MISCELLANEOUS EXERCISE

1. A and B are two events such that P (A) ≠ 0. Find P (B|A), if:
(i) A is a subset of B
(ii) A ∩ B = ϕ

Solution:

It is given that,

A and B are two events such that P (A) ≠ 0

We have, $A \cap B = A$

$\therefore P\ (A \cap B) = P\ (B \cap A) = P\ (A)$

Hence, $P(B|A) = \frac{P\ (B \cap A)}{P\ (A)}$

$= \frac{P\ (A)}{P\ (A)}$

= 1

(ii) We have,

$P\ (A \cap B) = 0$

$\therefore P(B|A) = \frac{P\ (A \cap B)}{P\ (A)}$

= 0

2. A couple has two children,
(i) Find the probability that both children are males, if it is known that at least one of the children is male.
(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Solution:

(i) According to the question, if the couple has two children then the sample space is:

S = {(b, b), (b, g), (g, b), (g, g)}

Assume that A denote the event of both children having male and B denote the event of having at least one of the male children

Thus, we have:

$A \cap B = \{(b, b)\}$

$P(A \cap B) = \frac{1}{4}$

P (A) = ¼

P (B) = 3/4

Hence, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

By substituting the values we get

$= \frac{\frac{1}{4}}{\frac{3}{4}}$

= 1/3

(ii) Assume that C denote the event having both children females and D denote the event of having elder child is female

∴ C = {(g, g)}

P (C) = ¼

And, D = {(g, b), (g, g)}

P (D) = (2/4)

Hence, $P(C|D) = \frac{P(C \cap D)}{P(D)}$

$= \frac{\frac{1}{4}}{\frac{2}{4}}$

= ½

3. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that

there are equal number of males and females.

Solution:

Given that, 5% of men and 0.25% of women have grey hair

∴ Total % of people having grey hair = 5 + 0.25

= 5.25 %

Hence, Probability of having a selected person male having grey hair, P = 5/25 = 20/21

4. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Solution:

Given that, 90% of the people are right handed

Let p denotes the probability of people that are right handed and q denotes the probability of people that are left handed

p = 9/10 and q = 1 – 9/10 = 1/10

Now by using the binomial distribution probability of having more than 6 right handed people can be given as:

$$\sum_{r=7}^{10} 10\ C_r p^r q^{n-r} = \sum_{r=7}^{10} 10\ C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Hence, the probability of having more than 6 right handed people:

= 1 – P (More than 6 people are right handed)

$= 1 - \sum_{r=7}^{10} 10\ C_r (0.9)^r (0.1)^{10-r}$

5. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that:

(i) All will bear 'X' mark.

(ii) Not more than 2 will bear 'Y' mark.

(iii) At least one ball will bear 'Y' mark.

(iv) The number of balls with 'X' mark and 'Y' mark will be equal.

Solution:

(i) It is given in the question that,

Total number of balls in the urn = 25
Number of balls bearing mark 'X' = 10
Number of balls bearing mark 'Y' = 15
Let p denotes the probability of balls bearing mark 'X' and q denotes the probability of balls bearing mark 'Y'
p = 10/25 = 2/5 and q = 15/25 = 3/5
Now, 6 balls are drawn with replacement. Hence, the number of trials are Bernoulli triangle.
Assume, Z be the random variable that represents the number of balls bearing 'Y' mark in the trials
∴ Z has a binomial distribution where n = 6 and p = 2/5

$$P\ (Z = z) = ^{n}C_{z}\, p^{n-z} q^{z}$$

Hence, P (All balls will bear mark 'X') = P (Z = 0)

$$= ^{6}C_{0}\left(\frac{2}{5}\right)^{6}$$

$$= \left(\frac{2}{5}\right)^{6}$$

(ii) Probability (Not more than 2 will bear 'Y' mark) = P (Z ≤ 2)

