

## EXERCISE 13.1

PAGE NO: 538

1. Given that E and F are events such that  $P(E) = 0.6$ ,  $P(F) = 0.3$  and  $P(E \cap F) = 0.2$ , find  $P(E|F)$  and  $P(F|E)$

**Solution:**

Given  $P(E) = 0.6$ ,  $P(F) = 0.3$  and  $P(E \cap F) = 0.2$

We know that by the definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

By substituting the values we get

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$\text{And } \Rightarrow P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$$

2. Compute  $P(A|B)$ , if  $P(B) = 0.5$  and  $P(A \cap B) = 0.32$

**Solution:**

Given:  $P(B) = 0.5$  and  $P(A \cap B) = 0.32$

We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now by substituting the values we get

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$$

3. If  $P(A) = 0.8$ ,  $P(B) = 0.5$  and  $P(B|A) = 0.4$ , find

(i)  $P(A \cap B)$

(ii)  $P(A|B)$

(iii)  $P(A \cup B)$

**Solution:**

Given  $P(A) = 0.8$ ,  $P(B) = 0.5$  and  $P(B|A) = 0.4$

(i) We know that by definition of conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B|A) P(A)$$

$$\Rightarrow P(A \cap B) = 0.4 \times 0.8$$

$$\Rightarrow P(A \cap B) = 0.32$$

(ii) We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now substituting the values we get

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$$

$$\Rightarrow P(A|B) = 0.64$$

(iii) Now,  $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Substituting the values we get

$$\Rightarrow P(A \cup B) = 0.8 + 0.5 - 0.32 = 1.3 - 0.32$$

$$\Rightarrow P(A \cup B) = 0.98$$

**4. Evaluate  $P(A \cup B)$ , if  $2P(A) = P(B) = 5/13$  and  $P(A|B) = 2/5$ .**

**Solution:**

Given  $2P(A) = P(B) = \frac{5}{13}$  and  $P(A|B) = \frac{2}{5}$

$$\Rightarrow P(B) = \frac{5}{13}, P(A) = \frac{5}{13 \times 2} = \frac{5}{26}, P(A|B) = \frac{2}{5} \dots\dots\dots (i)$$

We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A|B) P(B)$$

$$\Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13} \dots\dots\dots (ii)$$

Now,  $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5 + 10 - 4}{26} = \frac{15 - 4}{26}$$

$$\Rightarrow P(A \cup B) = \frac{11}{26}$$

5. If  $P(A) = 6/11$ ,  $P(B) = 5/11$  and  $P(A \cup B) = 7/11$ , find

(i)  $P(A \cap B)$

(ii)  $P(A|B)$

(iii)  $P(B|A)$

**Solution:**

Given:  $P(A) = \frac{6}{11}$ ,  $P(B) = \frac{5}{11}$ ,  $P(A \cup B) = \frac{7}{11}$

(i) We know that  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow P(A \cap B) = \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{11 - 7}{11}$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

(ii) Now, by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{4/11}{5/11}$$

$$\Rightarrow P(A|B) = \frac{4}{5}$$

(iii) Again, by definition of conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{4/11}{6/11} = \frac{4}{6} = \frac{2}{3}$$

$$\Rightarrow P(B|A) = \frac{2}{3}$$

Determine  $P(E|F)$  in Exercises 6 to 9.

6. A coin is tossed three times, where

- (i)  $E$  : head on third toss,  $F$  : heads on first two tosses
- (ii)  $E$  : at least two heads,  $F$  : at most two heads
- (iii)  $E$  : at most two tails,  $F$  : at least one tail

**Solution:**

The sample space of the given experiment will be:

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

(i) Here,  $E$ : head on third toss

And  $F$ : heads on first two tosses

$$\Rightarrow E = \{HHH, HTH, THH, TTH\} \text{ and } F = \{HHH, HHT\}$$

$$\Rightarrow E \cap F = \{HHH\}$$

$$\text{So, } P(E) = \frac{4}{8} = \frac{1}{2}, P(F) = \frac{2}{8} = \frac{1}{4}, P(E \cap F) = \frac{1}{8}$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/8}{1/4} = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

