

EXERCISE 1.3

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1. Make correct statements by filling in the symbols \subset or $\not\subset$ in the blank spaces:

- (i) $\{2, 3, 4\} \dots \{1, 2, 3, 4, 5\}$
- (ii) $\{a, b, c\} \dots \{b, c, d\}$
- (iii) $\{x: x \text{ is a student of Class XI of your school}\} \dots \{x: x \text{ student of your school}\}$
- (iv) $\{x: x \text{ is a circle in the plane}\} \dots \{x: x \text{ is a circle in the same plane with radius 1 unit}\}$
- (v) $\{x: x \text{ is a triangle in a plane}\} \dots \{x: x \text{ is a rectangle in the plane}\}$
- (vi) $\{x: x \text{ is an equilateral triangle in a plane}\} \dots \{x: x \text{ is a triangle in the same plane}\}$
- (vii) $\{x: x \text{ is an even natural number}\} \dots \{x: x \text{ is an integer}\}$

Solution:

- (i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$
- (ii) $\{a, b, c\} \not\subset \{b, c, d\}$
- (iii) $\{x: x \text{ is a student of Class XI of your school}\} \subset \{x: x \text{ student of your school}\}$
- (iv) $\{x: x \text{ is a circle in the plane}\} \not\subset \{x: x \text{ is a circle in the same plane with radius 1 unit}\}$
- (v) $\{x: x \text{ is a triangle in a plane}\} \not\subset \{x: x \text{ is a rectangle in the plane}\}$
- (vi) $\{x: x \text{ is an equilateral triangle in a plane}\} \subset \{x: x \text{ is a triangle in the same plane}\}$
- (vii) $\{x: x \text{ is an even natural number}\} \subset \{x: x \text{ is an integer}\}$

2. Examine whether the following statements are true or false:

- (i) $\{a, b\} \not\subset \{b, c, a\}$
- (ii) $\{a, e\} \subset \{x: x \text{ is a vowel in the English alphabet}\}$
- (iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$
- (iv) $\{a\} \subset \{a, b, c\}$
- (v) $\{a\} \in (a, b, c)$
- (vi) $\{x: x \text{ is an even natural number less than 6}\} \subset \{x: x \text{ is a natural number which divides 36}\}$

Solution:

(i) False.

Here each element of $\{a, b\}$ is an element of $\{b, c, a\}$.

(ii) True.

We know that a, e are two vowels of the English alphabet.

(iii) False.

$2 \in \{1, 2, 3\}$ where, $2 \notin \{1, 3, 5\}$

(iv) True.

Each element of $\{a\}$ is also an element of $\{a, b, c\}$.

(v) False.

Elements of $\{a, b, c\}$ are a, b, c . Hence, $\{a\} \subset \{a, b, c\}$

(vi) True.

$\{x: x \text{ is an even natural number less than } 6\} = \{2, 4\}$

$\{x: x \text{ is a natural number which divides } 36\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\} \in A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\Phi \in A$

(x) $\Phi \subset A$

(xi) $\{\Phi\} \subset A$

Solution:

It is given that $A = \{1, 2, \{3, 4\}, 5\}$

(i) $\{3, 4\} \subset A$ is incorrect

Here $3 \in \{3, 4\}$; where, $3 \notin A$.

(ii) $\{3, 4\} \in A$ is correct

$\{3, 4\}$ is an element of A .

(iii) $\{\{3, 4\}\} \subset A$ is correct

$\{3, 4\} \in \{\{3, 4\}\}$ and $\{3, 4\} \in A$.

(iv) $1 \in A$ is correct

1 is an element of A .

(v) $1 \subset A$ is incorrect

An element of a set can never be a subset of itself.

(vi) $\{1, 2, 5\} \subset A$ is correct

Each element of $\{1, 2, 5\}$ is also an element of A .

(vii) $\{1, 2, 5\} \in A$ is incorrect
 $\{1, 2, 5\}$ is not an element of A .

