} \left(\frac{-1}{x}\right)^r \\ &= (-1)^r {}^{20}C_r (2^{20-r}) (x^{40-2r-r}) \end{aligned}$$

For this term to contain  $x^{10}$ , we must have:

$$40 - 3r = 10$$

$$3r = 30$$

$$r = 10$$

$$\therefore \text{Coefficient of } x^{10} = (-1)^{10} {}^{20}C_{10} (2^{20-10}) = {}^{20}C_{10} (2^{10})$$

(ii)  $x^7$  in the expansion of  $(x - 1/x^2)^{40}$

Given:

$$(x - 1/x^2)^{40}$$

If  $x^7$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{40}C_r x^{40-r} \left(\frac{-1}{x^2}\right)^r \\ &= (-1)^r {}^{40}C_r x^{40-r-2r} \end{aligned}$$

For this term to contain  $x^7$ , we must have:

$$40 - 3r = 7$$

$$3r = 40 - 7$$

$$3r = 33$$

$$r = 33/3$$

$$= 11$$

$$\therefore \text{Coefficient of } x^7 = (-1)^{11} {}^{40}C_{11} = -{}^{40}C_{11}$$

(iii)  $x^{-15}$  in the expansion of  $(3x^2 - a/3x^3)^{10}$

Given:

$$(3x^2 - a/3x^3)^{10}$$

If  $x^{-15}$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{10}C_r (3x^2)^{10-r} \left(\frac{-a}{3x^3}\right)^r \\ &= (-1)^r {}^{10}C_r (3^{10-r-r}) (x^{20-2r-3r}) (a^r) \end{aligned}$$

For this term to contain  $x^{-15}$ , we must have:

$$20 - 5r = -15$$

$$5r = 20 + 15$$

$$5r = 35$$

$$r = 35/5$$

$$= 7$$

$$\therefore \text{Coefficient of } x^{-15} = (-1)^7 {}^{10}C_7 3^{10-14} (a^7) = -\frac{10 \times 9 \times 8}{3 \times 2 \times 9 \times 9} a^7 = -\frac{40}{27} a^7$$

(iv)  $x^9$  in the expansion of  $(x^2 - 1/3x)^9$

Given:

$$(x^2 - 1/3x)^9$$

If  $x^9$  occurs at the  $(r + 1)$ th term in the above expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^9C_r (x^2)^{9-r} \left(\frac{-1}{3x}\right)^r \\ &= (-1)^r {}^9C_r (x^{18-2r-r}) \left(\frac{1}{3^r}\right) \end{aligned}$$

For this term to contain  $x^9$ , we must have:

$$18 - 3r = 9$$

$$3r = 18 - 9$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

$$\therefore \text{Coefficient of } x^9 = (-1)^3 {}^9C_3 \frac{1}{3^3} = -\frac{9 \times 8 \times 7}{2 \times 9 \times 9} = -\frac{28}{9}$$

(v)  $x^m$  in the expansion of  $(x + 1/x)^n$

Given:

$$(x + 1/x)^n$$

If  $x^m$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$T_{r+1} = {}^nC_r x^{n-r} \frac{1}{x^r}$$

$$= {}^nC_r x^{n-2r}$$

For this term to contain  $x^m$ , we must have:

$$n - 2r = m$$

$$2r = n - m$$

$$r = (n - m)/2$$

$$\therefore \text{Coefficient of } x^m = {}^nC_{(n-m)/2} = \frac{n!}{\left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!}$$

(vi)  $x$  in the expansion of  $(1 - 2x^3 + 3x^5)(1 + 1/x)^8$

Given:

$$(1 - 2x^3 + 3x^5)(1 + 1/x)^8$$

If  $x$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$(1 - 2x^3 + 3x^5)(1 + 1/x)^8 = (1 - 2x^3 + 3x^5)({}^8C_0 + {}^8C_1(1/x) + {}^8C_2(1/x)^2 + {}^8C_3(1/x)^3 + {}^8C_4(1/x)^4 + {}^8C_5(1/x)^5 + {}^8C_6(1/x)^6 + {}^8C_7(1/x)^7 + {}^8C_8(1/x)^8)$$

So, 'x' occurs in the above expression at  $-2x^3 \cdot {}^8C_2(1/x^2) + 3x^5 \cdot {}^8C_4(1/x^4)$

$$\begin{aligned} \therefore \text{Coefficient of } x &= -2(8!/(2!6!)) + 3(8!/(4!4!)) \\ &= -56 + 210 \\ &= 154 \end{aligned}$$

(vii)  $a^5b^7$  in the expansion of  $(a - 2b)^{12}$

Given:

$$(a - 2b)^{12}$$

If  $a^5b^7$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{12}C_r a^{12-r} (-2b)^r \\ &= (-1)^r {}^{12}C_r (a^{12-r}) (b^r) (2^r) \end{aligned}$$

