Pair of Linear Equations in Two Variables

Basics Revisited

Equation

An equation is a statement that two mathematical expressions having one or more variables are equal.

Linear Equation

Equations in which the powers of all the variables involved are one are called linear equations. The degree of a linear equation is always one.

General form of a Linear Equation in Two Variables

The general form of a linear equation in two variables is $ax + by + c = 0$, where a and b cannot be zero simultaneously.

Representing linear equations for a word problem

To represent a word problem as a linear equation

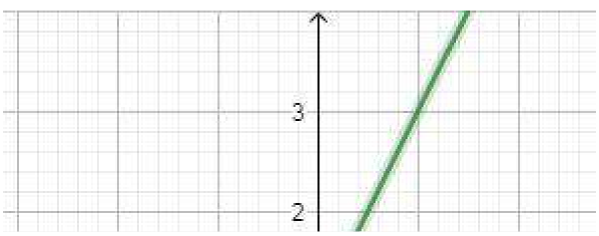
- Identify unknown quantities and denote them by variables.
- Represent the relationships between quantities in a mathematical form, replacing the unknowns with variables.

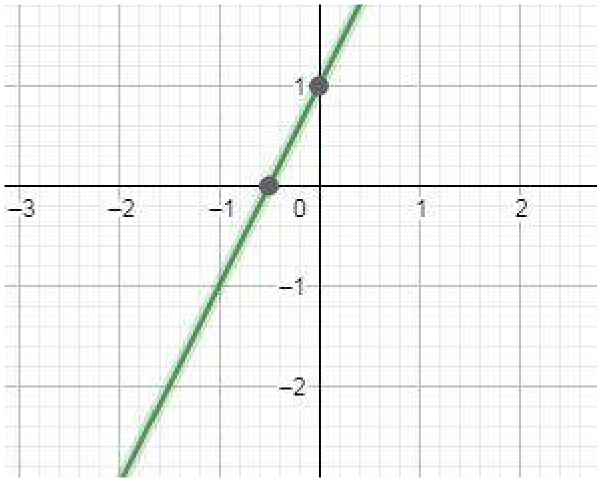
Solution of a Linear Equation in 2 variables

The solution of a linear equation in two variables is a pair of values, one for x and the other for y, which makes the two sides of the equation equal.
Eg: If 2x+y=4, then (0,4) is one of its solutions as it satisfies the equation. A linear equation in two variables has infinitely many solutions.

Geometrical Representation of a Linear Equation

Geometrically, a linear equation in two variables can be represented as a straight line.
$2x - y + 1 = 0$
$\Rightarrow y = 2x + 1$





Graph of y = 2x + 1

Plotting a Straight Line

The graph of a linear equation in two variables is a straight line. We plot the straight line as follows:

- Take any value for one of the variables($x_1 = 0$) and substitute it in the equation to get the corresponding value of the other variable (y_1).
- Repeat this again (put $y_2 = 0, get\ x_2$) to get two pairs of values for the variables which represent two points on the Cartesian plane. Draw a line through the two points.

Any additional points plotted in this manner will lie on the same line.

All about Lines

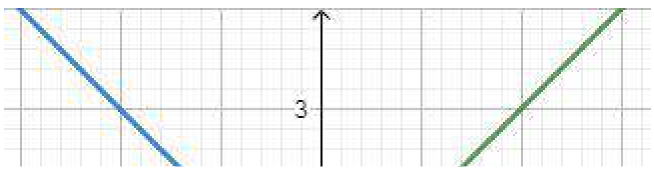
General form of a pair of linear equations in 2 variables

