Areas Related to Circles

Introduction

Area of a Circle

Area of a circle is πr^2, where $\pi = \frac{22}{7}$ or ≈ 3.14 (can be used interchangeably for problem solving purposes)and r is the radius of the circle.
π is the ratio of the circumference of a circle to its diameter.

Circumference of a circle

The perimeter of a circle is the distance covered by going around its boundary once. The perimeter of a circle has a special name: CIrcumference, which is π times the diameter which is given by the formula $2\pi r$

Segment of a circle

A circular segment is a region of a circle which is "cut off" from the rest of the circle by a secant or a chord

Sector of a circle

A circular sector or circle sector, is the portion of a circle enclosed by two radii and an arc, where the smaller area is known as the minor sector and the larger being the major sector.

Angle of a Sector

Angle of a sector is that angle which is enclosed between the two radii of the sector.

Length of arc of a sector

The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$

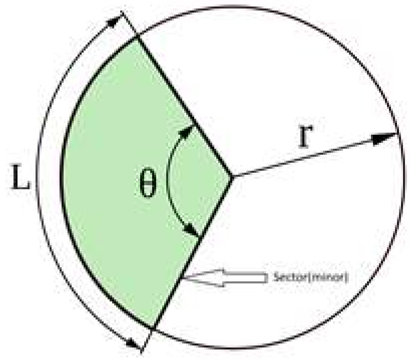
where θ is the angle of sector and r is the radius of the circle.

Area of a Sector of a Circle

Area of a sector is given by

$$\frac{\theta}{360^\circ} \times \pi r^2$$

where $\angle\theta$ is the angle of this sector(minor sector in the following case) and r is its radius

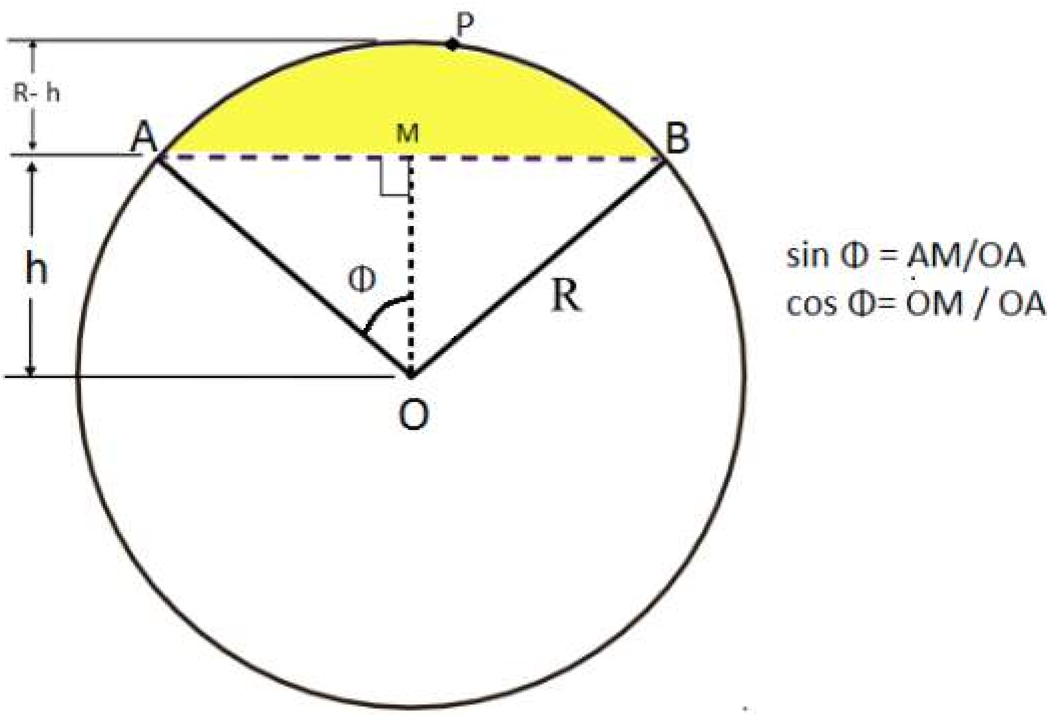


Area of a sector

Area of a Triangle

Area of a triangle is,
$Area = \frac{1}{2} \times base \times height$
If the triangle is a equilateral then
$Area = \frac{\sqrt{3}}{4} \times a^2$ where a is the side of the triangle.

Area of a Segment of a Circle



Area of the segment

Area of segment APB (highlighted in yellow)
= (Area of sector OAPB) - (Area of triangle AOB)

$$= (\frac{2\phi}{360^\circ} \times \pi r^2) - (\frac{1}{2} \times AB \times OM)$$

[To find the area of triangle AOB, use trigonometric ratios to find OM (height) and AB (base)]

Also, Area of segment APB can be calculated directly if the angle of the sector is known using the following formula.

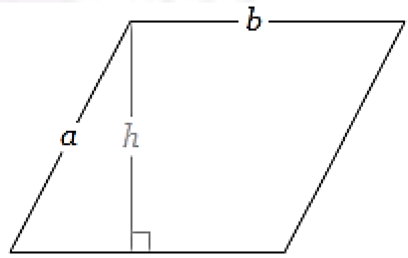
$$= (\frac{\theta}{360^\circ} \times \pi r^2) - (r^2 \times sin\frac{\theta}{2} \times cos\frac{\theta}{2})$$

where θ is the angle of the sector and r is the radius of the circle

Visualisations

Areas of different plane figures

- Area of a square (side l) $= l^2$
- Area of a rectangle $= l \times b$, where l and b are the length and breadth of the rectangle
- Area of a parallelogram $= b \times h$, where b is the base and h is perpendicular height.

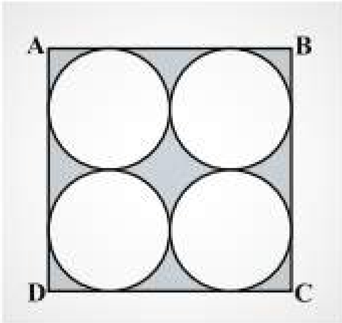


parallelogram

Area of a trapezium $= \frac{(a+b)}{2} \times h$, where a & b are the lengths of the parallel sides and h is the height of the trapezium
Area of a rhombus $= \frac{pq}{2}$, where p & q are the diagonals

Areas of Combination of Plane figures

For example : Find the area of the shaded part in the following figure : Given the ABCD is a square of side 28cm and has four equal circles enclosed within.



Area of shaded region

- Looking at the figure we can visualise that the required shaded area = $A(square\ ABCD)\ -\ 4\ \times A(Circle)$.
- Also, the diameter of each circle is 14 cm.

$= (l^2) - 4 \times (\pi r^2)$

$= (28^2) - [4 \times (\pi \times 49)]$

$= 784 - [4 \times \frac{22}{7} \times 49]$

$= 784 - 616$

$= 168 cm^2$