EXERCISE 13.2 PAGE NO: 546

1. If P (A) = 3/5 and P (B) = 1/5, find P (A ∩ B) if A and B are independent events.

Solution:
Given P (A) = 3/5 and P (B) = 1/5
As A and B are independent events.
⇒ P (A ∩ B) = P (A).P (B)
= 3/5 × 1/5 = 3/25

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Solution:
Given a pack of 52 cards.
As we know there are 26 cards in total which are black. Let A and B denotes respectively the events that the first and second drawn cards are black.
Now, P (A) = P (black card in first draw) = 26/52 = ½
Because the second card is drawn without replacement so, now the total number of black card will be 25 and total cards will be 51 that is the conditional probability of B given that A has already occurred.
Now, P (B/A) = P (black card in second draw) = 25/51
Thus the probability that both the cards are black
⇒ P (A ∩ B) = ½ × 25/51 = 25/102
Hence, the probability that both the cards are black = 25/102.

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Solution:
Given a box of oranges.
Let A, B and C denotes respectively the events that the first, second and third drawn orange is good.
Now, P (A) = P (good orange in first draw) = 12/15

Because the second orange is drawn without replacement so, now the total number of good oranges will be 11 and total oranges will be 14 that is the conditional probability of B given that A has already occurred.
Now, P (B/A) = P (good orange in second draw) = 11/14
Because the third orange is drawn without replacement so, now the total number of good oranges will be 10 and total orangs will be 13 that is the conditional probability of C given that A and B has already occurred.
Now, P (C/AB) = P (good orange in third draw) = 10/13
Thus the probability that all the oranges are good
⇒ $P(A \cap B \cap C) = 12/15 \times 11/14 \times 10/13 = 44/91$
Hence, the probability that a box will be approved for sale = 44/91

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Solution:
Given a fair coin and an unbiased die are tossed.
We know that the sample space S:
S = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}
Let A be the event head appears on the coin:
⇒ A = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}
⇒ P (A) = 6/12 = ½
Now, Let B be the event 3 on the die
⇒ B = {(H, 3), (T, 3)}
⇒ P (B) = 2/12 = 1/6
As, $A \cap B$ = {(H, 3)}
⇒ $P(A \cap B)$ = 1/12 …... (1)
And P (A). P (B) = ½ × 1/6 = 1/12 …… (2)
From (1) and (2) $P(A \cap B)$ = P (A). P (B)
Therefore, A and B are independent events.

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

Solution:
The sample space for the dice will be

S = {1, 2, 3, 4, 5, 6}
Let A be the event, the number is even:
⇒ A = {2, 4, 6}
⇒ P (A) = 3/6 = ½
Now, Let B be the event, the number is red:
⇒ B = {1, 2, 3}
⇒ P (B) = 3/6 = 1/2
As, A ∩ B = {2}
⇒ P (A ∩ B) = 1/6 .... (1)
And P (A). P (B) = ½ × ½ = ¼ ..... (2)
From (1) and (2) P (A ∩ B) ≠ P (A). P (B)
Therefore, A and B are not independent events.

6. Let E and F be events with P (E) = 3/5, P (F) = 3/10 and P (E ∩ F) = 1/5. Are E and F independent?

Solution:
Given P (E) = 3/5, P (F) = 3/10 and P (E ∩ F) = 1/5
P (E). P (F) = 3/5 × 3/10 = 9/50 ≠ 1/5
⇒ P (E ∩ F) ≠ P (E). P (F)
Therefore, E and F are not independent events.

7. Given that the events A and B are such that P (A) = 1/2, P (A ∪ B) = 3/5 and P (B) = p. Find p if they are (i) mutually exclusive (ii) independent.

Solution:
Given P (A) = ½, P (A ∪ B) = 1/5 and P (B) = p
(i) Mutually exclusive
When A and B are mutually exclusive.
Then (A ∩ B) = φ
⇒ P (A ∩ B) = 0
As we know, P (A ∪ B) = P (A) + P (B) - P (A ∩ B)
⇒ 3/5 = ½ + p -0
⇒ P = 3/5 – ½ = 1/10

(ii) Independent
When A and B are independent.