(ii) Here, E: at least two heads

And F: at most two heads

$$\Rightarrow E = \{HHH, HHT, HTH, THH\} \text{ and } F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$\Rightarrow E \cap F = \{HHT, HTH, THH\}$$

$$\text{So, } P(E) = \frac{4}{8} = \frac{1}{2}, P(F) = \frac{7}{8}, P(E \cap F) = \frac{3}{8}$$

Now, we know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{3/8}{7/8} = \frac{3}{7}$$

(iii) Here, E: at most two tails

And F: at least one tail

$$\Rightarrow E = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$$

$$\text{And } F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$\text{So, } P(E) = \frac{7}{8}, P(F) = \frac{7}{8}, P(E \cap F) = \frac{6}{8} = \frac{3}{4}$$

Now, we know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{3/4}{7/8} = \frac{6}{7}$$

**7. Two coins are tossed once, where**

**(i) E: tail appears on one coin, F: one coin shows head**

**(ii) E: no tail appears, F: no head appears**

**Solution:**

The sample space of the given experiment is  $S = \{HH, HT, TH, TT\}$

(i) Here, E: tail appears on one coin

And F: one coin shows head

$$\Rightarrow E = \{HT, TH\} \text{ and } F = \{HT, TH\}$$

$$\Rightarrow E \cap F = \{HT, TH\}$$

$$\text{So, } P(E) = \frac{2}{4} = \frac{1}{2}, P(F) = \frac{2}{4} = \frac{1}{2}, P(E \cap F) = \frac{2}{4} = \frac{1}{2}$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Substituting the values we get

$$\Rightarrow P(E|F) = \frac{1/2}{1/2}$$

$$\Rightarrow P(E|F) = 1$$

(ii) Here, E: no tail appears

And F: no head appears

$$\Rightarrow E = \{HH\} \text{ and } F = \{TT\}$$

$$\Rightarrow E \cap F = \varnothing$$

$$\text{So, } P(E) = \frac{1}{4}, P(F) = \frac{1}{4}, P(E \cap F) = \frac{0}{4} = 0$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Substituting the values we get

$$\Rightarrow P(E|F) = \frac{0}{1/4}$$

$$\Rightarrow P(E|F) = 0$$

**8. A die is thrown three times, E: 4 appears on the third toss, F: 6 and 5 appears respectively on first two tosses.**

**Solution:**

The sample space has 216 outcomes, where each element of the sample space has 3 entries and is of the form  $(x, y, z)$  where  $1 \leq x, y, z \leq 6$ .

Here, E: 4 appears on the third toss

$$\Rightarrow E = \left\{ \begin{array}{l} (1,1,4), (1,2,4), (1,3,4), (1,4,4), (1,5,4), (1,6,4), \\ (2,1,4), (2,2,4), (2,3,4), (2,4,4), (2,5,4), (2,6,4), \\ (3,1,4), (3,2,4), (3,3,4), (3,4,4), (3,5,4), (3,6,4), \\ (4,1,4), (4,2,4), (4,3,4), (4,4,4), (4,5,4), (4,6,4), \\ (5,1,4), (5,2,4), (5,3,4), (5,4,4), (5,5,4), (5,6,4), \\ (6,1,4), (6,2,4), (6,3,4), (6,4,4), (6,5,4), (6,6,4) \end{array} \right\}$$

Now, F: 6 and 5 appears respectively on first two tosses

$$\Rightarrow F = \{(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)\}$$

$$\Rightarrow E \cap F = \{(6, 5, 4)\}$$

$$\text{So, } P(E) = \frac{36}{216}, P(F) = \frac{6}{216}, P(E \cap F) = \frac{1}{216}$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Now by substituting the values we get

$$\Rightarrow P(E|F) = \frac{1/216}{6/216} = \frac{1}{6}$$

$$\Rightarrow P(E|F) = \frac{1}{6}$$

### 9. Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

#### Solution:

Let M denote mother, F denote father and S denote son.

Then, the sample space for the given experiment will be:

$$S = \{MFS, SFM, FSM, MSF, SMF, FMS\}$$

Here, E: Son on one end

And F: Father in middle

$$\Rightarrow E = \{MFS, SFM, SMF, FMS\} \text{ and } F = \{MFS, SFM\}$$

$$\Rightarrow E \cap F = \{MFS, SFM\}$$

$$\text{So, } P(E) = \frac{4}{6} = \frac{2}{3}, P(F) = \frac{2}{6} = \frac{1}{3}, P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Now by substituting the values we get

$$\Rightarrow P(E|F) = \frac{1/3}{1/3} = 1$$

$$\Rightarrow P(E|F) = 1$$

### 10. A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.



(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

**Solution:**

Let B denote black coloured die and R denote red coloured die.

