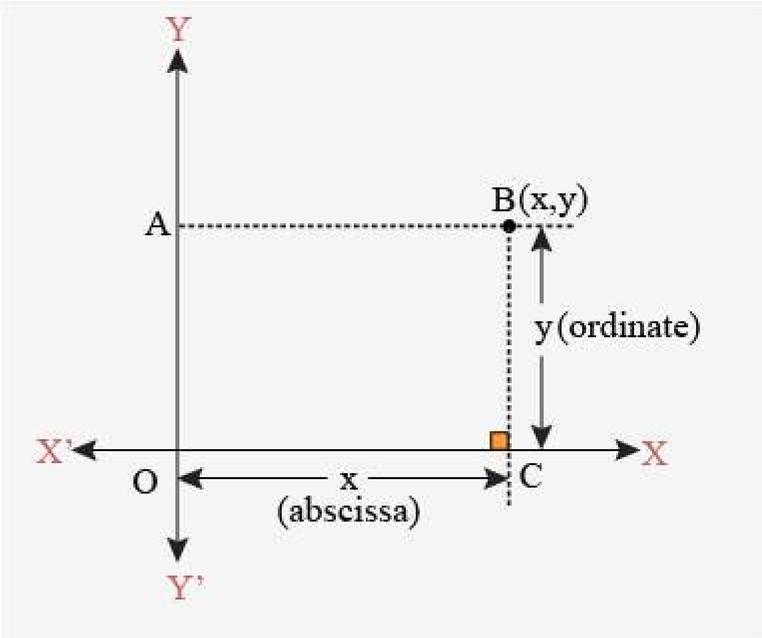
Coordinate Geometry

Basics Revisited

Points on a Cartesian Plane

Points on a plane are located by a pair of numbers called the **coordinates**. The distance of a point from the y-axis is called its x-coordinate, or **abscissa**. The distance of a point from the x-axis is called its y-coordinate, or **ordinate**

