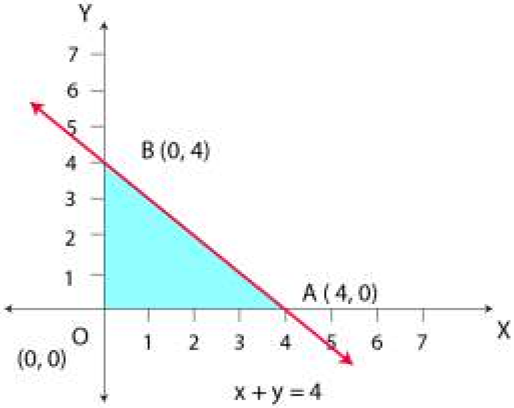
EXERCISE 12.1 PAGE NO: 513

1. Maximise Z = 3*x* + 4*y*

Subject to the constraints: $x + y \le 4, x \ge 0, y \ge 0.$

Solution:

The feasible region determined by the constraints, x + y ≤ 4, x ≥ 0, y ≥ 0, is given below



O (0, 0), A (4, 0), and B (0, 4) are the corner points of the feasible region. The values of Z at these points are given below

Corner point	Z = 3x + 4y	
O (0, 0)	0	
A (4, 0)	12	
B (0, 4)	16	Maximum

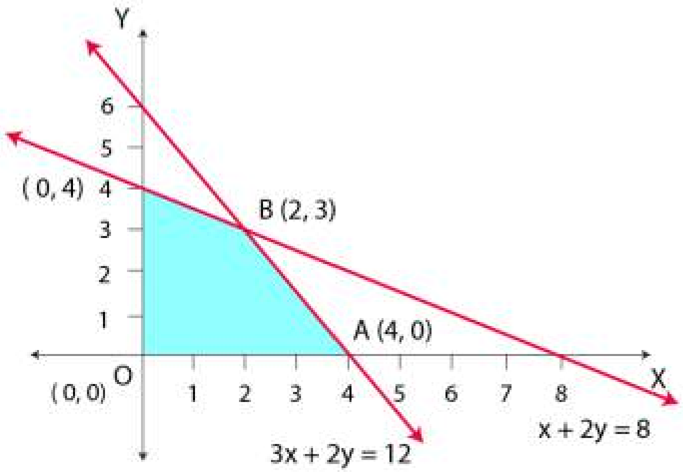
Hence, the maximum value of Z is 16 at the point B (0, 4)

2. Minimise Z = −3*x* + 4*y*

subject to $x+2y \le 8,\ 3x+2y \le 12,\ x \ge 0,\ y \ge 0$.

Solution:

The feasible region determined by the system of constraints, $x+2y \le 8,\ 3x+2y \le 12,\ x \ge 0,\ y \ge 0$ is given below

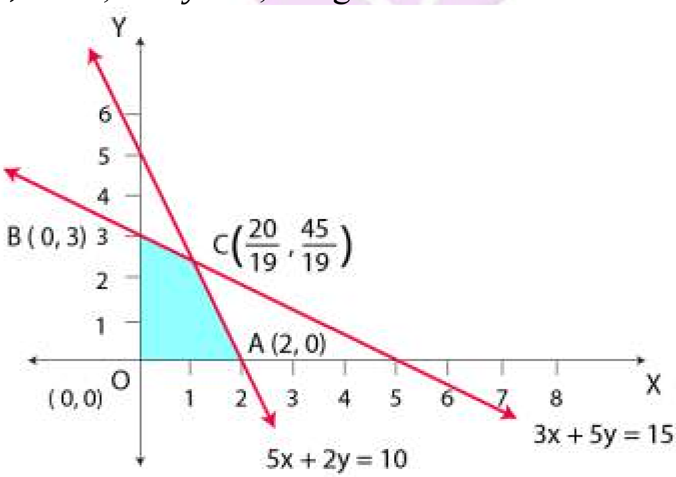


O (0, 0), A (4, 0), B (2, 3) and C (0, 4) are the corner points of the feasible region
The values of Z at these corner points are given below

Corner point	Z = - 3x + 4y	
O (0, 0)	0	
A (4, 0)	-12	Minimum
B (2, 3)	6	
C (0, 4)	16	

Hence, the minimum value of Z is – 12 at the point (4, 0)