For this term to contain  $a^5b^7$ , we must have:

$$12 - r = 5$$

$$r = 12 - 5$$

$$= 7$$

$$\begin{aligned} \therefore \text{Required coefficient} &= (-1)^7 {}^{12}C_7 (2^7) \\ &= -\frac{12 \times 11 \times 10 \times 9 \times 8 \times 128}{5 \times 4 \times 3 \times 2} \\ &= -101376 \end{aligned}$$

(viii)  $x$  in the expansion of  $(1 - 3x + 7x^2)(1 - x)^{16}$

Given:

$$(1 - 3x + 7x^2)(1 - x)^{16}$$

If  $x$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$\begin{aligned} (1 - 3x + 7x^2)(1 - x)^{16} &= (1 - 3x + 7x^2) ({}^{16}C_0 + {}^{16}C_1(-x) + {}^{16}C_2(-x)^2 + {}^{16}C_3(-x)^3 + {}^{16}C_4(-x)^4 \\ &+ {}^{16}C_5(-x)^5 + {}^{16}C_6(-x)^6 + {}^{16}C_7(-x)^7 + {}^{16}C_8(-x)^8 + {}^{16}C_9(-x)^9 + {}^{16}C_{10}(-x)^{10} + {}^{16}C_{11}(-x)^{11} \\ &+ {}^{16}C_{12}(-x)^{12} + {}^{16}C_{13}(-x)^{13} + {}^{16}C_{14}(-x)^{14} + {}^{16}C_{15}(-x)^{15} + {}^{16}C_{16}(-x)^{16}) \end{aligned}$$

So, ' $x$ ' occurs in the above expression at  ${}^{16}C_1(-x) - 3x{}^{16}C_0$

$$\begin{aligned} \therefore \text{Coefficient of } x &= -(16!/(1! 15!)) - 3(16!/(0! 16!)) \\ &= -16 - 3 \\ &= -19 \end{aligned}$$

**10. Which term in the expansion of  $\left\{ \left( \frac{x}{\sqrt{y}} \right)^{1/3} + \left( \frac{y}{x^{1/3}} \right)^{1/2} \right\}^{21}$  contains  $x$  and  $y$  to one and the same power.**

**Solution:**

Let us consider  $T_{r+1}$ th term in the given expansion contains  $x$  and  $y$  to one and the same power.

Then we have,

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{21}C_r \left[ \left( \frac{x}{\sqrt{y}} \right)^{1/3} \right]^{21-r} \left[ \left( \frac{y}{x^{1/3}} \right)^{1/2} \right]^r \\ &= {}^{21}C_r \left( \frac{x^{(21-r)/3}}{x^{r/6}} \right) \left( \frac{y^{r/2}}{y^{(21-r)/6}} \right) \end{aligned}$$

$$= {}^{21}C_r (x)^{7-r/2} (y)^{2r/3-7/2}$$

If x and y have the same power, then

$$7 - r/2 = 2r/3 - 7/2$$

$$2r/3 + r/2 = 7 + 7/2$$

$$(4r + 3r)/6 = (14+7)/2$$

$$7r/6 = 21/2$$

$$r = (21 \times 6)/(2 \times 7)$$

$$= 3 \times 3$$

$$= 9$$

Hence, the required term is the 10<sup>th</sup> term.

**11. Does the expansion of  $(2x^2 - 1/x)$  contain any term involving  $x^9$ ?**

**Solution:**

Given:

$$(2x^2 - 1/x)$$

If  $x^9$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{20}C_r (2x^2)^{20-r} \left(\frac{-1}{x}\right)^r \\ &= (-1)^r {}^{20}C_r (2)^{20-r} (x)^{40-2r-r} \end{aligned}$$

For this term to contain  $x^9$ , we must have

$$40 - 3r = 9$$

$$3r = 40 - 9$$

$$3r = 31$$

$$r = 31/3$$

It is not possible, since r is not an integer.

Hence, there is no term with  $x^9$  in the given expansion.

**12. Show that the expansion of  $(x^2 + 1/x)^{12}$  does not contain any term involving  $x^{-1}$ .**

**Solution:**

Given:

$$(x^2 + 1/x)^{12}$$

If  $x^{-1}$  occurs at the  $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$



$$\begin{aligned} T_{r+1} &= {}^{12}C_r (x^2)^{12-r} \left(\frac{1}{x}\right)^r \\ &= {}^{12}C_r x^{24-2r-r} \end{aligned}$$

For this term to contain  $x^{-1}$ , we must have

$$24 - 3r = -1$$

$$3r = 24 + 1$$

$$3r = 25$$

$$r = 25/3$$

It is not possible, since  $r$  is not an integer.

Hence, there is no term with  $x^{-1}$  in the given expansion.