= P (Z = 0) + P (Z = 1) + P (Z = 2)

$$= ^{6}C_{0}\,(p)^{6}(q)^{0} + ^{6}C_{1}(p)^{5}(q)^{1} + ^{6}C_{2}(p)^{4}(q)^{2}$$

$$= \left(\frac{2}{5}\right)^{6} + 6\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right) + 15\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{2}$$

$$= \left(\frac{2}{5}\right)^{4}\left[\left(\frac{2}{5}\right)^{2} + 6\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + 15\left(\frac{3}{5}\right)^{2}\right]$$

$$= \left(\frac{2}{5}\right)^{4}\left[\frac{175}{25}\right]$$

$$= 7\left(\frac{2}{5}\right)^{4}$$

(iii) Now, Probability (At least one ball will bear 'Y' mark) = P (Z ≥ 1)

= 1 – P (Z = 0)

$= 1 - \left(\frac{2}{5}\right)^6$

(iv) Probability (Having equal number of balls with 'X' mark and 'Y' mark) = P (Z = 3)

$= {}^6C_3 \left(\frac{2}{54}\right)^3 \left(\frac{3}{5}\right)^3$

$= \frac{20\times8\times27}{15625}$

$= \frac{864}{3125}$

6. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is 5/6. What is the probability that he will knock down fewer than 2 hurdles?

Solution:

Assume that p be the probability of player that will clear the hurdle while q be the probability of player that will knock down the hurdle

∴ p = 5/6 and q = 1 – 5/6 = 1/6

Let us also assume X be the random variable that represents the number of times the player will knock down the hurdle

∴ By binomial distribution, $P\ (X = x) = {}^n\ C_x\, p^{n-x} q^x$

Hence, probability (players knocking down less than 2 hurdles) = P (X < 2)

= P (X = 0) + P (X = 1)

$= {}^{10}C_0\ (q)^0 (p)^{10} + {}^{10}C_1 (q)(p)^9$

$= \left(\frac{5}{6}\right)^{10} \times \left[\frac{5}{6} + \frac{10}{6}\right]$

$= \frac{5}{2} \times \left(\frac{5}{6}\right)^9$

$= \frac{(5)^{10}}{2 \times (6)^6}$

7. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Solution:

From the given question, it is clear that
Probability of getting a six in a throw of die = 1/6
And, probability of not getting a six = 5/6
Let us assume, p = 1/6 and q = 5/6
Now, we have
Probability that the 2 sixes come in the first five throws of the die

$= {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

$= \frac{10 \times (5)^3}{(6)^5}$

Also, Probability that the six come in the sixth throw $= \frac{10 \times (5)^3}{(6)^5} \times \frac{1}{6}$

$= \frac{10 \times 125}{(6)^6}$

$= \frac{625}{23328}$

8. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Solution:

We know that, in a leap year there are total 366 days, 52 weeks and 2 days
Now, in 52 weeks there are total 52 Tuesdays
∴ Probability that the leap year will contain 53 Tuesdays is equal to the probability of remaining 2 days will be Tuesdays
Thus, the remaining two days can be

(Monday and Tuesday), (Tuesday and Wednesday), (Wednesday and Thursday), (Thursday and Friday), (Friday and Saturday), (Saturday and Sunday) and (Sunday and Monday)
∴ Total Number of cases = 7
Cases in which Tuesday can come = 2
Hence, probability (leap year having 53 Tuesdays) = 2/7

9. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

Solution:
Given that probability of failure = x
And, probability of success = 2x
∴ x + 2x = 1
3x = 1
X = 1/3
2x = 2/3
Assume p = 1/3 and q = 2/3
Also, X be the random variable that represents the number of trials
Hence, by binomial distribution we have:

$$P\ (X = x) = {}^n C_x\, p^{n-x} q^x$$

∴ Probability of having at least 4 successes $= P\ (X \geq 4)$

= P (X = 4) + P (X = 5) + P (X = 6)

$$= {}^6 C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6 C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + {}^6 C_6 \left(\frac{2}{3}\right)^6$$

$$= \frac{15\ (2)^4}{3^6} + \frac{6\ (2)^5}{3^6} + \frac{(2)^6}{3^6}$$

$$= \frac{31 \times (2)^4}{(3)^6}$$

$$= \frac{31}{9} \left(\frac{2}{3}\right)^4$$

10. How many times must a man toss a fair coin so that the probability of having at

least one head is more than 90%?

