

EXERCISE 6.3

Solve the following system of inequalities graphically:

1. $x \geq 3, y \geq 2$

Solution:

Given $x \geq 3$ (i)

$y \geq 2$ (ii)

Since $x \geq 3$ means for any value of y the equation will be unaffected so similarly for $y \geq 2$, for any value of x the equation will be unaffected.

Now putting $x = 0$ in the (i)

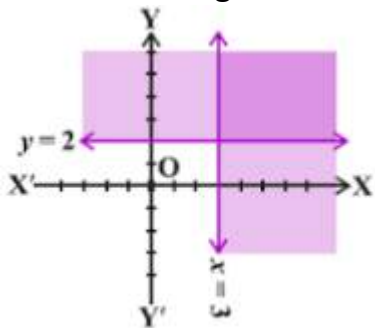
$0 \geq 3$ which is not true

Putting $y = 0$ in (ii)

$0 \geq 2$ which is not true again

This implies the origin doesn't satisfy in the given inequalities. The region to be included will be on the right side of the two equalities drawn on the graphs.

The shaded region is the desired region.



2. $3x + 2y \leq 12, x \geq 1, y \geq 2$

Solution:

Given $3x + 2y \leq 12$

Solving for the value of x and y by putting $x = 0$ and $y = 0$ one by one

We get

$y = 6$ and $x = 4$

So the points are $(0, 6)$ and $(4, 0)$

Now checking for $(0, 0)$

$0 \leq 12$ which is also true,

Hence the origin lies in the plane and the required area is toward the left of the equation.

Now checking for $x \geq 1$,

The value of x would be unaffected by any value of y

The origin would not lie on the plane

$\Rightarrow 0 \geq 1$ which is not true

The required area to be included would be on the left of the graph $x \geq 1$

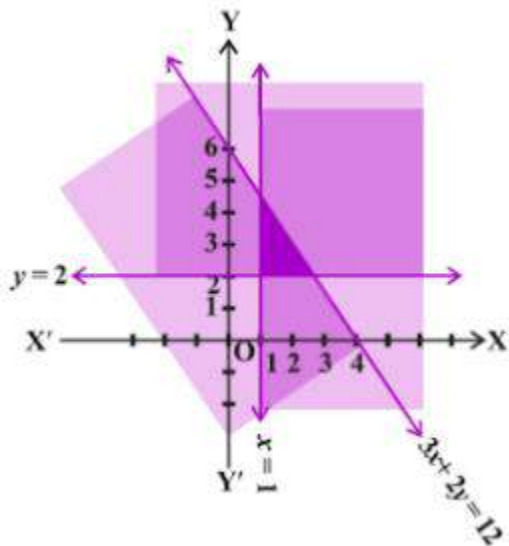
Similarly, for $y \geq 2$

Value of y will be unaffected by any value of x in the given equality. Also, the origin doesn't satisfy the given inequality.

$\Rightarrow 0 \geq 2$ which is not true, hence origin is not included in the solution of the inequality.

The region to be included in the solution would be towards the left of the equality $y \geq 2$

The shaded region in the graph will give the answer to the required inequalities as it is the region which is covered by all the given three inequalities at the same time satisfying all the given conditions.



3. $2x + y \geq 6$, $3x + 4y \leq 12$

Solution:

Given $2x + y \geq 6$ (i)

$3x + 4y \leq 12$ (ii)

$2x + y \geq 6$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 6$ and $x = 3$

So the point for the (0, 6) and (3, 0)

Now checking for (0, 0)

$0 \geq 6$ which is not true, hence the origin does not lie in the solution of the equality. The required region is on the right side of the graph.

Checking for $3x + 4y \leq 12$

Putting value of $x = 0$ and $y = 0$ one by one in equation

We get $y = 3$, $x = 4$

The points are (0, 3), (4, 0)

Now checking for origin (0, 0)

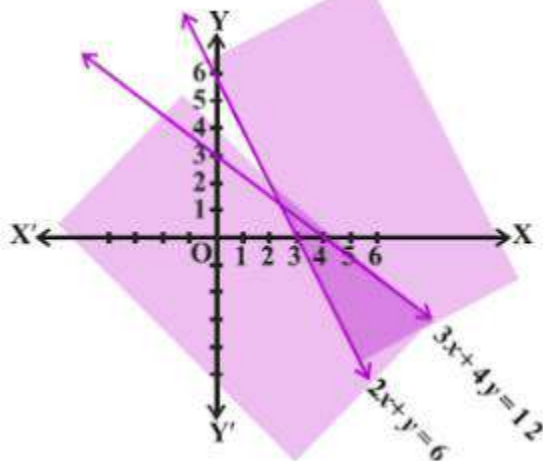
$0 \leq 12$ which is true,

So the origin lies in solution of the equation.