Then, the sample space for the given experiment will be:

$$S = \left\{ \begin{array}{l} (B1, R1), (B1, R2), (B1, R3), (B1, R4), (B1, R5), (B1, R6), \\ (B2, R1), (B2, R2), (B2, R3), (B2, R4), (B2, R5), (B2, R6), \\ (B3, R1), (B3, R2), (B3, R3), (B3, R4), (B3, R5), (B3, R6), \\ (B4, R1), (B4, R2), (B4, R3), (B4, R4), (B4, R5), (B4, R6), \\ (B5, R1), (B5, R2), (B5, R3), (B5, R4), (B5, R5), (B5, R6), \\ (B6, R1), (B6, R2), (B6, R3), (B6, R4), (B6, R5), (B6, R6) \end{array} \right\}$$

(a) Let A be the event of 'obtaining a sum greater than 9' and B be the event of 'getting a 5 on black die'.

Then,  $A = \{(B4, R6), (B5, R5), (B5, R6), (B6, R4), (B6, R5), (B6, R6)\}$

And  $B = \{(B5, R1), (B5, R2), (B5, R3), (B5, R4), (B5, R5), (B5, R6)\}$

$\Rightarrow A \cap B = \{(B5, R5), (B5, R6)\}$

So,  $P(A) = \frac{6}{36} = \frac{1}{6}$ ,  $P(B) = \frac{6}{36} = \frac{1}{6}$ ,  $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Now, we know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now by substituting the values we get

$$\Rightarrow P(A|B) = \frac{1/18}{1/6} = \frac{6}{18} = \frac{1}{3}$$

$$\Rightarrow P(A|B) = \frac{1}{3}$$

(b) Let A be the event of 'obtaining a sum 8' and B be the event of 'getting a number less than 4 on red die'.

Then,  $A = \{(B2, R6), (B3, R5), (B4, R4), (B5, R3), (B6, R2)\}$

$$B = \left\{ \begin{array}{l} (B1, R1), (B2, R1), (B3, R1), (B4, R1), (B5, R1), (B6, R1), \\ (B1, R2), (B2, R2), (B3, R2), (B4, R2), (B5, R2), (B6, R2), \\ (B1, R3), (B2, R3), (B3, R3), (B4, R3), (B5, R3), (B6, R3) \end{array} \right\}$$

And

$$\Rightarrow A \cap B = \{(B5, R3), (B6, R2)\}$$

$$\text{So, } P(A) = \frac{5}{36}, P(B) = \frac{18}{36} = \frac{1}{2}, P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Now, we know that

By definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now by substituting the values we get

$$\Rightarrow P(A|B) = \frac{1/18}{1/2} = \frac{2}{18} = \frac{1}{9}$$

$$\Rightarrow P(A|B) = \frac{1}{9}$$

**11. A fair die is rolled. Consider events  $E = \{1,3,5\}$ ,  $F = \{2,3\}$  and  $G = \{2,3,4,5\}$**

**Find**

**(i)  $P(E|F)$  and  $P(F|E)$**

**(ii)  $P(E|G)$  and  $P(G|E)$**

**(iii)  $P((E \cup F)|G)$  and  $P((E \cap F)|G)$**

**Solution:**

The sample space for the given experiment is  $S = \{1, 2, 3, 4, 5, 6\}$

Here,  $E = \{1, 3, 5\}$ ,  $F = \{2, 3\}$  and  $G = \{2, 3, 4, 5\}$  ..... (i)

$$\Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}, P(F) = \frac{2}{6} = \frac{1}{3}, P(G) = \frac{4}{6} = \frac{2}{3}$$
 ..... (ii)

Now,  $E \cap F = \{3\}$ ,  $F \cap G = \{2, 3\}$ ,  $E \cap G = \{3, 5\}$  ..... (iii)

$$\Rightarrow P(E \cap F) = \frac{1}{6}, P(F \cap G) = \frac{2}{6} = \frac{1}{3}, P(E \cap G) = \frac{2}{6} = \frac{1}{3}$$
 ..... (iv)

