EXERCISE 19.6 PAGE NO: 19.46

1. Find the A.M. between:
(i) 7 and 13 (ii) 12 and - 8 (iii) (x - y) and (x + y)
Solution:
(i) Let A be the Arithmetic mean
Then 7, A, 13 are in AP
Now, let us solve
A-7 = 13-A
2A = 13 + 7
A = 10

(ii) Let A be the Arithmetic mean
Then 12, A, - 8 are in AP
Now, let us solve
A - 12 = - 8 - A
2A = 12 + 8
A = 2

(iii) Let A be the Arithmetic mean
Then x - y, A, x + y are in AP
Now, let us solve
A - (x - y) = (x + y) – A
2A = x + y + x - y
A = x

2. Insert 4 A.M.s between 4 and 19.
Solution:
Let A_1, A_2, A_3, A_4 be the 4 AM Between 4 and 19
Then, 4, A_1, A_2, A_3, A_4, 19 are in AP.
By using the formula,
d = (b-a) / (n+1)
= (19 - 4) / (4 + 1)
= 15/5
= 3
So,
A_1 = a + d = 4 + 3 = 7
A_2 = A1 + d = 7 + 3 = 10
A_3 = A2 + d = 10 + 3 = 13

$A_4 = A3 + d = 13 + 3 = 16$

3. Insert 7 A.M.s between 2 and 17.

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be the 7 AMs between 2 and 17

Then, $2, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 17$ are in AP

By using the formula,

$a_n = a + (n - 1)d$

$a_n = 17, a = 2, n = 9$

so,

$17 = 2 + (9 - 1)d$

$17 = 2 + 9d – d$

$17 = 2 + 8d$

$8d = 17 – 2$

$8d = 15$

$d = 15/8$

So,

$A_1 = a + d = 2 + 15/8 = 31/8$

$A_2 = A_1 + d = 31/8 + 15/8 = 46/8$

$A_3 = A_2 + d = 46/8 + 15/8 = 61/8$

$A_4 = A_3 + d = 61/8 + 15/8 = 76/8$

$A_5 = A_4 + d = 76/8 + 15/8 = 91/8$

$A_6 = A_5 + d = 91/8 + 15/8 = 106/8$

$A_7 = A_6 + d = 106/8 + 15/8 = 121/8$

∴ the 7 AMs between 2 and 7 are 31/8, 46/8, 61/8, 76/8, 91/8, 106/8, 121/8

4. Insert six A.M.s between 15 and - 13.

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the 7 AM between 15 and - 13

Then, $15, A_1, A_2, A_3, A_4, A_5, A_6, - 13$ are in AP

By using the formula,

$a_n = a + (n - 1)d$

$a_n = -13, a = 15, n = 8$

so,

$-13 = 15 + (8 - 1)d$

$-13 = 15 + 7d$

$7d = -13 – 15$

$7d = -28$

$d = -4$

So,
$A_1 = a + d = 15 - 4 = 11$
$A_2 = A1 + d = 11 - 4 = 7$
$A_3 = A2 + d = 7 - 4 = 3$
$A_4 = A3 + d = 3 - 4 = -1$
$A_5 = A4 + d = -1 - 4 = -5$
$A_6 = A5 + d = -5 - 4 = -9$

5. There are n A.M.s between 3 and 17. The ratio of the last mean to the first mean is 3: 1. Find the value of n.
Solution:
Let the series be 3, A_1, A_2, A_3, ..., A_n, 17
Given, $a_n/a_1 = 3/1$
We know total terms in AP are n + 2
So, 17 is the (n + 2)th term
By using the formula,
$A_n = a + (n - 1)d$
$A_n = 17, a = 3$
So, $17 = 3 + (n + 2 - 1)d$
$17 = 3 + (n + 1)d$
$17 – 3 = (n + 1)d$
$14 = (n + 1)d$
$d = 14/(n+1)$
Now,
$A_n = 3 + 14/(n+1) = (17n + 3) / (n+1)$
$A_1 = 3 + d = (3n+17)/(n+1)$
Since,
$a_n/a_1 = 3/1$
$(17n + 3)/ (3n+17) = 3/1$
$17n + 3 = 3(3n + 17)$
$17n + 3 = 9n + 51$
$17n – 9n = 51 – 3$
$8n = 48$
$n = 48/8$
$= 6$
∴ There are 6 terms in the AP

6. Insert A.M.s between 7 and 71 in such a way that the 5^{th} A.M. is 27. Find the number of A.M.s.

Solution:

Let the series be 7, A_1, A_2, A_3, ..., A_n, 71

We know total terms in AP are n + 2

So 71 is the (n + 2)th term

By using the formula,

$A_n = a + (n - 1)d$

$A_n = 71$, n = 6

$A_6 = a + (6 - 1)d$

a + 5d = 27 (5^{th} term)

d = 4

so,

71 = (n + 2)th term

71 = a + (n + 2 - 1)d

71 = 7 + n(4)

n = 15

∴ There are 15 terms in AP

7. If n A.M.s are inserted between two numbers, prove that the sum of the means equidistant from the beginning and the end is constant.

Solution:

Let a and b be the first and last terms

The series be a, A_1, A_2, A_3, ..., A_n, b

We know, Mean = (a+b)/2

Mean of A_1 and A_n = $(A_1 + A_n)/2$

A_1 = a+d

A_n = a – d

So, AM = (a+d+b-d)/2

= (a+b)/2

AM between A_2 and A_{n-1} = (a+2d+b-2d)/2

= (a+b)/2

Similarly, (a + b)/2 is constant for all such numbers

Hence, AM = (a + b)/2

8. If x, y, z are in A.P. and $A_1$is the A.M. of x and y, and A_2 is the A.M. of y and z, then prove that the A.M. of A_1 and A_2 is y.

Solution:

Given that,

A_1 = AM of x and y

And A_2 = AM of y and z

So, $A_1 = (x+y)/2$
$A_2 = (y+x)/2$
AM of A_1 and $A_2 = (A_1 + A_2)/2$
$= [(x+y)/2 + (y+z)/2]/2$
$= [x+y+y+z]/2$
$= [x+2y+z]/2$
Since x, y, z are in AP, $y = (x+z)/2$
$AM = [(x + z/2) + (2y/2)]/2$
$= (y + y)/2$
$= 2y/2$
$= y$
Hence proved.

9. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P
Solution:
Let A_1, A_2, A_3, A_4, A_5 be the 5 numbers between 8 and 26
Then, 8, A_1, A_2, A_3, A_4, A_5, 26 are in AP
By using the formula,
$A_n = a + (n - 1)d$
$A_n = 26$, $a = 8$, $n = 7$
$26 = 8 + (7 - 1)d$
$26 = 8 + 6d$
$6d = 26 – 8$
$6d = 18$
$d = 18/6$
$= 3$
So,
$A_1 = a + d = 8 + 3 = 11$
$A_2 = A_1 + d = 11 + 3 = 14$
$A_3 = A_2 + d = 14 + 3 = 17$
$A_4 = A_3 + d = 17 + 3 = 20$
$A_5 = A_4 + d = 20 + 3 = 23$