3. Maximise Z = 5*x* + 3*y*
subject to $3x+5y \leq 15,\ 5x+2y \leq 10,\ x \geq 0,\ y \geq 0$.
Solution:
The feasible region determined by the system of constraints, $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, and $y \geq 0$, are given below

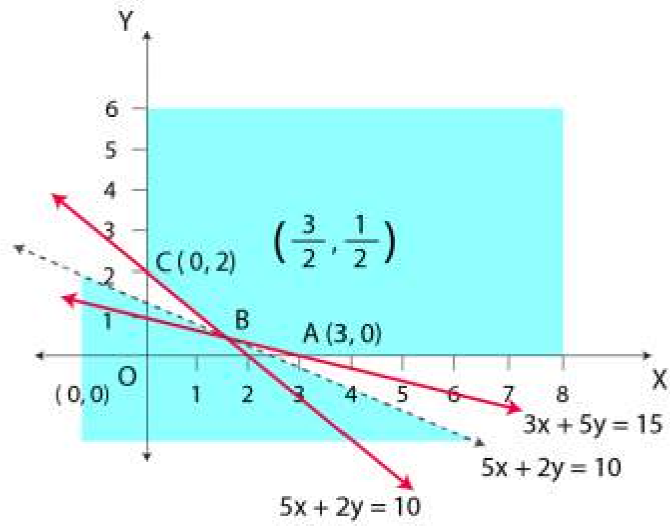


O (0, 0), A (2, 0), B (0, 3) and C (20 / 19, 45 / 19) are the corner points of the feasible region. The values of Z at these corner points are given below

Corner point	Z = 5x + 3y	
O (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
C (20 / 19, 45 / 19)	235 / 19	Maximum

Hence, the maximum value of Z is 235 / 19 at the point (20 / 19, 45 / 19)

4. Minimise Z = 3*x* + 5*y*
such that $x+3y\geq 3,\ x+y\geq 2,\ x,y\geq 0$.
Solution:
The feasible region determined by the system of constraints, x + 3y ≥ 3, x + y ≥ 2, and x, y ≥ 0 is given below



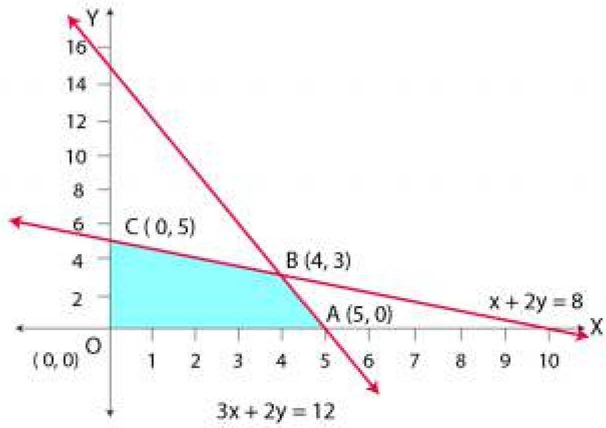
It can be seen that the feasible region is unbounded.
The corner points of the feasible region are A (3, 0), B (3 / 2, 1 / 2) and C (0, 2)
The values of Z at these corner points are given below

Corner point	Z = 3x + 5y	
A (3, 0)	9	
B (3 / 2, 1 / 2)	7	Smallest
C (0, 2)	10	

7 may or may not be the minimum value of Z because the feasible region is unbounded
For this purpose, we draw the graph of the inequality, 3x + 5y < 7 and check the resulting half plane have common points with the feasible region or not
Hence, it can be seen that the feasible region has no common point with 3x + 5y < 7

Thus, the minimum value of Z is 7 at point B (3 / 2, 1 / 2)

5. Maximise Z = 3*x* + 2*y*
subject to $x+2y\le 10, 3x+y\le 15, x, y\ge 0$.
Solution:
The feasible region determined by the constraints, $x + 2y \le 10$, $3x + y \le 15$, $x \ge 0$, and $y \ge 0$, is given below