The region on the right of the equation is the region required.

The solution is the region which is common to the graphs of both the inequalities.

The shaded region is the required region.



4. $x + y \geq 4$, $2x - y < 0$

Solution:

Given $x + y \geq 4$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 4$ and $x = 4$

The points for the line are (0, 4) and (4, 0)

Checking for the origin (0, 0)

$0 \geq 4$

This is not true,

So the origin would not lie in the solution area. The required region would be on the

right of line's graph.

$$2x - y < 0$$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 0$ and $x = 0$

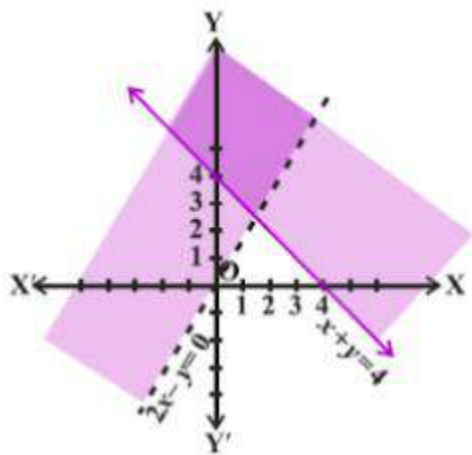
Putting $x = 1$ we get $y = 2$

So the points for the given inequality are $(0, 0)$ and $(1, 2)$

Now that the origin lies on the given equation we will check for $(4, 0)$ point to check which side of the line's graph will be included in the solution.

$\Rightarrow 8 < 0$ which is not true, hence the required region would be on the left side of the line $2x - y < 0$

The shaded region is the required solution of the inequalities.



5. $2x - y > 1, x - 2y < -1$

Solution:

Given $2x - y > 1$ (i)

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = -1$ and $x = 1/2 = 0.5$

The points are $(0, -1)$ and $(0.5, 0)$

Checking for the origin, putting $(0, 0)$

$0 > 1$, which is false

Hence the origin does not lie in the solution region. The required region would be on the right of the line's graph.

$x - 2y < -1$ (ii)

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 1/2 = 0.5$ and $x = -1$

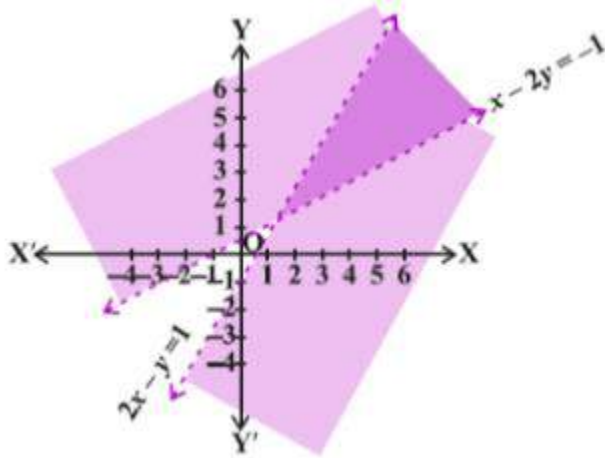
The required points are $(0, 0.5)$ and $(-1, 0)$

Now checking for the origin, (0, 0)

$0 < -1$ which is false

Hence the origin does not lie in the solution area, the required area would be on the left side of the line's graph.

∴ the shaded area is the required solution of the given inequalities.



6. $x + y \leq 6$, $x + y \geq 4$

Solution:

Given $x + y \leq 6$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of

$Y = 6$ and $x = 6$

The required points are (0, 6) and (6, 0)

Checking further for origin (0, 0)

We get $0 \leq 6$, this is true.

Hence the origin would be included in the area of the line's graph. So the required solution of the equation would be on the left side of the line graph which will be including origin.