(viii) $\{1, 2, 3\} \subset A$ is incorrect
 $3 \in \{1, 2, 3\}$; where, $3 \notin A$.

(ix) $\Phi \in A$ is incorrect
 Φ is not an element of A .

(x) $\Phi \subset A$ is correct
 Φ is a subset of every set.

(xi) $\{\Phi\} \subset A$ is incorrect
 $\Phi \in \{\Phi\}$; where, $\Phi \in A$.

4. Write down all the subsets of the following sets:

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) $\{1, 2, 3\}$

(iv) Φ

Solution:

(i) Subsets of $\{a\}$ are
 Φ and $\{a\}$.

(ii) Subsets of $\{a, b\}$ are
 Φ , $\{a\}$, $\{b\}$, and $\{a, b\}$.

(iii) Subsets of $\{1, 2, 3\}$ are
 Φ , $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$, and $\{1, 2, 3\}$.

(iv) Only subset of Φ is Φ .

5. How many elements has $P(A)$, if $A = \Phi$?

Solution:

If A is a set with m elements

$$n(A) = m \text{ then } n[P(A)] = 2^m$$

If $A = \Phi$ we get $n(A) = 0$

$$n[P(A)] = 2^0 = 1$$

Therefore, $P(A)$ has one element.

6. Write the following as intervals:

(i) $\{x: x \in \mathbf{R}, -4 < x \leq 6\}$

(ii) $\{x: x \in \mathbf{R}, -12 < x < -10\}$

(iii) $\{x: x \in \mathbf{R}, 0 \leq x < 7\}$

(iv) $\{x: x \in \mathbf{R}, 3 \leq x \leq 4\}$

Solution:

(i) $\{x: x \in \mathbf{R}, -4 < x \leq 6\} = (-4, 6]$

(ii) $\{x: x \in \mathbf{R}, -12 < x < -10\} = (-12, -10)$

(iii) $\{x: x \in \mathbf{R}, 0 \leq x < 7\} = [0, 7)$

(iv) $\{x: x \in \mathbf{R}, 3 \leq x \leq 4\} = [3, 4]$

7. Write the following intervals in set-builder form:

(i) $(-3, 0)$

(ii) $[6, 12]$

(iii) $(6, 12]$

(iv) $[-23, 5)$

Solution:

(i) $(-3, 0) = \{x: x \in \mathbf{R}, -3 < x < 0\}$

(ii) $[6, 12] = \{x: x \in \mathbf{R}, 6 \leq x \leq 12\}$

(iii) $(6, 12] = \{x: x \in \mathbf{R}, 6 < x \leq 12\}$

(iv) $[-23, 5) = \{x: x \in \mathbf{R}, -23 \leq x < 5\}$

8. What universal set (s) would you propose for each of the following:

(i) The set of right triangles

(ii) The set of isosceles triangles

Solution:

(i) Among the set of right triangles, the universal set is the set of triangles or the set of polygons.

(ii) Among the set of isosceles triangles, the universal set is the set of triangles or the set of polygons or the set of two-dimensional figures.

9. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C

(i) $\{0, 1, 2, 3, 4, 5, 6\}$

(ii) Φ

(iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Solution:

(i) We know that $A \subset \{0, 1, 2, 3, 4, 5, 6\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6\}$

So $C \not\subset \{0, 1, 2, 3, 4, 5, 6\}$

Hence, the set $\{0, 1, 2, 3, 4, 5, 6\}$ cannot be the universal set for the sets A, B, and C.

(ii) $A \not\subset \Phi, B \not\subset \Phi, C \not\subset \Phi$

Hence, Φ cannot be the universal set for the sets A, B, and C.

(iii) $A \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$C \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Hence, the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the universal set for the sets A, B, and C.

(iv) $A \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

So $C \not\subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

Hence, the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ cannot be the universal set for the sets A, B, and C.