### 13. Find the middle term in the expansion of:

(i)  $(2/3x - 3/2x)^{20}$

(ii)  $(a/x + bx)^{12}$

(iii)  $(x^2 - 2/x)^{10}$

(iv)  $(x/a - a/x)^{10}$

**Solution:**

(i)  $(2/3x - 3/2x)^{20}$

We have,

$(2/3x - 3/2x)^{20}$  where,  $n = 20$  (even number)

So the middle term is  $(n/2 + 1) = (20/2 + 1) = (10 + 1) = 11$ . ie., 11<sup>th</sup> term

Now,

$$\begin{aligned} T_{11} &= T_{10+1} \\ &= {}^{20}C_{10} (2/3x)^{20-10} (3/2x)^{10} \\ &= {}^{20}C_{10} 2^{10}/3^{10} \times 3^{10}/2^{10} x^{10-10} \\ &= {}^{20}C_{10} \end{aligned}$$

Hence, the middle term is  ${}^{20}C_{10}$ .

(ii)  $(a/x + bx)^{12}$

We have,

$(a/x + bx)^{12}$  where,  $n = 12$  (even number)

So the middle term is  $(n/2 + 1) = (12/2 + 1) = (6 + 1) = 7$ . ie., 7<sup>th</sup> term

Now,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^{12}C_6 \left(\frac{a}{x}\right)^{12-6} (bx)^6 \\ &= {}^{12}C_6 a^6 b^6 \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2} a^6 b^6 \end{aligned}$$

$$= 924 a^6 b^6$$

Hence, the middle term is  $924 a^6 b^6$ .

(iii)  $(x^2 - 2/x)^{10}$

We have,

$$(x^2 - 2/x)^{10} \text{ where, } n = 10 \text{ (even number)}$$

So the middle term is  $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$ . i.e., 6<sup>th</sup> term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned} &= {}^{10}C_5 (x^2)^{10-5} \left(\frac{-2}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 32x^5 \\ &= -8064x^5 \end{aligned}$$

Hence, the middle term is  $-8064x^5$ .

(iv)  $(x/a - a/x)^{10}$

We have,

$$(x/a - a/x)^{10} \text{ where, } n = 10 \text{ (even number)}$$

So the middle term is  $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$ . i.e., 6<sup>th</sup> term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned} &= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\ &= -252 \end{aligned}$$

Hence, the middle term is  $-252$ .

#### 14. Find the middle terms in the expansion of:

(i)  $(3x - x^3/6)^9$

(ii)  $(2x^2 - 1/x)^7$

(iii)  $(3x - 2/x^2)^{15}$

(iv)  $(x^4 - 1/x^3)^{11}$

**Solution:**

(i)  $(3x - x^3/6)^9$

We have,

$$(3x - x^3/6)^9 \text{ where, } n = 9 \text{ (odd number)}$$

So the middle terms are  $((n+1)/2) = ((9+1)/2) = 10/2 = 5$  and

$$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$$

The terms are 5<sup>th</sup> and 6<sup>th</sup>.

Now,

$$\begin{aligned}
 T_5 &= T_{4+1} \\
 &= {}^9C_4 (3x)^{9-4} \left(\frac{-x^3}{6}\right)^4 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 27 \times 9 \times \frac{1}{36 \times 36} x^{17} \\
 &= \frac{189}{8} x^{17}
 \end{aligned}$$

And,

$$\begin{aligned}
 T_6 &= T_{5+1} \\
 &= {}^9C_5 (3x)^{9-5} \left(\frac{-x^3}{6}\right)^5 \\
 &= -\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 81 \times \frac{1}{216 \times 36} x^{19} \\
 &= -\frac{21}{16} x^{19}
 \end{aligned}$$

Hence, the middle term are  $\frac{189}{8} x^{17}$  and  $-\frac{21}{16} x^{19}$ .

(ii)  $(2x^2 - 1/x)^7$

We have,

$(2x^2 - 1/x)^7$  where,  $n = 7$  (odd number)

So the middle terms are  $((n+1)/2) = ((7+1)/2) = 8/2 = 4$  and

$((n+1)/2 + 1) = ((7+1)/2 + 1) = (8/2 + 1) = (4 + 1) = 5$

The terms are 4<sup>th</sup> and 5<sup>th</sup>.

Now,

$$\begin{aligned}
 T_4 &= T_{3+1} \\
 &= {}^7C_3 (2x^2)^{7-3} \left(\frac{-1}{x}\right)^3 \\
 &= -\frac{7 \times 6 \times 5}{3 \times 2} \times 16 x^8 \times \frac{1}{x^3} \\
 &= -560 x^5
 \end{aligned}$$

And,

$$\begin{aligned}
 T_5 &= T_{4+1} \\
 &= {}^7C_4 (2x^2)^{7-4} \left(\frac{-1}{x}\right)^4 \\
 &= 35 \times 8 \times x^6 \times \frac{1}{x^4} \\
 &= 280 x^2
 \end{aligned}$$

Hence, the middle term are  $-560x^5$  and  $280x^2$ .