$x + y \geq 4$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of

$y = 4$ and $x = 4$

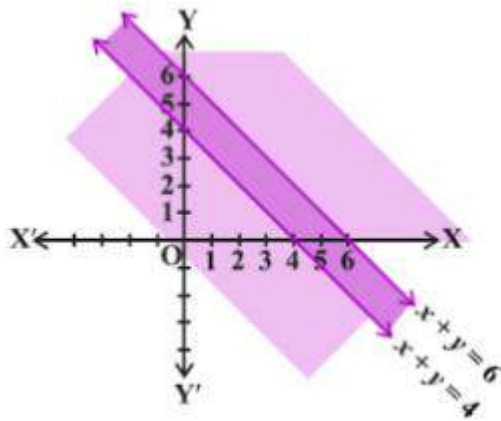
The required points are (0, 4) and (4, 0)

Checking for the origin (0, 0)

$0 \geq 4$ which is false

So the origin would not be included in the required area. The solution area will be above the line graph or the area on the right of line graph.

Hence the shaded area in the graph is required graph area.



7. $2x + y \geq 8$, $x + 2y \geq 10$

Solution:

Given $2x + y \geq 8$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 8$ and $x = 4$

The required points are $(0, 8)$ and $(4, 0)$

Checking if the origin is included in the line's graph $(0, 0)$

$0 \geq 8$, which is false

Hence the origin is not included in the solution area and the required area would be the area to the right of line's graph.

$x + 2y \geq 10$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 5$ and $x = 10$

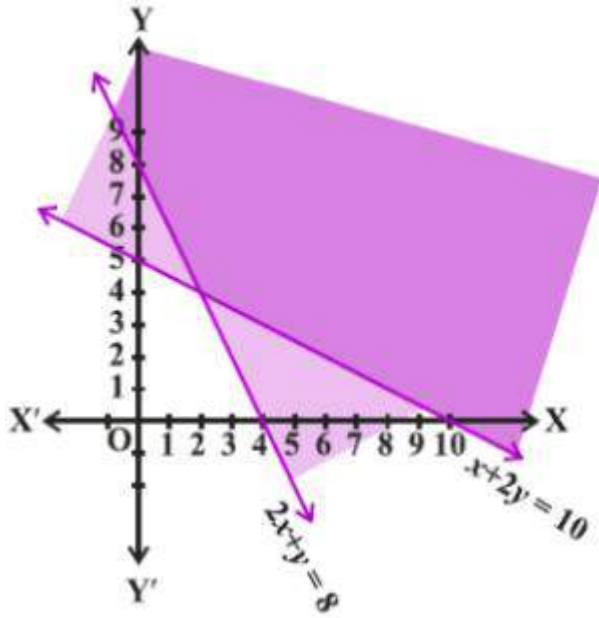
The required points are $(0, 5)$ and $(10, 0)$

Checking for the origin $(0, 0)$

$0 \geq 10$ which is false,

Hence the origin would not lie in the required solution area. The required area would be to the left of the line graph.

The shaded area in the graph is the required solution of the given inequalities.



8. $x + y \leq 9, y > x, x \geq 0$

Solution:

Given $x + y \leq 9$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 9$ and $x = 9$

The required points are $(0, 9)$ and $(9, 0)$

Checking if the origin is included in the line's graph $(0, 0)$

$0 \leq 9$

Which is true, so the required area would be including the origin and hence will lie on the left side of the line's graph.

$y > x$,

Solving for $y = x$

We get $x = 0, y = 0$ so the origin lies on the line's graph.

The other points would be $(0, 0)$ and $(2, 2)$

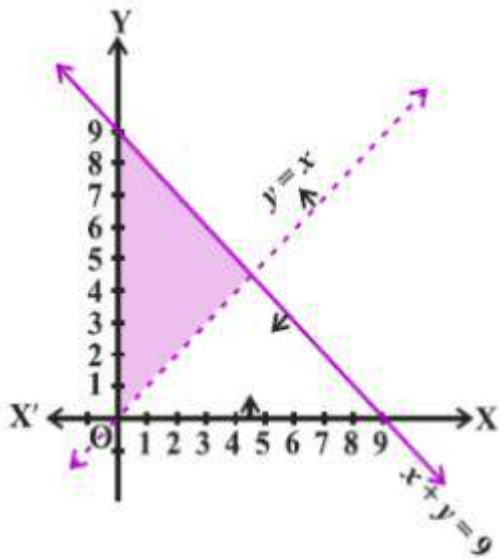
Checking for $(9, 0)$ in $y > x$,

We get $0 > 9$ which is false, since the area would not include the area below the line's graph and hence would be on the left side of the line.

