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1. If X and Y are two sets such that *n*(X) = 17, *n*(Y) = 23 and *n*(X ∪ Y) = 38, find *n*(X ∩ Y).
Solution:

Given
n (X) = 17
n (Y) = 23
n (X U Y) = 38
We can write it as
n (X U Y) = n (X) + n (Y) – n (X ∩ Y)
Substituting the values
38 = 17 + 23 – n (X ∩ Y)
By further calculation
n (X ∩ Y) = 40 – 38 = 2
So we get
n (X ∩ Y) = 2

2. If X and Y are two sets such that X ∪Y has 18 elements, X has 8 elements and Y has 15 elements; how many elements does X ∩ Y have?
Solution:

Given
n (X U Y) = 18
n (X) = 8
n (Y) = 15
We can write it as
n (X U Y) = n (X) + n (Y) – n (X ∩ Y)
Substituting the values
18 = 8 + 15 - n (X ∩ Y)
By further calculation
n (X ∩ Y) = 23 – 18 = 5
So we get
n (X ∩ Y) = 5

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?
Solution:

Consider H as the set of people who speak Hindi
E as the set of people who speak English
We know that
n(H ∪ E) = 400

$n(H) = 250$
$n(E) = 200$
It can be written as
$n(H \cup E) = n(H) + n(E) – n(H \cap E)$
By substituting the values
$400 = 250 + 200 – n(H \cap E)$
By further calculation
$400 = 450 – n(H \cap E)$
So we get
$n(H \cap E) = 450 – 400$
$n(H \cap E) = 50$

Therefore, 50 people can speak both Hindi and English.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and S ∩ T has 11 elements, how many elements does S ∪ T have?
Solution:

We know that
$n(S) = 21$
$n(T) = 32$
$n(S \cap T) = 11$
It can be written as
$n (S \cup T) = n (S) + n (T) – n (S \cap T)$
Substituting the values
$n (S \cup T) = 21 + 32 – 11$
So we get
$n (S \cup T)= 42$

Therefore, the set (S ∪ T) has 42 elements.

5. If X and Y are two sets such that X has 40 elements, X ∪Y has 60 elements and X ∩Y has 10 elements, how many elements does Y have?
Solution:

We know that
$n(X) = 40$
$n(X \cup Y) = 60$
$n(X \cap Y) = 10$
It can be written as
$n(X \cup Y) = n(X) + n(Y) – n(X \cap Y)$
By substituting the values

$60 = 40 + n(Y) – 10$
On further calculation
$n(Y) = 60 – (40 – 10) = 30$

Therefore, the set Y has 30 elements.

6. In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?
Solution:

Consider C as the set of people who like coffee
T as the set of people who like tea
$n(C \cup T) = 70$
$n(C) = 37$
$n(T) = 52$
It is given that
$n(C \cup T) = n(C) + n(T) – n(C \cap T)$
Substituting the values
$70 = 37 + 52 – n(C \cap T)$
By further calculation
$70 = 89 – n(C \cap T)$
So we get
$n(C \cap T) = 89 – 70 = 19$

Therefore, 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?
Solution:

Consider C as the set of people who like cricket
T as the set of people who like tennis
$n(C \cup T) = 65$
$n(C) = 40$
$n(C \cap T) = 10$
It can be written as
$n(C \cup T) = n(C) + n(T) – n(C \cap T)$
Substituting the values
$65 = 40 + n(T) – 10$
By further calculation
$65 = 30 + n(T)$
So we get

n(T) = 65 – 30 = 35

Hence, 35 people like tennis.

We know that,
(T – C) ∪ (T ∩ C) = T
So we get,
(T – C) ∩ (T ∩ C) = Φ
Here
n (T) = n (T – C) + n (T ∩ C)
Substituting the values
35 = n (T – C) + 10
By further calculation
n (T – C) = 35 – 10 = 25

Therefore, 25 people like only tennis.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?
Solution:

Consider F as the set of people in the committee who speak French
S as the set of people in the committee who speak Spanish
n(F) = 50
n(S) = 20
n(S ∩ F) = 10
It can be written as
n(S ∪ F) = n(S) + n(F) – n(S ∩ F)
By substituting the values
n(S ∪ F) = 20 + 50 – 10
By further calculation
n(S ∪ F) = 70 – 10
n(S ∪ F) = 60

Therefore, 60 people in the committee speak at least one of the two languages.