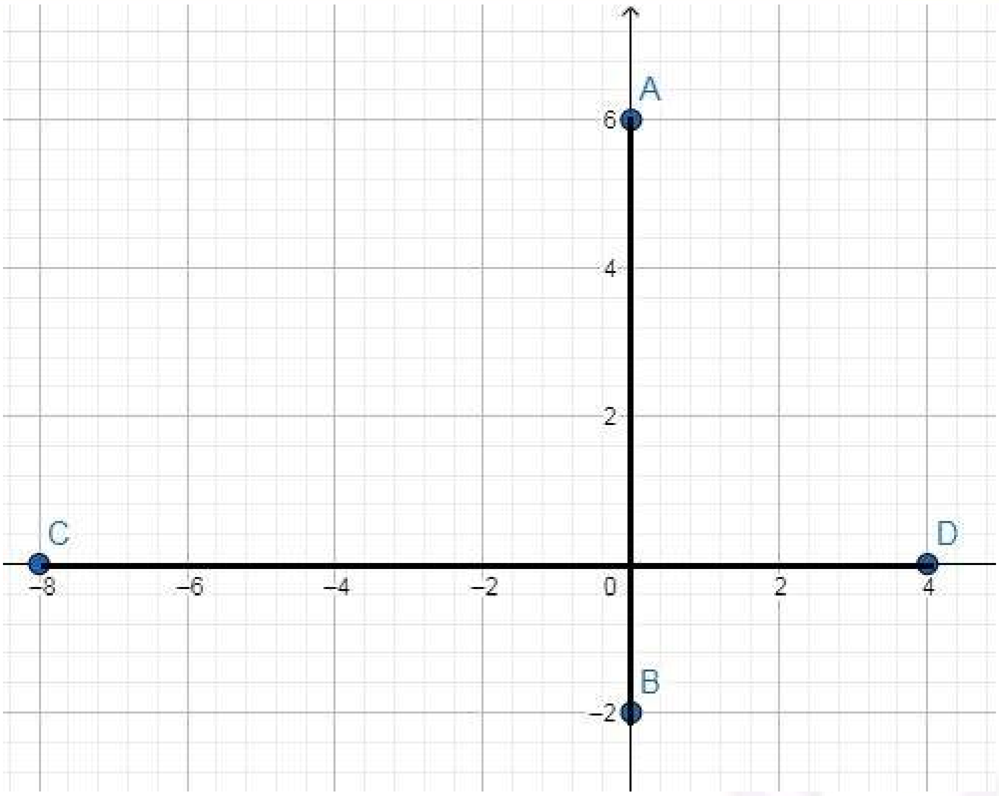


Representation of $(x,\ y)$ on the cartesian plane

Distance Formula

Distance between Two Points on the Same Coordinate Axes

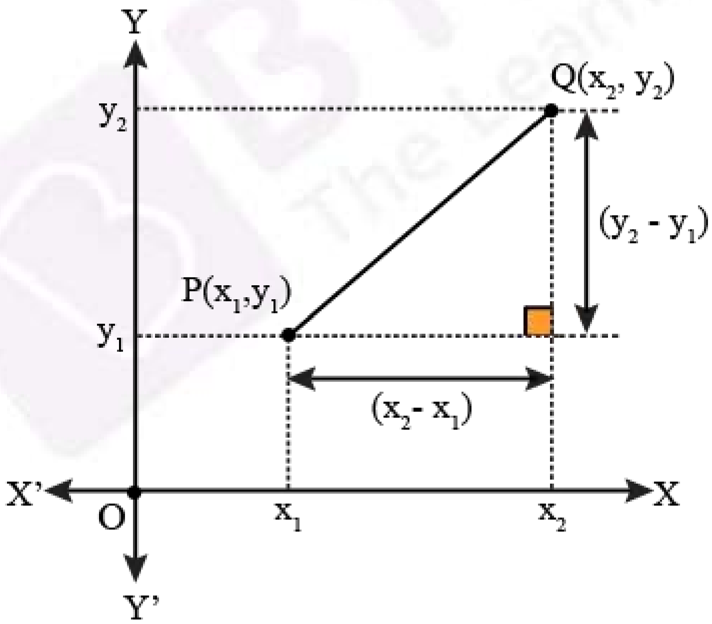
The distance between two points which are on the same axis (x-axis or y-axis), is given by the difference between their ordinates if they are on the y-axis, else by the difference between their abscissa if they are on the x-axis.



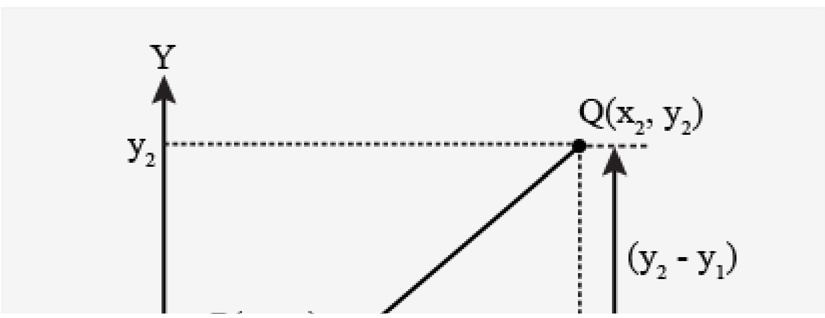
Distance AB = 6 - (-2) = 8 units

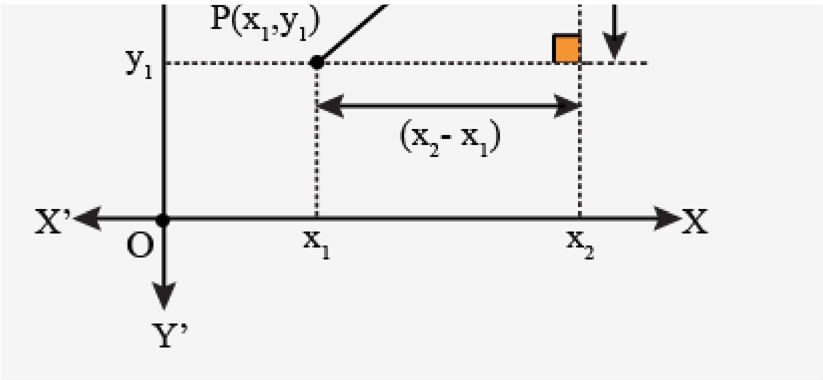
Distance CD = 4 - (-8) = 12 units

Distance between Two Points Using Pythagoras Theorem



Transparent BG





Finding distance between 2 points using Pythagoras Theorem

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points on the cartesian plane.
Draw lines parallel to the axes through P and Q to meet at T. ΔPTQ is right angled at T.
From **Pythagoras Theorem**,
$PQ^2 = PT^2 + QT^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance Formula

Distance between any two points (x_1, y_1) and (x_2, y_2) is given by
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Where d is the distance between the points (x_1, y_1) and (x_2, y_2).

Section Formula

Section Formula

If the point P(x,y) **divides** the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ **internally** in the **ratio m:n**, then, the coordinates of P are given by the **section formula** as
$P(x, y) = (\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n})$

Finding ratio given the points

To find the ratio in which a given point P(x,y) divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$,

- Assume that the ratio is k :1
- Substitute the ratio in the section formula for any of the coordinates to get the value of k.

$x = \frac{kx_2 + x_1}{k+1}$
Since, x_1, x_2 and x are known, k can be calculated. The same can be calculated from the y-

coordinates also.

Mid Point

The **midpoint** of any line segment divides it in the ratio **1:1**.
The coordinates of the midpoint(P) of line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by
$P(x, y) = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

Points of Trisection

To find the points of trisection P and Q which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ into three equal parts:

i) **AP : PB = 1 : 2**
So, $P = (\frac{x_2+2x_1}{3}, \frac{y_2+2y_1}{3})$

ii) **AQ : QB = 2 : 1**
So, $Q = (\frac{2x_2+x_1}{3}, \frac{2y_2+y_1}{3})$

Centroid of a triangle

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC, then the coordinates of its centroid(P) is given by
$P(x, y) = (\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$

Area from Coordinates

Area of a triangle given its vertices

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a Δ ABC, then its area is given by

$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
Where A is the area of the Δ ABC.

Collinearity Condition

If three points A, B and C are collinear and B lies between A and C, then, AB + BC = AC. AB, BC, and AC can be calculated using the distance formula.

- The ratio in which B divides AC, calculated using section formula for both the x and y coordinates separately will be equal.
- Area of triangle formed by the three points is zero.