We have $x \geq 0$

The area of the required line's graph would be on the right side of the line's graph.

Therefore the shaded area is the required solution of the given inequalities.



9. $5x + 4y \leq 20$, $x \geq 1$, $y \geq 2$

Solution:

Given $5x + 4y \leq 20$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 5$ and $x = 4$

The required points are $(0, 5)$ and $(4, 0)$

Checking if the origin lies in the solution area $(0, 0)$

$$0 \leq 20$$

Which is true, hence the origin would lie in the solution area. The required area of the line's graph is on the left side of the graph.

We have $x \geq 1$,

For all the values of y , x would be 1,

The required points would be $(1, 0)$, $(1, 2)$ and so on.

Checking for origin $(0, 0)$

$$0 \geq 1, \text{ which is not true}$$

So the origin would not lie in the required area. The required area on the graph will be on the right side of the line's graph.

Consider $y \geq 2$

Similarly for all the values of x , y would be 2.

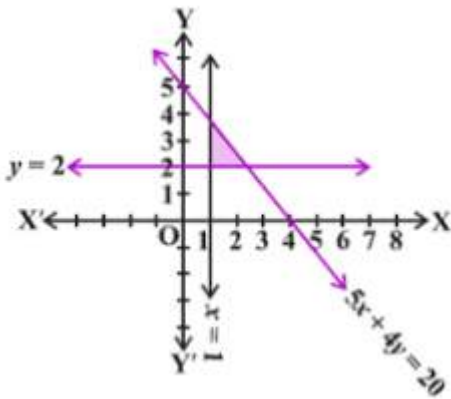
The required points would be $(0, 2)$, $(1, 2)$ and so on.

Checking for origin $(0, 0)$

$$0 \geq 2, \text{ this is no true}$$

Hence the required area would be on the right side of the line's graph.

The shaded area on the graph shows the required solution of the given inequalities.



10. $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$

Solution:

Given $3x + 4y \leq 60$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 15$ and $x = 20$

The required points are $(0, 15)$ and $(20, 0)$

Checking if the origin lies in the required solution area $(0, 0)$

$0 \leq 60$, this is true.

Hence the origin would lie in the solution area of the line's graph.

The required solution area would be on the left of the line's graph.

We have $x + 3y \leq 30$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 10$ and $x = 30$

The required points are $(0, 10)$ and $(30, 0)$

Checking for the origin $(0, 0)$

$0 \leq 30$, this is true.

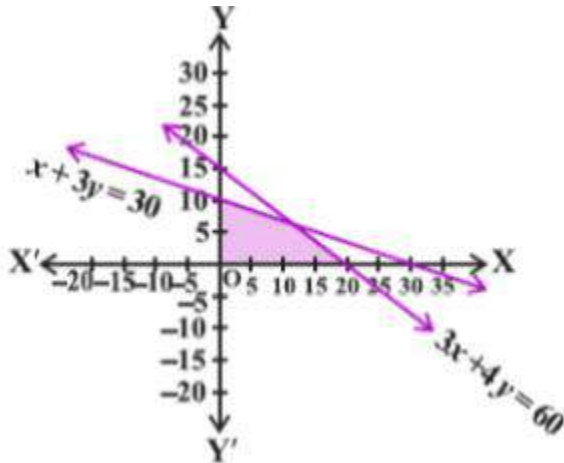
Hence the origin lies in the solution area which is given by the left side of the line's graph.

Consider $x \geq 0$,

$y \geq 0$,

The given inequalities imply the solution lies in the first quadrant only.

Hence the solution of the inequalities is given by the shaded region in the graph.



11. $2x + y \geq 4$, $x + y \leq 3$, $2x - 3y \leq 6$

Solution:

Given $2x + y \geq 4$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 4$ and $x = 2$

The required points are $(0, 4)$ and $(2, 0)$

Checking for origin $(0, 0)$

$0 \geq 4$, this is not true

Hence the origin doesn't lie in the solution area of the line's graph. The solution area would be given by the right side of the line's graph.

$x + y \leq 3$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 3$ and $x = 3$

The required points are $(0, 3)$ and $(3, 0)$

Checking for the origin $(0, 0)$

$0 \leq 3$, this is true

Hence the solution area would include the origin and hence would be on the left side of the line's graph.