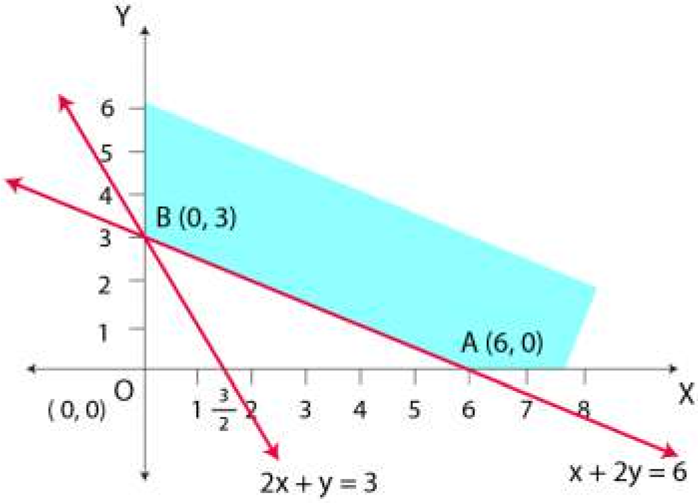


A (5, 0), B (4, 3), C (0, 5) and D (0, 0) are the corner points of the feasible region.
The values of Z at these corner points are given below

Corner point	Z = 3x + 2y	
A (5, 0)	15	
B (4, 3)	18	Maximum
C (0, 5)	10	

Hence, the maximum value of Z is 18 at the point (4, 3)

6. Minimise Z = *x* + 2*y*
subject to $2x+y\ge 3,\ x+2y\ge 6,\ x, y\ge 0$.
Solution:
The feasible region determined by the constraints, $2x + y \ge 3$, $x + 2y \ge 6$, $x \ge 0$, and $y \ge 0$, is given below

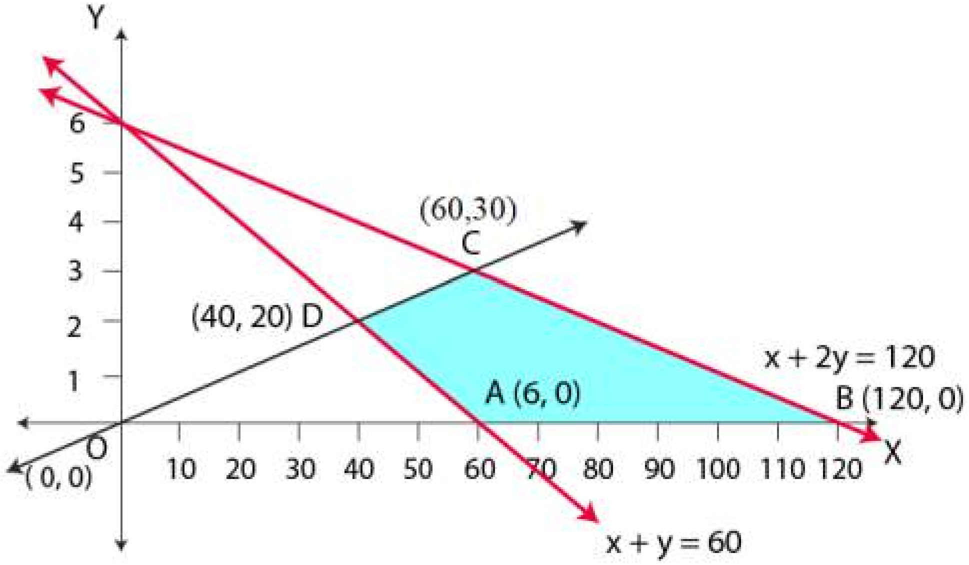


A (6, 0) and B (0, 3) are the corner points of the feasible region
The values of Z at the corner points are given below

Corner point	Z = x + 2y
A (6, 0)	6
B (0, 3)	6

Here, the values of Z at points A and B are same. If we take any other point such as (2, 2) on line x + 2y = 6, then Z = 6
Hence, the minimum value of Z occurs for more than 2 points.
Therefore, the value of Z is minimum at every point on the line x + 2y = 6

7. Minimise and Maximise Z = 5*x* + 10*y*
subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$.
Solution:
The feasible region determined by the constraints, $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, and $y \geq 0$, is given below