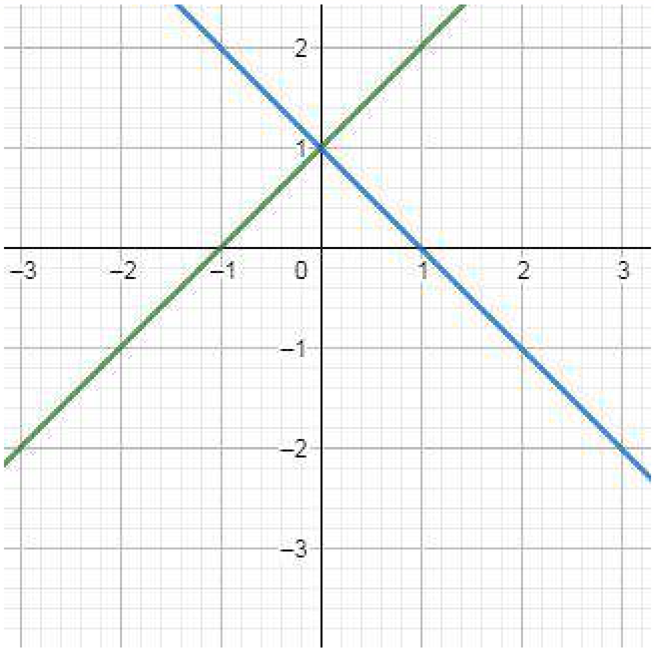
A pair of linear equations in two variables can be represented as follows
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
The coefficients of x and y cannot be zero simultaneously for an equation.

Nature of 2 straight lines in a plane

For a pair of straight lines on a plane, there are three possibilities

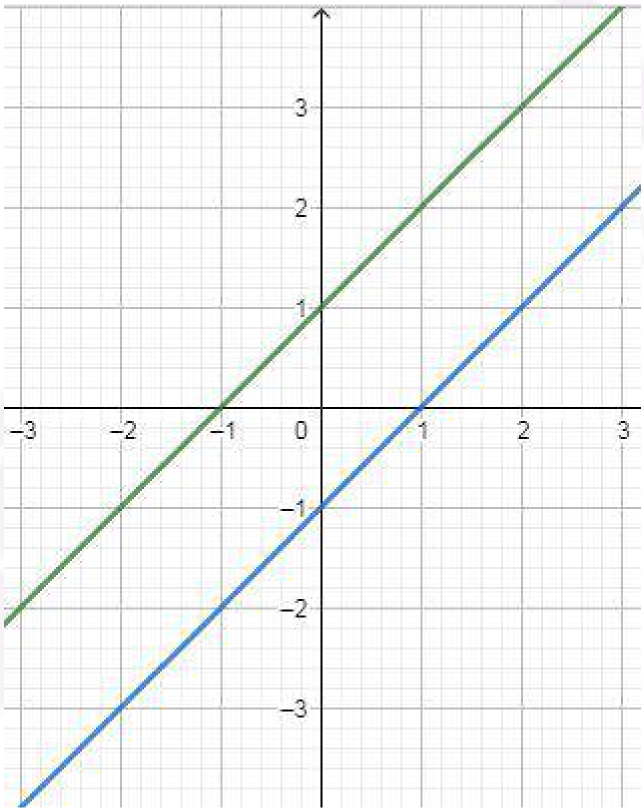
i) They intersect at exactly one point





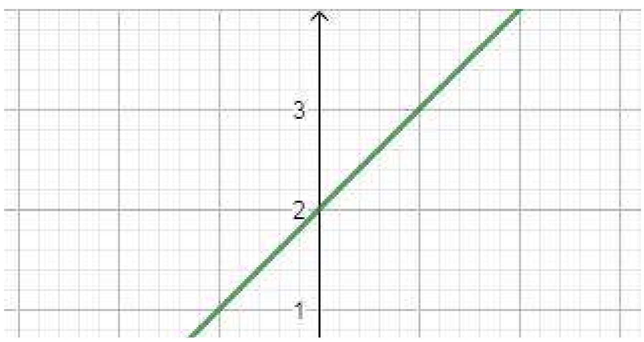
pair of linear equations which intersect at a single point.