(iii)  $(3x - 2/x^2)^{15}$

We have,

$(3x - 2/x^2)^{15}$  where,  $n = 15$  (odd number)

So the middle terms are  $((n+1)/2) = ((15+1)/2) = 16/2 = 8$  and

$((n+1)/2 + 1) = ((15+1)/2 + 1) = (16/2 + 1) = (8 + 1) = 9$

The terms are 8<sup>th</sup> and 9<sup>th</sup>.

Now,

$T_8 = T_{7+1}$

$$\begin{aligned} &= {}^{15}C_7 (3x)^{15-7} \left(\frac{-2}{x^2}\right)^7 \\ &= -\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \times 3^8 \times 2^7 x^{8-14} \\ &= \frac{-6435 \times 3^8 \times 2^7}{x^6} \end{aligned}$$

And,

$T_9 = T_{8+1}$

$$\begin{aligned} &= {}^{15}C_8 (3x)^{15-8} \left(\frac{-2}{x^2}\right)^8 \\ &= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \times 3^7 \times 2^8 \times x^{7-16} \\ &= \frac{6435 \times 3^7 \times 2^8}{x^9} \end{aligned}$$

Hence, the middle term are  $(-6435 \times 3^8 \times 2^7)/x^6$  and  $(6435 \times 3^7 \times 2^8)/x^9$ .

(iv)  $(x^4 - 1/x^3)^{11}$

We have,

$(x^4 - 1/x^3)^{11}$

where,  $n = 11$  (odd number)

So the middle terms are  $((n+1)/2) = ((11+1)/2) = 12/2 = 6$  and

$((n+1)/2 + 1) = ((11+1)/2 + 1) = (12/2 + 1) = (6 + 1) = 7$

The terms are 6<sup>th</sup> and 7<sup>th</sup>.

Now,

$T_6 = T_{5+1}$

$$\begin{aligned} &= {}^{11}C_5 (x^4)^{11-5} \left(\frac{-1}{x^3}\right)^5 \\ &= -\frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} \times (x)^{24-15} \\ &= -462 x^9 \end{aligned}$$

And,

$T_7 = T_{6+1}$

$$\begin{aligned}
 &= {}^{11}C_6 (x^4)^{11-6} \left(\frac{-1}{x^3}\right)^6 \\
 &= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} (x)^{20-18} \\
 &= 462 x^2
 \end{aligned}$$

Hence, the middle term are  $-462x^9$  and  $462x^2$ .

**15. Find the middle terms in the expansion of:**

- (i)  $(x - 1/x)^{10}$
- (ii)  $(1 - 2x + x^2)^n$
- (iii)  $(1 + 3x + 3x^2 + x^3)^{2n}$
- (iv)  $(2x - x^2/4)^9$
- (v)  $(x - 1/x)^{2n+1}$
- (vi)  $(x/3 + 9y)^{10}$
- (vii)  $(3 - x^3/6)^7$
- (viii)  $(2ax - b/x^2)^{12}$
- (ix)  $(p/x + x/p)^9$
- (x)  $(x/a - a/x)^{10}$

**Solution:**

(i)  $(x - 1/x)^{10}$

We have,

$(x - 1/x)^{10}$  where,  $n = 10$  (even number)

So the middle term is  $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$ . ie., 6<sup>th</sup> term

Now,

$$\begin{aligned}
 T_6 &= T_{5+1} \\
 &= {}^{10}C_5 x^{10-5} \left(\frac{-1}{x}\right)^5 \\
 &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\
 &= -252
 \end{aligned}$$

Hence, the middle term is -252.

(ii)  $(1 - 2x + x^2)^n$

We have,

$(1 - 2x + x^2)^n = (1 - x)^{2n}$  where,  $n$  is an even number.

So the middle term is  $(2n/2 + 1) = (n + 1)$ th term.

Now,

$$\begin{aligned}
 T_n &= T_{n+1} \\
 &= {}^{2n}C_n (-1)^n (x)^n \\
 &= (2n)!/(n!)^2 (-1)^n x^n
 \end{aligned}$$



Hence, the middle term is  $(2n)!/(n!)^2 (-1)^n x^n$ .

**(iii)**  $(1 + 3x + 3x^2 + x^3)^{2n}$

We have,

$(1 + 3x + 3x^2 + x^3)^{2n} = (1 + x)^{6n}$  where,  $n$  is an even number.

So the middle term is  $(n/2 + 1) = (6n/2 + 1) = (3n + 1)$ th term.

Now,

$$\begin{aligned} T_{2n} &= T_{3n+1} \\ &= {}^{6n}C_{3n} x^{3n} \\ &= (6n)!/(3n!)^2 x^{3n} \end{aligned}$$

Hence, the middle term is  $(6n)!/(3n!)^2 x^{3n}$ .

**(iv)**  $(2x - x^2/4)^9$

We have,

$(2x - x^2/4)^9$  where,  $n = 9$  (odd number)

So the middle terms are  $((n+1)/2) = ((9+1)/2) = 10/2 = 5$  and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5<sup>th</sup> and 6<sup>th</sup>.