⇒ P (A ∩ B) = P (A). P (B)
⇒ P (A ∩ B) = ½ p
As we know, P (A ∪ B) = P (A) + P (B) - P (A ∩ B)
⇒ 3/5 = ½ + 2 – p/2
⇒ p/2 = 3/5 – ½
⇒ p = 2 × 1/10 = 1/5

8. Let A and B be independent events with P (A) = 0.3 and P (B) = 0.4. Find
(i) P (A ∩ B)
(ii) P (A ∪ B)
(iii) P (A|B)
(iv) P (B|A)

Solution:
Given P (A) = 0.3 and P(B) = 0.4
(i) P (A ∩ B)
When A and B are independent.
⇒ P (A ∩ B) = P (A). P (B)
⇒ P (A ∩ B) = 0.3 × 0.4
⇒ P (A ∩ B) = 0.12

(ii) P (A ∪ B)
As we know, P (A ∪ B) = P (A) + P (B) - P (A ∩ B)
⇒ P (A ∪ B) = 0.3 + 0.4 – 0.12
⇒ P (A ∪ B) = 0.58

(iii) P (A|B)

As we know $P(A|B) = \frac{P(A \cap B)}{P(B)}$

⇒ $P(A|B) = \frac{0.12}{0.4}$

⇒ P (A|B) = 0.3

(iv) P (B|A)

As we know $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(B|A) = \frac{0.12}{0.3}$

$\Rightarrow P(B|A) = 0.4$

9. If A and B are two events such that P (A) = 1/4 , P (B) = 1/2 and P (A ∩ B) = 1/8, find P (not A and not B).

Solution:

Given P (A) = ¼, P (B) = ½ and P (A ∩ B) = 1/8

P (not A and not B) = $P(A' \cap B')$

As, $\{A' \cap B' = (A \cup B)'\}$

⇒ P (not A and not B) = $P((A \cup B)')$

= 1 - P (A ∪ B)

= 1- [P (A) + P (B) - P (A ∩ B)]

$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right]$

$= 1 - \left[\frac{5}{8}\right] = \frac{3}{8}$

10. Events A and B are such that P (A) = 1/2, P (B) = 7/12 and P (not A or not B) = 1/4. State whether A and B are independent?

Solution:

Given P (A) = ½, P (B) =7/12 and P (not A or not B) = 1/4

⇒ $P(A' \cup B') = 1/4$

⇒ $P(A \cap B)' = 1/4$

⇒ 1 - P (A ∩ B) = 1/4

⇒ P (A ∩ B) = 1 - 1/4

⇒ P (A ∩ B) = 3/4…….. (1)

And P (A). P (B) = ½ × 7/12 = 7/24 …. (2)

From (1) and (2) P (A ∩ B) ≠ P (A). P (B)

Therefore, A and B are not independent events.

11. Given two independent events A and B such that P (A) = 0.3, P(B) = 0.6.
Find
(i) P (A and B)
(ii) P (A and not B)
(iii) P (A or B)
(iv) P (neither A nor B)

Solution:
Given P (A) = 0.3, P (B) = 0.6.
(i) P (A and B)
As A and B are independent events.
$\Rightarrow$ P (A and B) = P (A ∩ B) = P (A). P (B)
= 0.3 × 0.6
= 0.18

(ii) P (A and not B)
$\Rightarrow$ P (A and not B) = P (A ∩ B′) = P (A) – P (A ∩ B)
= 0.3 - 0.18
= 0.12

(iii) P (A or B)
$\Rightarrow$ P (A or B) = P (A ∪ B)
As we know, P (A ∪ B) = P (A) + P (B) - P (A ∩ B)
$\Rightarrow$ P (A ∪ B) = 0.3 + 0.6 – 0.18
$\Rightarrow$ P (A ∪ B) = 0.72

(iv) P (neither A nor B)
P (neither A nor B) = P (A′ ∩ B′)
As, { A′ ∩ B′ = (A ∪ B)′}
$\Rightarrow$ P (neither A nor B) = P ((A ∪ B)′)
= 1 - P (A ∪ B)
= 1 - 0.72
= 0.28

