Constructions

Introduction to Constructions

Linear Pair axiom

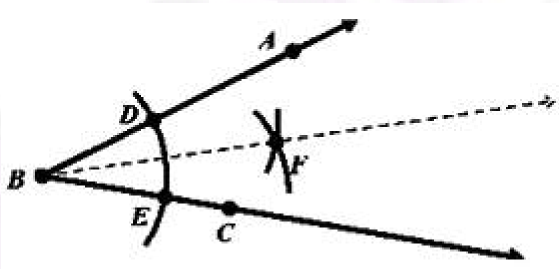
- If a ray stands on a line then the adjacent angles form a linear pair of angles.
- If two angles form a linear pair, then uncommon arms of both the angles form a straight line.

Angle Bisector

Construction of an Angle bisector

Suppose we want to draw the angle bisector of ∠ABC we will do it as follows:

- Taking B as center and any radius, draw an arc to intersect AB and BC to intersect at D and E respectively.
- Taking D and E as centers and with radius more than $\frac{DE}{2}$, draw arcs to intersect each other at a point F.
- Draw the ray BF. This ray BF is the required bisector of the ∠ABC.



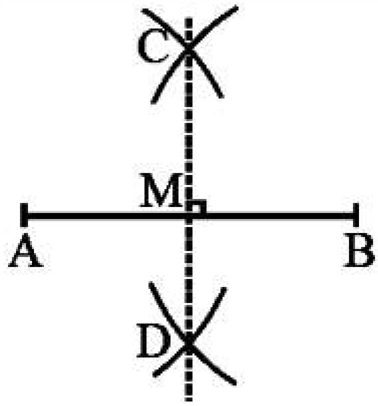
Construction of angle bisector

Perpendicular Bisector

Construction of a perpendicular bisector

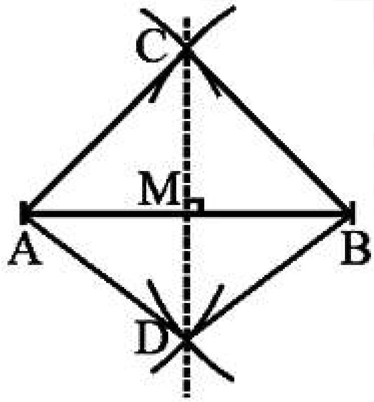
Steps of construction of a perpendicular bisector on the line segment AB:

- Take A and B as centers and radius more than $\frac{AB}{2}$ draw arcs on both sides of the line.
- Arcs intersect at the points C and D. Join CD.
- CD intersects AB at M. CMD is the required perpendicular bisector of the line segment AB.



Proof of validity of construction of a perpendicular bisector

Proof of the validity of construction of the perpendicular bisector:
ΔDAC and ΔDBC are congruent by SSS congruency. ($\because$ AC = BC, AD = BD and CD = CD)
$\angle ACM$ and $\angle BCM$are equal (cpct)
ΔAMC and ΔBMC are congruent by SAS congruency. ($\because$ AC = BC, $\angle ACM$ = $\angle BCM$ and CM = CM)
AM = BM and ∠AMC = ∠BMC (cpct)
∠AMC + ∠BMC = 180° (Linear Pair Axiom)
$\therefore$ ∠AMC = ∠BMC = 90°
Therefore, CMD is the perpendicular bisector.

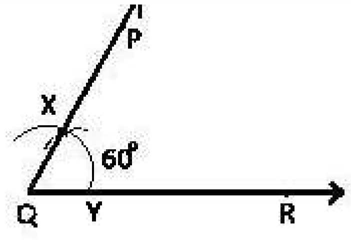


Constructing Angles

Construction of an Angle of 60 degrees

Steps of construction of an angle of 60 degrees:

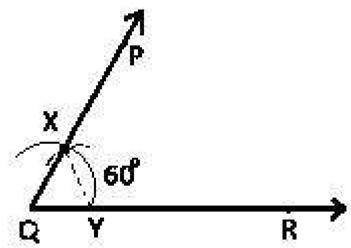
- Draw a ray QR.
- Take Q as the center and some radius draw an arc of a circle, which intersects QR at a point Y.
- Take Y as the center with the same radius draw an arc intersecting the previously drawn arc at point X.
- Draw a ray QP passing through X
- $\angle PQR = 60^\circ$



Constructing 60 degrees.

Proof for validity of construction of an Angle of 60 degrees

Proof for the validity of construction of the 60°angle:
Join XY
XY = XQ = YQ (By construction)
$\therefore \triangle XQY$ is an equilateral triangle.
Therefore, $\angle XQY = \angle PQR = 60^\circ$



Triangle Constructions

Construction of triangles

At least three parts of a triangle have to be given for constructing it but not all combinations of three parts are sufficient for the purpose.
Therefore a unique triangle can be constructed if the following parts of a triangle are given:

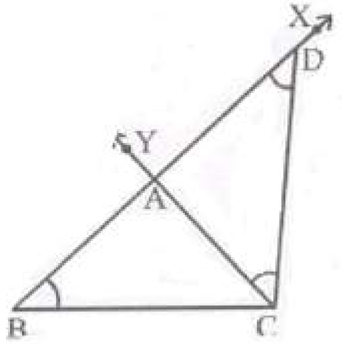
- two sides and the included angle is given.
- three sides are given.
- two angles and the included side is given.
- In a right triangle, hypotenuse and one side is given.
- If two sides and an angle (not the included angle) are given, then it is not always possible to construct such a triangle uniquely.