Now,

$T_5 = T_{4+1}$

$$\begin{aligned} &= {}^9C_4 (2x)^{9-4} \left(\frac{-x^2}{4}\right)^4 \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 2^5 \frac{1}{4^4} x^{5+8} \\ &= \frac{63}{4} x^{13} \end{aligned}$$

And,

$T_6 = T_{5+1}$

$$\begin{aligned} &= {}^9C_5 (2x)^{9-5} \left(\frac{-x^2}{4}\right)^5 \\ &= -\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 2^4 \frac{1}{4^5} x^{4+10} \\ &= -\frac{63}{32} x^{14} \end{aligned}$$

Hence, the middle term is  $63/4 x^{13}$  and  $-63/32 x^{14}$ .

**(v)**  $(x - 1/x)^{2n+1}$

We have,

$(x - 1/x)^{2n+1}$  where,  $n = (2n + 1)$  is an (odd number)

So the middle terms are  $((n+1)/2) = ((2n+1+1)/2) = (2n+2)/2 = (n + 1)$  and

$$((n+1)/2 + 1) = ((2n+1+1)/2 + 1) = ((2n+2)/2 + 1) = (n + 1 + 1) = (n + 2)$$

The terms are  $(n + 1)^{\text{th}}$  and  $(n + 2)^{\text{th}}$ .

Now,

$$\begin{aligned} T_n &= T_{n+1} \\ &= {}^{2n+1}C_n x^{2n+1-n} \times \frac{(-1)^n}{x^n} \\ &= (-1)^n {}^{2n+1}C_n x \end{aligned}$$

And,

$$\begin{aligned} T_{n+2} &= T_{n+1+1} \\ &= {}^{2n+1}C_n x^{2n+1-n-1} \frac{(-1)^{n+1}}{x^{n+1}} \\ &= (-1)^{n+1} {}^{2n+1}C_n \times \frac{1}{x} \end{aligned}$$

Hence, the middle term is  $(-1)^n \cdot {}^{2n+1}C_n x$  and  $(-1)^{n+1} \cdot {}^{2n+1}C_n (1/x)$ .

**(vi)**  $(x/3 + 9y)^{10}$

We have,

$(x/3 + 9y)^{10}$  where,  $n = 10$  is an even number.

So the middle term is  $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$ . i.e., 6th term.

Now,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5 \\ &= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times \frac{1}{3^5} \times 9^5 \times x^5 y^5 \\ &= 61236 x^5 y^5 \end{aligned}$$

Hence, the middle term is  $61236x^5y^5$ .

**(vii)**  $(3 - x^3/6)^7$

We have,

$(3 - x^3/6)^7$  where,  $n = 7$  (odd number).

So the middle terms are  $((n+1)/2) = ((7+1)/2) = 8/2 = 4$  and

$((n+1)/2 + 1) = ((7+1)/2 + 1) = (8/2 + 1) = (4 + 1) = 5$

The terms are 4<sup>th</sup> and 5<sup>th</sup>.

Now,

$$\begin{aligned} T_4 &= T_{3+1} \\ &= {}^7C_3 (3)^{7-3} (-x^3/6)^3 \\ &= -105/8 x^9 \end{aligned}$$

And,

$$T_5 = T_{4+1}$$

$$\begin{aligned}
 &= {}^9C_4 (3)^{9-4} (-x^3/6)^4 \\
 &= \frac{7 \times 6 \times 5}{3 \times 2} \times 3^5 \times \frac{1}{6^4} x^{12} \\
 &= \frac{35}{48} x^{12}
 \end{aligned}$$

Hence, the middle terms are  $-105/8 x^9$  and  $35/48 x^{12}$ .

**(viii)**  $(2ax - b/x^2)^{12}$

We have,

$(2ax - b/x^2)^{12}$  where,  $n = 12$  is an even number.

So the middle term is  $(n/2 + 1) = (12/2 + 1) = (6 + 1) = 7$ . i.e., 7th term.

Now,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^{12}C_6 (2ax)^{12-6} \left(\frac{-b}{x^2}\right)^6 \\
 &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \times \left(\frac{2ab}{x}\right)^6 \\
 &= \frac{59136 a^6 b^6}{x^6}
 \end{aligned}$$

Hence, the middle term is  $(59136a^6b^6)/x^6$ .

**(ix)**  $(p/x + x/p)^9$

We have,

$(p/x + x/p)^9$  where,  $n = 9$  (odd number).

So the middle terms are  $((n+1)/2) = ((9+1)/2) = 10/2 = 5$  and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5<sup>th</sup> and 6<sup>th</sup>.

