Area of a Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

Types of triangles:
Based on sides - a) Equilateral b) Isosceles c) Scalene
Based on angles - a) Acute angled triangle b) Right-angled triangle c) Obtuse angled triangle

Area of a triangle

\[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]

In case of equilateral and isosceles triangles, if the length of the sides of triangles are given then, we use Pythagoras theorem in order to find the height of a triangle.

Area of an equilateral triangle

Consider an equilateral \( \Delta ABC \), with each side as \( a \) units. Let AO be perpendicular bisector of BC. In order to derive the formula for the area of equilateral triangle, we need to find height AO.

Using Pythagoras theorem,
\[ AC^2 = OA^2 + OC^2 \]
\[ OA^2 = AC^2 - OC^2 \]
Substitute \( AC = a, OC = \frac{a}{2} \) to find OA
We know the area of triangle is
\[ A = \frac{1}{2} \times \text{base} \times \text{height}, \]
\[ A = \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2} \]

\[ \therefore \text{Area of Equilateral triangle} = \frac{\sqrt{3}a^2}{4} \]

**Area of an isosceles triangle**

Consider an isosceles \( \triangle ABC \) with equal sides as \( a \) units and base as \( b \) unit.

The height of the triangle can be found by Pythagoras' Theorem:
\[ CD^2 = AC^2 - AD^2 \]
\[ \Rightarrow h = a^2 - \frac{b^2}{4} = \frac{4a^2 - b^2}{4} \]
\[ \Rightarrow h = \frac{1}{2} \sqrt{4a^2 - b^2} \]

Area of triangle is \( A = \frac{1}{2}bh \)
\[ \therefore A = \frac{1}{2} \times b \times \frac{1}{2} \sqrt{4a^2 - b^2} \]
\[ \therefore A = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2} \]

**Area of a triangle - By Heron's formula**

Area of a \( \triangle ABC \), given sides \( a, b, c \) by Heron's formula (Also known as Hero's Formula):

\[ \text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \]

\[ s = \frac{a + b + c}{2} \]
Find semi perimeter \((s) = \frac{a+b+c}{2}\)

\[\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}\]

This formula is helpful to find area of a scalene triangle, given the lengths of all its sides.

### Area of any polygon - By Heron’s formula

Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using the Heron’s formula.

**Example:** A park, in the shape of a quadrilateral \(ABCD\), has \(\angle C = 90^\circ\), \(AB = 9\) m, \(BC = 12\) m, \(CD = 5\) m and \(AD = 8\) m. How much area does it occupy?

⇒ We draw the figure according to the information given.

The figure can be split into 2 triangles \(\Delta BCD\) and \(\Delta ABD\)

From \(\Delta BCD\), we can find \(BD\) (Using Pythagoras’ Theorem)

\[BD^2 = 12^2 + 5^2 = 169\]

\[BD = 13\text{ cm}\]

Semi-perimeter for \(\Delta BCD\) \(S_1 = \frac{12 + 5 + 13}{2} = 15\)

Semi-perimeter \(\Delta ABD\) \(S_2 = \frac{9 + 8 + 13}{2} = 15\)

Using Heron’s formula we find \(A_1\) and \(A_2\)

\[A_1 = \sqrt{15(15 - 12)(15 - 5)(15 - 13)} = \sqrt{15 \times 3 \times 10 \times 2}\]
\[ A_1 = \sqrt{900} = 30\text{cm}^2 \]

Similarly we find \( A_2 \) to be 35.49\( \text{cm}^2 \).

The area of the quadrilateral \( ABCD = A_1 + A_2 = 65.49\text{ cm}^2 \)