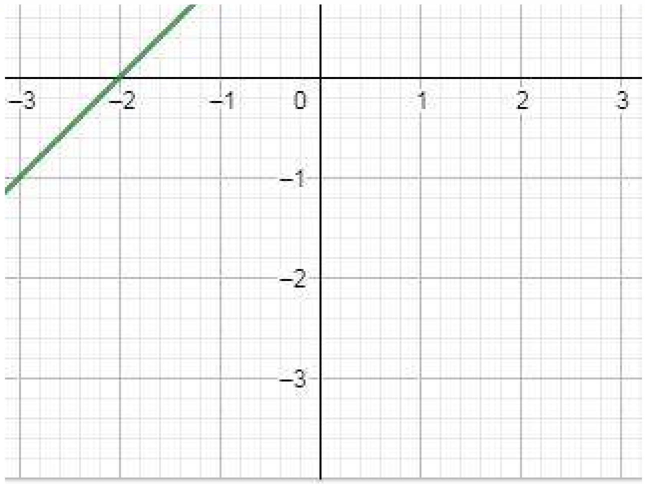
ii) They are parallel



pair of linear equations which are parallel.

iii) They are coincident





pair of linear equations which are coincident.

Graphical Solution

Representing pair of LE in 2 variables graphically

Graphically, a pair of linear equations in two variables can be represented by a pair of straight lines.

Graphical method of finding solution of a pair of Linear Equations

Graphical Method of finding the solution to a pair of linear equations is as follows:

- Plot both the equations (two straight lines)
- Find the point of intersection of the lines.

The point of intersection is the solution.

Comparing the ratios of coefficients of a Linear Equation

i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are said to be **consistent**. Graphs of the two equations intersect at a unique point. The pair of linear equations have **exactly one solution**.
ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the equations are said to be **dependent**. One equation can be obtained by multiplying the other equation with a non-zero constant. In this case, graphs of both the equations coincide. Dependent equations are consistent. The pair linear equations have **infinitely many solutions.**
iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the equations are said to be **inconsistent**. The graphs of the equations are parallel to each other. The pair of linear equations have **no solution**.

Algebraic Solution

Finding solution for consistent pair of Linear Equations

The solution of a pair of linear equations is of the form (x,y) which satisfies both the equations simultaneously. Solution for a consistent pair of linear equations can be found out using

i) Elimination method

ii) Substitution Method

iii) Cross-multiplication method

iv) Graphical method

Substitution Method of finding solution of a pair of Linear Equations

Substitution method:
$y-2x=1$
$x+2y=12$
(i) express one variable in terms of the other using one of the equations. In this case,
$y=2x+1$
(ii) substitute for this variable (y) in the second equation to get a linear equation in one variable, x.
$x+2\times(2x+1)=12$
$\Rightarrow 5x+2=12$
(iii) Solve the linear equation in one variable to find the value of that variable.
$5x+2=12$
$\Rightarrow x=2$
(iv) Substitute this value in one of the equations to get the value of the other variable.
$y=2\times 2+1=5$
So, (2,5) is the required solution of the pair of linear equations $y-2x=1$ and $x+2y=12$.

Elimination method of finding solution of a pair of Linear Equations

Elimination method
Consider x+2y=8 and 2x-3y=2
Step 1: Make the coefficients of any variable same by multiplying the equations with constants. Multiplying the first equation by 2, we get,
2x+4y=16
Step 2: Add or subtract the equations to eliminate one variable, giving a single variable equation.
Subtract second equation from the previous equation
2x + 4y = 16

2x - 3y = 2
\- + -
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0(x)+7y=14

Step 3: Solve for one variable and substitute this in any equation to get the other variable.

$y = 2;$

$x = 8 - 2y = 8 - 4 = 4$

(4,2) is the solution.

Cross-multiplication Method of finding solution of a pair of Linear Equations

For the pair of linear equations

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0,$

x and y can be calculated as

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Solving Linear Equations

Equations reducible to a pair of Linear Equations in 2 variables

Some equations may be in a form which can be reduced to a linear equation through substitution.

$\frac{2}{x} + \frac{3}{y} = 4$

$\frac{5}{x} - \frac{4}{y} = 9$

In this case, we may make the substitution $\frac{1}{x} = u \text{ and } \frac{1}{y} = v$

The pair of equations reduces to

2u +3v=4

5u-4v=9

The above pair of equations may be solved. After solving, back substitute to get the values of x and y.