Now,

$$\begin{aligned}
 T_5 &= T_{4+1} \\
 &= {}^9C_4 \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \left(\frac{p}{x}\right) \\
 &= \frac{126 p}{x}
 \end{aligned}$$

And,

$$\begin{aligned}
 T_6 &= T_{5+1} \\
 &= {}^9C_5 (p/x)^{9-5} (x/p)^5 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \left(\frac{x}{p}\right) \\
 &= \frac{126 x}{p}
 \end{aligned}$$

Hence, the middle terms are  $126p/x$  and  $126x/p$ .

$$(x) (x/a - a/x)^{10}$$

We have,

$$(x/a - a/x)^{10} \text{ where, } n = 10 \text{ (even number)}$$

So the middle term is  $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$ . ie., 6<sup>th</sup> term

Now,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\ &= -252 \end{aligned}$$

Hence, the middle term is -252.

**16. Find the term independent of x in the expansion of the following expressions:**

(i)  $(3/2 x^2 - 1/3x)^9$

(ii)  $(2x + 1/3x^2)^9$

(iii)  $(2x^2 - 3/x^3)^{25}$

(iv)  $(3x - 2/x^2)^{15}$

(v)  $((\sqrt{x}/3) + \sqrt{3/2x^2})^{10}$

(vi)  $(x - 1/x^2)^{3n}$

(vii)  $(1/2 x^{1/3} + x^{-1/5})^8$

(viii)  $(1 + x + 2x^3)(3/2x^2 - 3/3x)^9$

(ix)  $(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}$ ,  $x > 0$

(x)  $(3/2x^2 - 1/3x)^6$

**Solution:**

(i)  $(3/2 x^2 - 1/3x)^9$

Given:

$$(3/2 x^2 - 1/3x)^9$$

If  $(r + 1)$ th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r \\ &= (-1)^r {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}} \times x^{18-2r-r} \end{aligned}$$

For this term to be independent of x, we must have

$$18 - 3r = 0$$

$$3r = 18$$

$$r = 18/3$$

$$= 6$$

So, the required term is 7<sup>th</sup> term.

We have,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 \times (3^{9-12})/(2^{9-6}) \\ &= (9 \times 8 \times 7)/(3 \times 2) \times 3^{-3} \times 2^{-3} \\ &= 7/18 \end{aligned}$$

Hence, the term independent of x is 7/18.

**(ii)**  $(2x + 1/3x^2)^9$

Given:

$$(2x + 1/3x^2)^9$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^9C_r (2x)^{9-r} \left(\frac{1}{3x^2}\right)^r \\ &= {}^9C_r \cdot \frac{2^{9-r}}{3^r} x^{9-r-2r} \end{aligned}$$

For this term to be independent of x, we must have

$$9 - 3r = 0$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

So, the required term is 4<sup>th</sup> term.

We have,

$$\begin{aligned} T_4 &= T_{3+1} \\ &= {}^9C_3 \times (2^6)/(3^3) \\ &= {}^9C_3 \times 64/27 \end{aligned}$$

Hence, the term independent of x is  ${}^9C_3 \times 64/27$ .

**(iii)**  $(2x^2 - 3/x^3)^{25}$

Given:

$$(2x^2 - 3/x^3)^{25}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^{25}C_r (2x^2)^{25-r} (-3/x^3)^r \\ &= (-1)^r {}^{25}C_r \times 2^{25-r} \times 3^r x^{50-2r-3r} \end{aligned}$$

For this term to be independent of x, we must have

$$50 - 5r = 0$$

$$5r = 50$$

$$r = 50/5$$

$$= 10$$

So, the required term is 11<sup>th</sup> term.

We have,

$$\begin{aligned} T_{11} &= T_{10+1} \\ &= (-1)^{10} {}^{25}C_{10} \times 2^{25-10} \times 3^{10} \\ &= {}^{25}C_{10} (2^{15} \times 3^{10}) \end{aligned}$$

Hence, the term independent of x is  ${}^{25}C_{10} (2^{15} \times 3^{10})$ .

**(iv)**  $(3x - 2/x^2)^{15}$

Given:

$$(3x - 2/x^2)^{15}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^{15}C_r (3x)^{15-r} (-2/x^2)^r \\ &= (-1)^r {}^{15}C_r \times 3^{15-r} \times 2^r x^{15-r-2r} \end{aligned}$$

For this term to be independent of x, we must have

$$15 - 3r = 0$$

$$3r = 15$$

$$r = 15/3$$

$$= 5$$

So, the required term is 6<sup>th</sup> term.

We have,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= (-1)^5 {}^{15}C_5 \times 3^{15-5} \times 2^5 \\ &= -3003 \times 3^{10} \times 2^5 \end{aligned}$$

Hence, the term independent of x is  $-3003 \times 3^{10} \times 2^5$ .

**(v)**  $((\sqrt{x}/3) + \sqrt{3}/2x^2)^{10}$

Given:

$$((\sqrt{x}/3) + \sqrt{3}/2x^2)^{10}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$



$$\begin{aligned}
 &= {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r \\
 &= {}^{10}C_r \cdot \frac{3^{\frac{r}{2}-\frac{10-r}{2}}}{2^r} x^{\frac{10-r}{2}-2r}
 \end{aligned}$$

For this term to be independent of  $x$ , we must have

$$(10-r)/2 - 2r = 0$$

$$10 - 5r = 0$$

$$5r = 10$$

$$r = 10/5$$

$$= 2$$

So, the required term is 3<sup>rd</sup> term.