$2x - 3y \leq 6$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = -2$ and $x = 3$

The required points are $(0, -2)$, $(3, 0)$

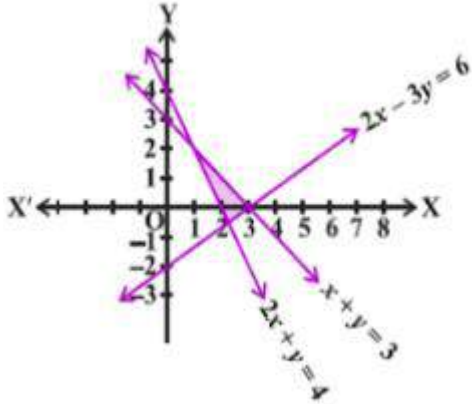
Checking for the origin $(0, 0)$

$0 \leq 6$ this is true

So the origin lies in the solution area and the area would be on the left of the line's

graph.

Hence the shaded area in the graph is the required solution area for the given inequalities.



12. $x - 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$

Solution:

Given, $x - 2y \leq 3$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = -3/2 = -1.5$ and $x = 3$

The required points are $(0, -1.5)$ and $(3, 0)$

Checking for the origin $(0, 0)$

$0 \leq 3$, this is true.

Hence the solution area would be on the left of the line's graph

$3x + 4y \geq 12$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 3$ and $x = 4$

The required points are $(0, 3)$ and $(4, 0)$

Checking for the origin $(0, 0)$

$0 \geq 12$, this is not true

So the solution area would of include the origin and the required solution area would be on the right side of the line's graph.

We have $x \geq 0$,

For all the values of y , the value of x would be same in the given inequality, which would be the region above the x axis on the graph.

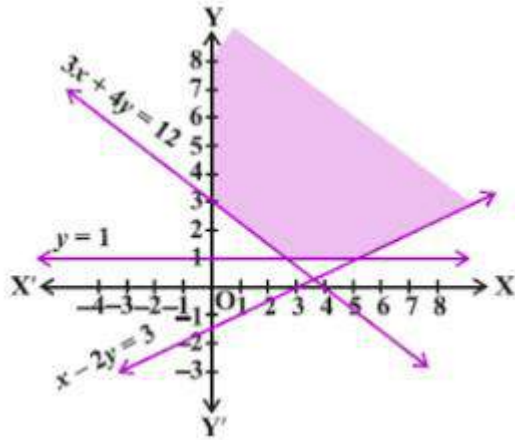
Consider, $y \geq 1$

For all the values of x , the value of y would be same in the given inequality.

The solution area of the line would be not include origin as $0 \geq 1$ is not true.

The solution area would be on the left side of the line's graph.

The shaded area in the graph is the required solution area which satisfies all the given inequalities at the same time.



13. $4x + 3y \leq 60$, $y \geq 2x$, $x \geq 3$, $x, y \geq 0$

Solution:

Given, $4x + 3y \leq 60$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 20$ and $x = 15$

The required points are $(0, 20)$ and $(15, 0)$

Checking for the origin $(0, 0)$

$0 \leq 60$, this is true.

Hence the origin would lie in the solution area. The required area would include be on the left of the line's graph.

We have $y \geq 2x$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 0$ and $x = 0$

Hence the line would pass through origin.

To check which side would be included in the line's graph solution area, we would check for point $(15, 0)$

$\Rightarrow 0 \geq 15$, this is not true so the required solution area would be to the left of the line's graph.

Consider, $x \geq 3$,

For any value of y , the value of x would be same.

Also the origin $(0, 0)$ doesn't satisfies the inequality as $0 \geq 3$

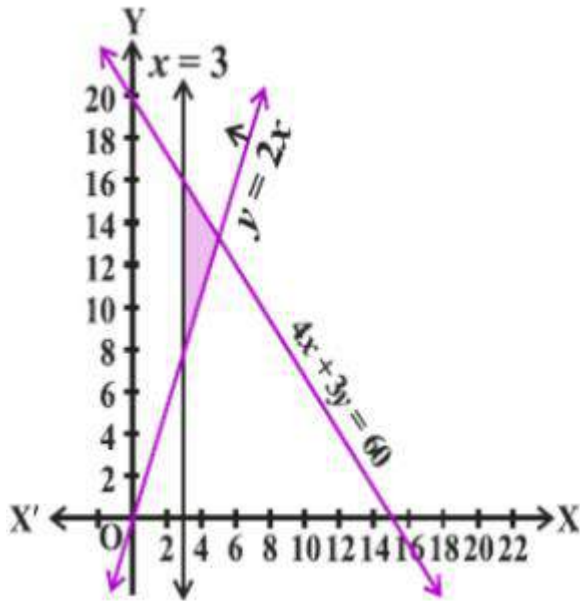
So the origin doesn't lie in the solution area, hence the required solution area would be the right of the line's graph.