12. A die is tossed thrice. Find the probability of getting an odd number at least once.

Solution:

Given a die is tossed thrice.
Then the sample space S = {1, 2, 3, 4, 5, 6}
Let P (A) = probability of getting an odd number in first throw.
⇒ P (A) = 3/6 = ½.
Let P (B) = probability of getting an even number.
⇒ P (B) =3/6 = ½.
Now, probability of getting an even number in three times = ½ × ½ × ½ = 1/8
So, probability of getting an odd number at least once
= 1 – probability of getting an odd number in no throw
= 1 - probability of getting an even number in three times
= 1 – 1/8
∴ Probability of getting an odd number at least once = 7/8

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red.
(ii) First ball is black and second is red.
(iii) One of them is black and other is red.

Solution:
Given A box containing 10 black and 8 red balls.
Total number of balls in box = 18
(i) Both balls are red.
Probability of getting a red ball in first draw = 8/18 = 4/9
As the ball is replaced after first throw,
Hence, Probability of getting a red ball in second draw = 8/18 = 4/9
Now, Probability of getting both balls red = 4/9 × 4/9 = 16/81

(ii) First ball is black and second is red.
Probability of getting a black ball in first draw = 10/18 = 5/9
As the ball is replaced after first throw,
Hence, Probability of getting a red ball in second draw = 8/18 = 4/9
Now, Probability of getting first ball is black and second is red = 5/9 × 5/9 = 20/81

(iii) One of them is black and other is red.
Probability of getting a black ball in first draw = 10/18 = 5/9
As the ball is replaced after first throw,

Hence, Probability of getting a red ball in second draw = 8/18 = 4/9
Now, Probability of getting first ball is black and second is red = 5/9 × 4/9 = 20/81
Probability of getting a red ball in first draw = 8/18 = 4/9
As the ball is replaced after first throw,
Hence, Probability of getting a black ball in second draw = 10/18 = 5/9
Now, Probability of getting first ball is red and second is black = 5/9 × 4/9 = 20/81
Therefore, Probability of getting one of them is black and other is red:
= Probability of getting first ball is black and second is red + Probability of getting first ball is red and second is black
= 20/81 + 20 /81 = 40/81

14. Probability of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that
(i) The problem is solved
(ii) Exactly one of them solves the problem.

Solution:
Given,
P (A) = Probability of solving the problem by A = 1/2
P (B) = Probability of solving the problem by B = 1/3
Because A and B both are independent.
⇒ P (A ∩ B) = P (A). P (B)
⇒ P (A ∩ B) = ½ × 1/3 = 1/6
P (A′) = 1 – P (A) = 1 – 1/2 = 1/2
P (B′) = 1 – P (B) = 1 – 1/3 = 2/3
(i) The problem is solved
The problem is solved, i.e. it is either solved by A or it is solved by B.
= P (A ∪ B)
As we know, P (A ∪ B) = P (A) + P (B) - P (A ∩ B)
⇒ P (A ∪ B) = ½ + 1/3 – 1/6 = 4/6
⇒ P (A ∪ B) = 2/3

(ii) Exactly one of them solves the problem
That is either problem is solved by A but not by B or vice versa
That is P (A).P (B′) + P (A′).P (B)
= ½ (2/3) + ½ (1/3)
= 1/3 + 1/6 = 3/6

⇒ P (A).P (B′) + P (A′).P (B) = ½

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?
(i) E: 'the card drawn is a spade' F: 'the card drawn is an ace'
(ii) E: 'the card drawn is black' F: 'the card drawn is a king'
(iii) E: 'the card drawn is a king or queen' F: 'the card drawn is a queen or jack'.

Solution:
Given: A deck of 52 cards.
(i) In a deck of 52 cards, 13 cards are spade and 4 cards are ace and only one card is there which is spade and ace both.
Hence, P (E) = the card drawn is a spade = 13/52 = 1/4
P (F) = the card drawn is an ace = 4/52 = 1/13
P (E ∩ F) = the card drawn is a spade and ace both = 1/52….. (1)
And P (E). P (F)
= ¼ × 1/13 = 1/52…. (2)
From (1) and (2)
⇒ P (E ∩ F) = P (E). P (F)
Hence, E and F are independent events.