Solution:

Let us assume that, man tosses the coin n times. Thus, n tosses are the Bernoulli trials

∴ Probability of getting head at the toss of the coin = ½

Let us assume, p = ½ and q = ½

$\therefore P\ (X = x) = {}^nC_x\, p^{n-x} q^x$

$= {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x$

$= {}^nC_x \left(\frac{1}{2}\right)^n$

It is given in the question that,

Probability of getting at least one head > 90/100

$\therefore P\ (x \geq 1) > 0.9$

1 – P (x = 0) > 0.9

$1 - {}^nC_0 . \frac{1}{2^n} > 0.9$

$\frac{1}{2^n} < 0.1$

$2^n > \frac{1}{0.1}$

$2^n > 10$

Hence, the minimum value of n satisfying the given inequality = 4

∴ The man have to toss the coin 4 or more times

11. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Solution:

For the situation given in the equation, we have
Probability of getting a six in a throw of a die = 1/6
Also, probability of not getting a 6 = 5/6
Now, there are three cases from which the expected value of the amount which he wins can be calculated:
(i) First case is that, if he gets a six on his first through then the required probability will be 1/6
∴ Amount received by him = Rs. 1
(ii) Secondly, if he gets six on his second throw then the probability = (5/6 × 1/6)
= 5/36
∴ Amount received by him = - Rs. 1 + Rs. 1
= 0

(iii) Lastly, if he does not get six in first two throws and gets six in his third throw then the probability = 5/6 × 5/6 × 1/6
= 25/216
∴ Amount received by him = - Rs. 1 – Rs. 1 + Rs. 1
= - 1
Hence, expected value that he can win = 1/6 – 25/216
= (36 – 25)/216
= 11/216

12. Suppose we have four boxes A, B, C and D containing coloured marbles as given below:

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Solution:

Let us assume R be the event of drawing the red marbles

Let us also assume E_A, E_B and E_C denote the boxes A, B and C respectively

Given that,

Total number of marbles = 40

Also, total number of red marbles = 15

P (R) = 15/40

= 3/8

Probability of taking out the red marble from box A,

$$P\ (E_A|R) = \frac{P\ (E_A \cap R)}{P\ (R)}$$

$$= \frac{\frac{1}{40}}{\frac{3}{8}}$$

= 1/15

Also, probability of taking out the red marble from box B,

$$P\ (E_B|R) = \frac{P\ (E_B \cap R)}{P\ (R)}$$

$$= \frac{\frac{6}{40}}{\frac{3}{8}}$$

= 2/5

And, Probability of taking out the red marble from box C,

$$P\ (E_C|R) = \frac{P\ (E_C \cap R)}{P\ (R)}$$

$$= \frac{\frac{8}{40}}{\frac{3}{8}}$$

= 8/15

13. Assume that the chances of a patient having a heart attack are 40%. It is also

assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Solution:

Let us assume, X denotes the events having a person heart attack

A_1 denote events having the selected person followed the course of yoga and meditation

And, A_2 denote the events having the person adopted the drug prescription

It is given in the question that,

P (X) = 0.40

And, $P\,(A_1) = P\,(A_2) = \frac{1}{2}$

$$P\,(X|A_1) = 0.40 \times 0.70 = 0.28$$

$$P\,(X|A_2) = 0.40 \times 0.75 = 0.30$$

∴ Probability (The patient suffering from a heart attack and followed a course of meditation and yoga):

$$P\,(A_1|X) = \frac{P(A_1)\,P\,(X|A_1)}{P(A_1)P\,(X|A_1) + P\,(A_2)P\,(X|A_2)}$$

$$= \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30}$$

$$= \frac{14}{29}$$

14. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability 1/2).