(i) We know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/6}{1/3} = \frac{3}{6} = \frac{1}{2} \text{ [Using (II) and (IV)]}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

Similarly, we have

$$P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{1/6}{1/2} = \frac{2}{6} = \frac{1}{3} \text{ [Using (ii) and (iv)]}$$

$$\Rightarrow P(F|E) = \frac{1}{3}$$

(ii) We know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{1/3}{2/3} = \frac{1}{2}$$

$$\Rightarrow P(E|G) = \frac{1}{2}$$

Similarly, we have

$$P(G|E) = \frac{P(G \cap E)}{P(E)} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$\Rightarrow P(G|E) = \frac{2}{3}$$

(iii) Clearly, from (i), we have

$$E = \{1, 3, 5\}, F = \{2, 3\} \text{ and } G = \{2, 3, 4, 5\}$$

$$\Rightarrow E \cup F = \{1, 2, 3, 5\}$$

$$\Rightarrow (E \cup F) \cap G = \{2, 3, 5\}$$

$$\Rightarrow P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow P((E \cup F) \cap G) = \frac{1}{2} \dots\dots\dots (v)$$

Now, we know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P((E \cup F)|G) = \frac{P((E \cup F) \cap G)}{P(G)} = \frac{1/2}{2/3} = \frac{3}{4} \text{ [Using (ii) and (v)]}$$

$$\Rightarrow P((E \cup F)|G) = \frac{3}{4}$$

Similarly, we have  $E \cap F = \{3\}$  [Using (iii)]

And  $G = \{2, 3, 4, 5\}$  [Using (i)]

$$\Rightarrow (E \cap F) \cap G = \{3\}$$

$$\Rightarrow P((E \cap F) \cap G) = \frac{1}{6} \dots\dots\dots (vi)$$

So,

$$P((E \cap F)|G) = \frac{P((E \cap F) \cap G)}{P(G)} = \frac{1/6}{2/3} = \frac{1}{4} \text{ [Using (ii) and (vi)]}$$

$$\Rightarrow P((E \cap F)|G) = \frac{1}{4}$$

**12. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?**

**Solution:**

Let B denote boy and G denote girl.

Then, the sample space of the given experiment is  $S = \{GG, GB, BG, BB\}$

Let E be the event that 'both are girls'.

$$\Rightarrow E = \{GG\}$$

$$\Rightarrow P(E) = \frac{1}{4}$$

(i) Let F be the event that 'the youngest is a girl'.

$$\Rightarrow F = \{GG, BG\}$$

$$\Rightarrow P(F) = \frac{2}{4} = \frac{1}{2} \dots\dots\dots (i)$$

Now,  $E \cap F = \{GG\}$

$$\Rightarrow P(E \cap F) = \frac{1}{4} \dots\dots\dots (ii)$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2} \text{ [Using (i) and (ii)]}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

(ii) Let H be the event that 'at least one is a girl'.

$$\Rightarrow H = \{GG, GB, BG\}$$

$$\Rightarrow P(H) = \frac{3}{4} \dots\dots\dots (iii)$$

Now,  $E \cap H = \{GG\}$

$$\Rightarrow P(E \cap H) = \frac{1}{4} \dots\dots\dots (iv)$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

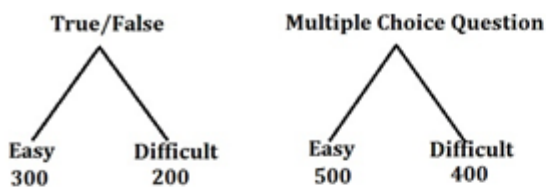
$$\Rightarrow P(E|H) = \frac{P(E \cap H)}{P(H)} = \frac{1/4}{3/4} = \frac{1}{3} \text{ [Using (iii) and (iv)]}$$

$$\Rightarrow P(E|H) = \frac{1}{3}$$

**13. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?**

**Solution:**

Here, there are two types of questions, True/False or Multiple Choice Questions (T/F or MCQ), and each of them are divided into Easy and Difficult type, as shown below in the tree diagram.



So, in all, there are, 500 T/F questions and 900 MCQs.

Also, there are 800 Easy questions and 600 difficult questions.

$\Rightarrow$  the sample space of this experiment has  $500 + 900 = 1400$  outcomes.