We have,

$$T_3 = T_{2+1}$$

$$\begin{aligned}
 &= {}^{10}C_2 \times \frac{3^{\frac{2}{2}-\frac{10-2}{2}}}{2^2} \\
 &= \frac{10 \times 9}{2 \times 4 \times 9} \\
 &= 90/72 \\
 &= 15/12 \\
 &= 5/4
 \end{aligned}$$

Hence, the term independent of  $x$  is  $5/4$ .

**(vi)**  $(x - 1/x^2)^{3n}$

Given:

$$(x - 1/x^2)^{3n}$$

If  $(r + 1)$ th term in the given expression is independent of  $x$ .

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^{3n}C_r x^{3n-r} (-1/x^2)^r \\
 &= {}^{3n}C_r x^{3n-r} (-1)^r x^{-2r} \\
 &= (-1)^r {}^{3n}C_r x^{3n-r-2r}
 \end{aligned}$$

For this term to be independent of  $x$ , we must have

$$3n - 3r = 0$$

$$r = n$$

So, the required term is  $(n+1)$ th term.

We have,

$$(-1)^n {}^{3n}C_n$$

Hence, the term independent of  $x$   $(-1)^n {}^{3n}C_n$

**(vii)**  $(1/2 x^{1/3} + x^{-1/5})^8$

Given:

$$\left(\frac{1}{2}x^{1/3} + x^{-1/5}\right)^8$$

If  $(r + 1)$ th term in the given expression is independent of  $x$ .

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^n C_r x^{n-r} a^r \\ &= {}^8 C_r \left(\frac{1}{2}x^{1/3}\right)^{8-r} \left(x^{-1/5}\right)^r \\ &= {}^8 C_r \cdot \frac{1}{2^{8-r}} x^{\frac{8-r}{3} - \frac{r}{5}} \end{aligned}$$

For this term to be independent of  $x$ , we must have

$$(8-r)/3 - r/5 = 0$$

$$(40 - 5r - 3r)/15 = 0$$

$$40 - 5r - 3r = 0$$

$$40 - 8r = 0$$

$$8r = 40$$

$$r = 40/8$$

$$= 5$$

So, the required term is 6th term.

We have,

$$\begin{aligned} T_5 &= T_{5+1} \\ &= {}^8 C_5 \times 1/(2^{8-5}) \\ &= (8 \times 7 \times 6)/(3 \times 2 \times 8) \\ &= 7 \end{aligned}$$

Hence, the term independent of  $x$  is 7.

**(viii)**  $(1 + x + 2x^3)(3/2x^2 - 3/3x)^9$

Given:

$$(1 + x + 2x^3)(3/2x^2 - 3/3x)^9$$

If  $(r + 1)$ th term in the given expression is independent of  $x$ .

Then, we have:

$$\begin{aligned} &(1 + x + 2x^3)(3/2x^2 - 3/3x)^9 = \\ &= (1 + x + 2x^3) \left[ \left(\frac{3}{2}x^2\right)^9 {}^{-9} C_1 \left(\frac{3}{2}x^2\right)^8 \frac{1}{3x} \dots + {}^9 C_6 \left(\frac{3}{2}x^2\right)^3 \left(\frac{1}{3x}\right)^6 - {}^9 C_7 \left(\frac{3}{2}x^2\right)^2 \left(\frac{1}{3x}\right)^7 \right] \end{aligned}$$

By computing we get,

The term independent of  $x$

$$\begin{aligned} &= 1 \left[ {}^9 C_6 \frac{3^3}{2^3} \times \frac{1}{3^6} \right] - 2x^3 \left[ {}^9 C_7 \frac{3^3}{2^3} \times \frac{1}{3^7} \times \frac{1}{x^3} \right] \\ &= \left[ \frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{1}{8 \times 27} \right] - 2 \left[ \frac{9 \times 8}{1 \times 2} - \frac{1}{4 \times 243} \right] \end{aligned}$$

$$\begin{aligned}
 &= 7/18 - 2/27 \\
 &= (189 - 36)/486 \\
 &= 153/486 \text{ (divide by 9)} \\
 &= 17/54
 \end{aligned}$$

Hence, the term independent of  $x$  is  $17/54$ .

**(ix)**  $(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}$ ,  $x > 0$

Given:

$$(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}, x > 0$$

If  $(r + 1)$ th term in the given expression is independent of  $x$ .

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^nC_r x^{n-r} a^r \\
 &= {}^{18}C_r \left(x^{1/3}\right)^{18-r} \left(\frac{1}{2x^{1/3}}\right)^r \\
 &= {}^{18}C_r \times \frac{1}{2^r} x^{\frac{18-r}{3} - \frac{r}{3}}
 \end{aligned}$$

For this term to be independent of  $r$ , we must have

$$(18-r)/3 - r/3 = 0$$

$$(18 - r - r)/3 = 0$$

$$18 - 2r = 0$$

$$2r = 18$$

$$r = 18/2$$

$$= 9$$

So, the required term is 10th term.