Solution:

From the question, we have:

Total number of determinants of second order where the element being or 1 = $(2)^4$ = 16

Now, we have the value of determinants is positive in following cases:

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 11 \\ 0 & 11 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

∴ Required probability = 3/16

15. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

P (A fails) = 0.2

P (B fails alone) = 0.15

P (A and B fail) = 0.15

Evaluate the following probabilities:

(i) P (A fails | B has failed)

(ii) P (A fails alone)

Solution:

(i) Let us assume the event which is failed by A is denoted by E_A

And, event which is failed by B is denoted by E_B

It is given in the question that,

Event failed by A, P (E_A) = 0.2

Event failed by both, $P\ (E_A \cap E_B) = \ 0.15$

And, event failed by B alone $= \ P\ (E_B) - \ P\ (E_A \cap E_B)$

0.15 = P (E_B) – 0.15

∴ P (E_B) = 0.30

Hence, $P\ (E_A|E_B) = \frac{P\ (E_A \cap E_B)}{P\ (E_A)}$

$= \frac{0.15}{0.3}$

= 0.5

(ii) We have, probability where A fails alone $= P\ (E_A) - \ P\ (E_A \cap E_B)$

= 0.2 – 0.15
= 0.05

16. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.
Choose the correct answer in each of the following:

Solution:
Let us firstly assume, A_1 denote the events that a red ball is transferred from bag I to II
And, A_2 denote the event that a black ball is transferred from bag I to II
∴ P (A_1) = 3/7
And, P (A_2) = 4/7
Let X be the event that the drawn ball is red
∴ when red ball is transferred from bag I to II,

$$P(X|A_1) = \frac{5}{10}$$

$$= \frac{1}{2}$$

And, when black ball is transferred from bag I to II,

$$P(X|A_2) = \frac{4}{10}$$

$$= \frac{2}{5}$$

Hence, $P\ (A_2|X) = \frac{P(A_2)\ P(X|A_2)}{P(A_1)\ P(X|A_1)+P(A_2)\ P(X|A_2)}$

$$= \frac{\frac{4}{7}\times\frac{2}{5}}{\frac{3}{7}\times\frac{1}{2}+\frac{4}{7}\times\frac{2}{5}}$$

$$= \frac{16}{31}$$

17. If A and B are two events such that P(A) ≠ 0 and P(B | A) = 1, then
A. A ⊂ B

B. B ⊂ A
C. B = ϕ
D. A = ϕ

Solution:
A. A ⊂ B

Explanation:
It is given in the question that,

A and B are two events where,

$P\ (A) \neq 0$

And, P (B|A) = 1

$\therefore P\ (B|A) = \frac{P\ (B \cap A)}{P\ (A)}$

$1 = \frac{P\ (B\ \cap A)}{P\ (A)}$

$P\ (A) = \ P\ (B \cap A)$

$\therefore A\ \subset B$

Hence, option A is correct

18. If P (A|B) > P (A), then which of the following is correct:
A. P (B|A) < P (B)
B. P (A ∩ B) < P (A) . P (B)
C. P (B|A) > P (B)
D. P (B|A) = P (B)

Solution:
C. P (B|A) > P (B)

Explanation:

Given that,

P (A|B) > P (A)

$\therefore \frac{P\ (A \cap B)}{P\ (B)} > P\ (A)$

$P\ (A\ \cap B) > \ P\ (A).P\ (B)$

$\frac{P\ (A\ \cap B)}{P\ (A)} > P\ (B)$

P (B|A) > P (B)

Hence, option C is correct

19. **If A and B are any two events such that P(A) + P(B) – P(A and B) = P(A), then**
A. P(B|A) = 1
B. P(A|B) = 1
C. P(B|A) = 0
D. P(A|B) = 0

Solution:
B. P (A|B) = 1
Explanation:
Given that,

A and B are any two events where,

P (A) + P (B) – P (A and B) = P (A)

$P\ (A) + \ P\ (B) - \ P\ (A\ \cap B) = P\ (A)$

$P\ (B) - \ P\ (A \cap B) = \ 0$

$P\ (A \cap B) = \ P\ (B)$

$\therefore P(A|B) = \frac{P\ (A \cap B)}{P\ (B)}$

= 1

Hence, option B is correct