Given base, base angle and sum of other two sides

Steps for construction of a triangle given base, base angle, and the sum of other two sides:

- Draw the base BC and at point B make an angle say XBC equal to the given angle.
- Cut the line segment BD equal to AB $+$ AC from ray BX.
- Join DC and make an angle DCY equal to $\angle BDC$.
- Let CY intersect BX at A.

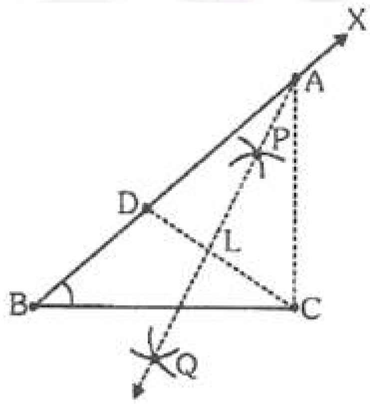
- ABC is the required triangle.



Given base(BC), base angle(ABC) and AB-AC

Steps of construction of a triangle given base(BC), base angle(∠ABC) and difference of the other two sides(AB-AC):

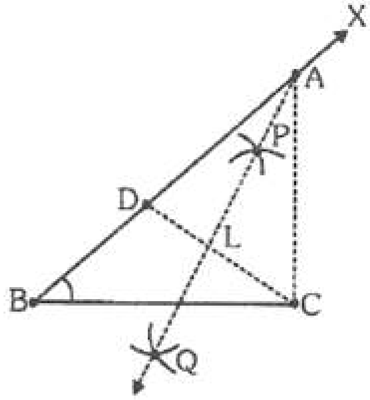
- Draw base BC and with point B as the vertex make an angle XBC equal to the given angle.
- Cut the line segment BD equal to AB – AC(AB > AC) on the ray BX.
- Join DC and draw the perpendicular bisector PQ of DC.
- Let it intersect BX at a point A. Join AC.
- Then △ABC is the required triangle.



Proof for validation for Construction of a triangle with given base, base angle and difference between two sides

Validation of the steps of construction of a triangle with given base, base angle and difference between two sides

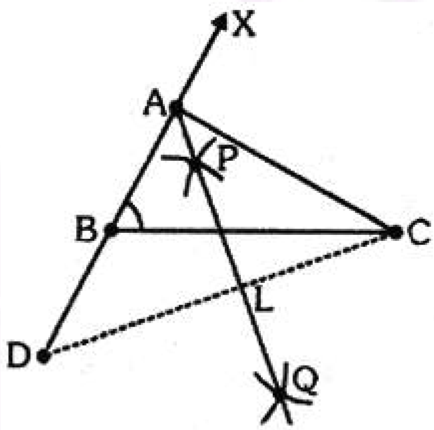
- Base BC and $\angle B$ are drawn as given.
- Point A lies on the perpendicular bisector of DC. So, AD = AC.
- BD = AB – AD = AB – AC   (∵ AD = AC).
- Therefore ABC is the required triangle.



Given base(BC), base angle(ABC) and AC-AB

Steps of construction of a triangle given base(BC), base angle(∠ABC) and difference of the other two sides(AC-AB):

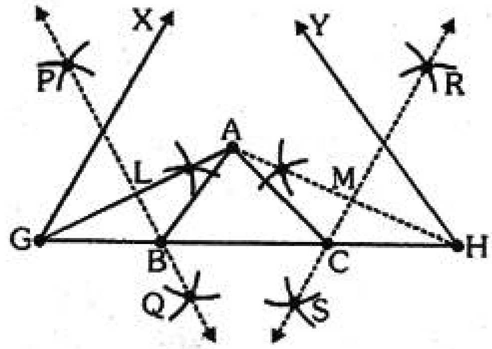
- Draw the base BC and at point B make an angle XBC equal to the given angle.
- Cut the line segment BD equal to AC – AB from the line BX extended on opposite side of line segment BC.
- Join DC and draw the perpendicular bisector, say PQ of DC.
- Let PQ intersect BX at A. Join AC.
- △ABC is the required triangle.



Given perimeter and two base angles

Steps of construction of a triangle with given perimeter and two base angles.

- Draw a line segment, say GH equal to BC + CA + AB.
- Make angles XGH equal to $\angle B$ and YHG equal to $\angle C$, where angle B and C are the given base angles.
- Draw the angle bisector of $\angle XGH$ and $\angle YHG$. Let these bisectors intersect at a point A.
- Draw perpendicular bisectors PQ of AG and RS of AH.
- Let PQ intersect GH at B and RS intersect GH at C. Join AB and AC
- △ABC is the required triangle.



Proof for validation for Construction of a triangle with given perimeter and two base angles

Validating the steps of construction of a triangle with given perimeter and two base angles:

- B lies on the perpendicular bisector PQ of AG and C lies on the perpendicular bisector RS of AH. So,GB = AB and CH = AC.
- BC + CA + AB = BC + GB + CH = GH. (∵ GB = AB and CH = AC)
- $\angle BAG = \angle AGB$ $(\because \Delta AGB, AB = GB)$
- $\angle ABC = \angle BAG + \angle AGB = 2\angle AGB = \angle XGH$
- Similarly, $\angle ACB = \angle YHG$