We have,

$$\begin{aligned}
 T_{10} &= T_{9+1} \\
 &= {}^{18}C_9 \times 1/2^9
 \end{aligned}$$

Hence, the term independent of  $x$  is  ${}^{18}C_9 \times 1/2^9$ .

**(x)**  $(3/2x^2 - 1/3x)^6$

Given:

$$(3/2x^2 - 1/3x)^6$$

If  $(r + 1)$ th term in the given expression is independent of  $x$ .

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^nC_r x^{n-r} a^r \\
 &= {}^6C_r \left(\frac{3}{2}x^2\right)^{6-r} \left(\frac{-1}{3x}\right)^r \\
 &= (-1)^r {}^6C_r \times \frac{3^{6-r-r}}{2^{6-r}} x^{12-2r-r}
 \end{aligned}$$

For this term to be independent of  $r$ , we must have

$$12 - 3r = 0$$

$$3r = 12$$

$$r = 12/3$$

$$= 4$$

So, the required term is 5th term.

We have,

$$T_5 = T_{4+1}$$

$$= {}^6C_4 \times \frac{3^{6-4-4}}{2^{6-4}}$$

$$= \frac{6 \times 5}{2 \times 1 \times 4 \times 9}$$

$$= \frac{5}{12}$$

Hence, the term independent of  $x$  is  $5/12$ .

**17. If the coefficients of  $(2r + 4)$ th and  $(r - 2)$ th terms in the expansion of  $(1 + x)^{18}$  are equal, find  $r$ .**

**Solution:**

Given:

$$(1 + x)^{18}$$

We know, the coefficient of the  $r$  term in the expansion of  $(1 + x)^n$  is  ${}^nC_{r-1}$

So, the coefficients of the  $(2r + 4)$  and  $(r - 2)$  terms in the given expansion are  ${}^{18}C_{2r+4-1}$  and  ${}^{18}C_{r-2-1}$

For these coefficients to be equal, we must have

$${}^{18}C_{2r+4-1} = {}^{18}C_{r-2-1}$$

$${}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$$2r + 3 = r - 3 \text{ (or) } 2r + 3 + r - 3 = 18 \text{ [Since, } {}^nC_r = {}^nC_s \Rightarrow r = s \text{ (or) } r + s = n]$$

$$2r - r = -3 - 3 \text{ (or) } 3r = 18 - 3 + 3$$

$$r = -6 \text{ (or) } 3r = 18$$

$$r = -6 \text{ (or) } r = 18/3$$

$$r = -6 \text{ (or) } r = 6$$

$\therefore r = 6$  [since,  $r$  should be a positive integer.]

**18. If the coefficients of  $(2r + 1)$ th term and  $(r + 2)$ th term in the expansion of  $(1 + x)^{43}$  are equal, find  $r$ .**

**Solution:**

Given:

$$(1 + x)^{43}$$

We know, the coefficient of the  $r$  term in the expansion of  $(1 + x)^n$  is  ${}^nC_{r-1}$

So, the coefficients of the  $(2r + 1)$  and  $(r + 2)$  terms in the given expansion are  ${}^{43}C_{2r+1-1}$  and  ${}^{43}C_{r+2-1}$

For these coefficients to be equal, we must have

$${}^{43}C_{2r+1-1} = {}^{43}C_{r+2-1}$$

$${}^{43}C_{2r} = {}^{43}C_{r+1}$$

$$2r = r + 1 \text{ (or) } 2r + r + 1 = 43 \text{ [Since, } {}^nC_r = {}^nC_s \Rightarrow r = s \text{ (or) } r + s = n]$$

$$2r - r = 1 \text{ (or) } 3r + 1 = 43$$

$$r = 1 \text{ (or) } 3r = 43 - 1$$

$$r = 1 \text{ (or) } 3r = 42$$

$$r = 1 \text{ (or) } r = 42/3$$

$$r = 1 \text{ (or) } r = 14$$

$\therefore r = 14$  [since, value '1' gives the same term]

**19. Prove that the coefficient of  $(r + 1)$ th term in the expansion of  $(1 + x)^{n+1}$  is equal to the sum of the coefficients of  $r$ th and  $(r + 1)$ th terms in the expansion of  $(1 + x)^n$ .**

**Solution:**

We know, the coefficients of  $(r + 1)$ th term in  $(1 + x)^{n+1}$  is  ${}^{n+1}C_r$

So, sum of the coefficients of the  $r$ th and  $(r + 1)$ th terms in  $(1 + x)^n$  is

$$\begin{aligned} (1 + x)^n &= {}^nC_{r-1} + {}^nC_r \\ &= {}^{n+1}C_r \text{ [since, } {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}] \end{aligned}$$

Hence proved.