(ii) In a deck of 52 cards, 26 cards are black and 4 cards are king and only two card are there which are black and king both.
Hence, P (E) = the card drawn is of black = 26/52 = ½
P (F) = the card drawn is a king = 4/52 = 1/13
P (E ∩ F) = the card drawn is a black and king both = 2/52 = 1/26.... (1)
And P (E). P (F)
= ½ × 1/13 = 1/26…. (2)
From (1) and (2)
⇒ P (E ∩ F) = P (E). P (F)
Hence, E and F are independent events.

(iii) In a deck of 52 cards, 4 cards are queen, 4 cards are king and 4 cards are jack.
Hence, P (E) = the card drawn is either king or queen = 8/52 = 2/13
P (F) = the card drawn is either queen or jack = 8/52 = 2/13
There are 4 cards which are either king or queen and either queen or jack.
P (E ∩ F) = the card drawn is either king or queen and either queen or jack = 4/52 = 1/13

… (1)
And P (E). P (F)
= 2/13 × 2/13 = 4/169…. (2)
From (1) and (2)
⇒ P (E ∩ F) ≠ P (E). P (F)
Hence, E and F are not independent events.

16. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
(a) Find the probability that she reads neither Hindi nor English newspapers.
(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.
(c) If she reads English newspaper, find the probability that she reads Hindi newspaper

Solution:
Given:
Let H and E denote the number of students who read Hindi and English newspaper respectively.
Hence, P (H) = Probability of students who read Hindi newspaper= 60/100 = 3/5
P (E) = Probability of students who read English newspaper = 40/100 = 2/5
P (H ∩ E) = Probability of students who read Hindi and English both newspaper = 20/100 = 1/5
(a) Find the probability that she reads neither Hindi nor English newspapers.
P (neither H nor E)
P (neither H nor E) = P ($H' \cap E'$)
As, { $H' \cap E' = (H \cup E)'$}
⇒ P (neither A nor B) = P ($(H \cup E)'$)
= 1 - P (H ∪ E)
= 1- [P (H) + P (E) - P (H ∩ E)]
$$= 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right]$$
$$= 1 - \left[\frac{4}{5}\right] = \frac{1}{5}$$

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

P (E|H) = Hindi newspaper reading has already occurred and the probability that she reads English newspaper is to find.

As we know $P\ (E|H) = \frac{P\ (H \cap E)}{P(H)}$

$\Rightarrow P\ (E|H) = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{5} \times \frac{5}{3}$

⇒ P (E|H) = 1/3

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

P (H|E) = English newspaper reading has already occurred and the probability that she reads Hindi newspaper is to find.

As we know $P\ (H|E) = \frac{P\ (H \cap E)}{P(E)}$

$\Rightarrow P\ (H|E) = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{5} \times \frac{5}{2}$

⇒ P (H|E) = ½

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is
A. 0
B. 1/3
C. 1/12
D. 1/36

Solution:
D. 1/36

Explanation:

Given A pair of dice is rolled.
Hence the number of outcomes = 36
Let P (E) be the probability to get an even prime number on each die.
As we know the only even prime number is 2.
So, E = {2, 2}
⇒ P € = 1/36

18. Two events A and B will be independent, if
(A) A and B are mutually exclusive
(B) P (A'B') = [1 – P (A)] [1 – P (B)]
(C) P (A) = P (B)
(D) P (A) + P (B) = 1

Solution:
(B) P (A'B') = [1 – P (A)] [1 – P (B)]

Explanation:
P (A'B') = [1 – P (A)] [1 – P (B)]
⇒ P (A'∩ B') = 1 – P (A) – P (B) + P (A) P (B)
⇒ 1 - P (A ∪ B) =1 – P (A) – P (B) + P (A) P (B)
= - [P (A) + P (B) - P (A ∩ B)] = – P (A) – P (B) + P (A) P (B)
= - P (A) – P (B) + P (A ∩ B) = – P (A) – P (B) + P (A) P (B)
⇒ P (A ∩ B) = P (A). P (B)
Hence, it shows A and B are Independent events.