We have $x, y \geq 0$

Since given both x and y are greater than 0

\therefore the solution area would be in the first Ist quadrant only.

The shaded area in the graph shows the solution area for the given inequalities



14. $3x + 2y \leq 150, x + 4y \leq 80, x \leq 15, y \geq 0, x \geq 0$

Solution:

Given, $3x + 2y \leq 150$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 75$ and $x = 50$

The required points are $(0, 75)$ and $(50, 0)$

Checking for the origin $(0, 0)$

$0 \leq 150$, this is true

Hence the solution area for the line would be on the left side of the line's graph which would be including the origin too.

We have $x + 4y \leq 80$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 20$ and $x = 80$

The required points are $(0, 20)$ and $(80, 0)$

Checking for the origin $(0, 0)$

$0 \leq 80$, this is also true so the origin lies in the solution area.

The required solution area would be toward the left of the line's graph.

Given $x \leq 15$,

For all the values of y , x would be same

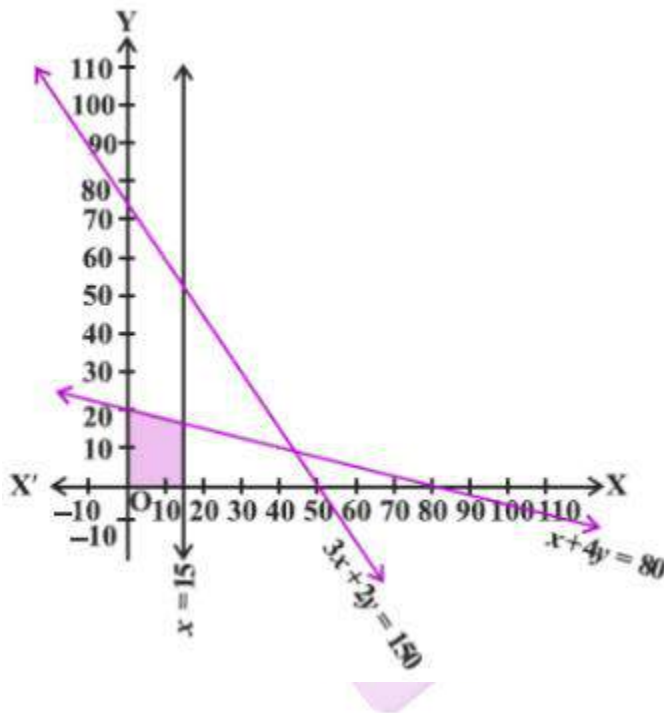
Checking for the origin $(0, 0)$

$0 \leq 15$, this is true so the origin would be included in the solution area. The required solution area would be towards the left of the line's graph.

Consider $y \geq 0, x \geq 0$

Since x and y are greater than 0, the solution would lie in the 1st quadrant.

The shaded area in the graph satisfies all the given inequalities and hence is the solution area for given inequalities.



15. $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

Solution:

Given, $x + 2y \leq 10$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 5$ and $x = 10$

The required points are $(0, 5)$ and $(10, 0)$

Checking for the origin $(0, 0)$

$0 \leq 10$, this is true.

Hence the solution area would be toward origin including the same. The solution area

would be toward the left of the line's graph.

We have $x + y \geq 1$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 1$ and $x = 1$

The required points are $(0, 1)$ and $(1, 0)$

Checking for the origin $(0, 0)$

$0 \geq 1$, this is not true.

Hence the origin would not be included into the solution area. The required solution area would be toward right of the line's graph.

Consider $x - y \leq 0$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 0$ and $x = 0$

Hence the origin would lie on the line.

To check which side of the line graph would be included in the solution area we would check for the $(10, 0)$

$10 \leq 0$ which is not true hence the solution area would be on the left side of the line's graph.

Again we have $x \geq 0, y \geq 0$

Since both x and y are greater than 0, the solution area would be in the 1st quadrant.

Hence, the solution area for the given inequalities would be the shaded area of the graph satisfying all the given inequalities.