Now, let E be the event of 'getting an Easy question' and F be the event of 'getting an MCQ'.

$$\Rightarrow P(E) = \frac{800}{1400} = \frac{8}{14} \text{ And } P(F) = \frac{900}{1400} = \frac{9}{14} \dots\dots\dots (i)$$

Now,  $E \cap F$  is the event of getting an MCQ which is Easy.

Clearly, from the diagram, we know that there are 500 MCQs that are easy.

$$\text{So } P(E \cap F) = \frac{500}{1400} = \frac{5}{14} \dots\dots\dots (ii)$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{5/14}{9/14} = \frac{5}{9} \text{ [Using (i) and (ii)]}$$

$$\Rightarrow P(E|F) = \frac{5}{9}$$

**14. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event ‘the sum of numbers on the dice is 4’.**

**Solution:**

The sample space of the given experiment is given below

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

Let E be the event that ‘the sum of numbers on the dice is 4’ and F be the event that ‘the two numbers appearing on throwing the two dice are different’.

$$\Rightarrow E = \{(1, 3), (2, 2), (3, 1)\}$$

$$F = \left\{ \begin{array}{l} (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5) \end{array} \right\}$$

And

$$\Rightarrow E \cap F = \{(1, 3), (3, 1)\}$$

$$\Rightarrow P(E) = \frac{3}{36} = \frac{1}{12}, P(F) = \frac{30}{36} = \frac{5}{6}, P(E \cap F) = \frac{2}{36} = \frac{1}{18} \dots\dots\dots (i)$$

Now, we know that by definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

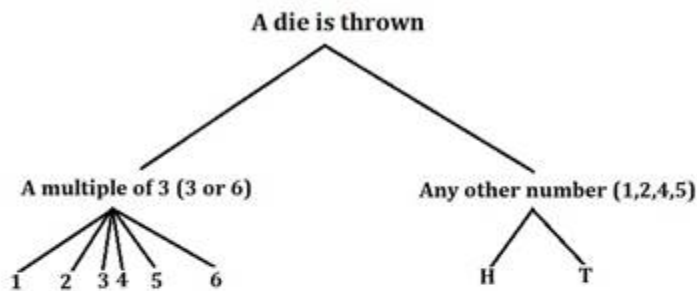
$$\Rightarrow P(E|F) = \frac{1/18}{5/6} = \frac{1}{15}$$

$$\Rightarrow P(E|F) = \frac{1}{15}$$

**15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.**

**Solution:**

The experiment is explained below in the tree diagram:



The sample space of the given experiment is given below

$$S = \left\{ \begin{array}{l} (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), \\ 1H, 2H, 4H, 5H, 1T, 2T, 4T, 5T \end{array} \right\}$$

Let E be the event that 'the coin shows a tail' and F be the event that 'at least one die shows a 3'.

$$\Rightarrow E = \{1T, 2T, 4T, 5T\} \text{ and } F = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$$

$$\Rightarrow E \cap F = \varnothing \Rightarrow P(E \cap F) = 0 \dots\dots\dots (i)$$

Now, we know that by definition of conditional probability,



$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{0}{P(F)} = 0 \quad [\text{Using (i)}]$$

$$\Rightarrow P(E|F) = 0$$

16. If  $P(A) = 1/2$ ,  $P(B) = 0$ , then  $P(A|B)$  is

- A. 0
- B.  $\frac{1}{2}$
- C. not defined
- D. 1

**Solution:**

C. Not defined

**Explanation:**

We know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \dots\dots\dots (i)$$

Given:  $P(A) = \frac{1}{2}$

And  $P(B) = 0$

$\Rightarrow$  Using (i), we have

$$P(A|B) = \frac{P(A \cap B)}{0} = (A \cap B) \times \frac{1}{0}, \text{ which is not defined.}$$

17. If A and B are events such that  $P(A|B) = P(B|A)$ , then

- A.  $A \subset B$  but  $A \neq B$
- B.  $A = B$
- C.  $A \cap B = \phi$
- D.  $P(A) = P(B)$

**Solution:**

D.  $P(A) = P(B)$

**Explanation:**

Given:  $P(A|B) = P(B|A)$  ..... (i)

Now, we know that by definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ ..... (ii)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \text{ ..... (iii)}$$

Using (i), we have

$$P(A|B) = P(B|A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A) = P(B)$$