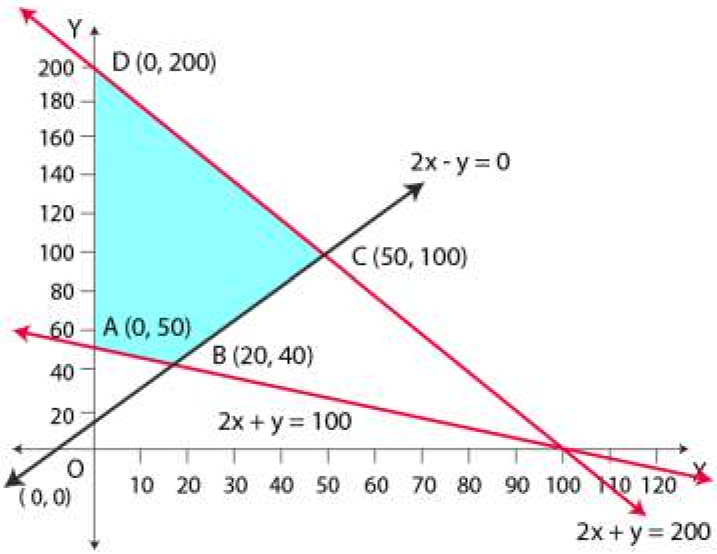


A (60, 0), B (120, 0), C (60, 30), and D (40, 20) are the corner points of the feasible region. The values of Z at these corner points are given

Corner point	Z = 5x + 10y	
A (60, 0)	300	Minimum
B (120, 0)	600	Maximum
C (60, 30)	600	Maximum
D (40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30)

8. Minimise and Maximise Z = $x + 2y$
subject to $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200;\ x, y \geq 0$.
Solution:
The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is given below



A (0, 50), B (20, 40), C (50, 100) and D (0, 200) are the corner points of the feasible region. The values of Z at these corner points are given below

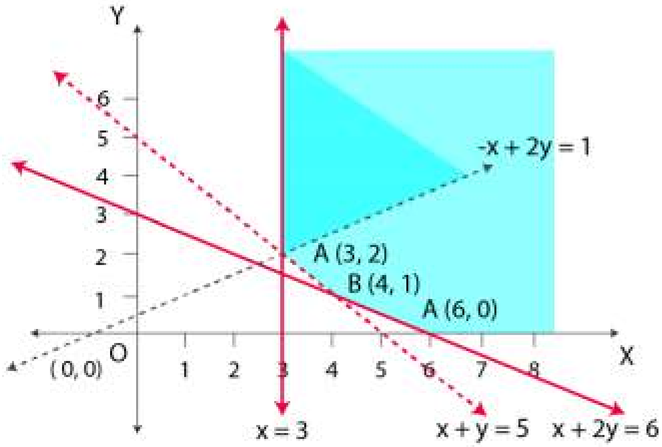
Corner point	Z = x + 2y	
A (0, 50)	100	Minimum
B (20, 40)	100	Minimum
C (50, 100)	250	
D (0, 200)	400	Maximum

The maximum value of Z is 400 at point (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40)

9. Maximise Z = − *x* + 2*y*, subject to the constraints:
$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

Solution:

The feasible region determined by the constraints, $x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$ is given below

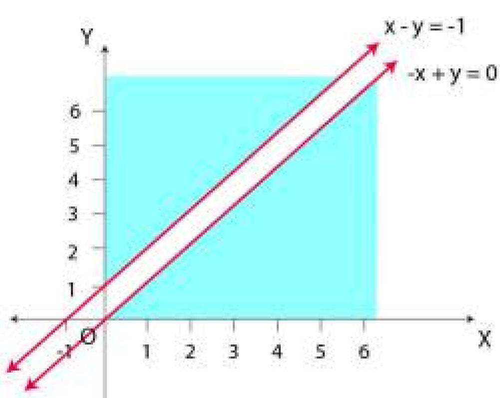


Here, it can be seen that the feasible region is unbounded.
The values of Z at corner points A (6, 0), B (4, 1) and C (3, 2) are given below

Corner point	Z = - x + 2y
A (6, 0)	Z = - 6
B (4, 1)	Z = - 2
C (3, 2)	Z = 1

Since the feasible region is unbounded, hence, z = 1 may or may not be the maximum value
For this purpose, we graph the inequality, - x + 2y > 1, and check whether the resulting half plane has points in common with the feasible region or not.
Here, the resulting feasible region has points in common with the feasible region
Hence, z = 1 is not the maximum value.
Z has no maximum value.

10. Maximise Z = *x* + *y*, subject to $x - y \le -1,\ -x + y \le 0,\ x, y \ge 0$.
Solution:
The region determined by the constraints, $x - y \le -1,\ -x + y \le 0,\ x, y \ge 0$ is given below



There is no feasible region and therefore, z has no maximum value.