MISCELLANEOUS EXERCISE PAGE: 26

1. Decide, among the following sets, which sets are subsets of one and another:

A= {x: x ∈ R and x satisfy $x^2 - 8x + 12 = 0$},

B = {2, 4, 6},

C = {2, 4, 6, 8…},

D = {6}.

Solution:

According to the question,

We have,

A = {x: x ∈ R and x satisfies $x^2 - 8x + 12 = 0$}

2 and 6 are the only solutions of $x^2 - 8x + 12 = 0$.

Hence, A = {2, 6}

B = {2, 4, 6}, C = {2, 4, 6, 8 …}, D = {6}

Hence, D ⊂ A ⊂ B ⊂ C

Hence, A ⊂ B, A ⊂ C, B ⊂ C, D ⊂ A, D ⊂ B, D ⊂ C

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If x ∈ A and A ∈ B, then x ∈ B

(ii) If A ⊂ B and B ∈ C, then A ∈ C

(iii) If A ⊂ B and B ⊂ C, then A ⊂ C

(iv) If A ⊄ B and B ⊄ C, then A ⊄ C

(v) If x ∈ A and A ⊄ B, then x ∈ B

(vi) If A ⊂ B and x ∉ B, then x ∉ A

Solution:

(i) False

According to the question,

A = {1, 2} and B = {1, {1, 2}, {3}}

Now, we have,

2 ∈ {1, 2} and {1, 2} ∈ {1, {1, 2}, {3}}

Hence, we get,

A ∈ B

We also know,

{2} ∉ {1, {1, 2}, {3}}

(ii) False

According to the question

Let us assume that,

A {2}
B = {0, 2}
And, C = {1, {0, 2}, 3}
From the question,
$A \subset B$
Hence,
$B \in C$
But, we know,
$A \notin C$

(iii) True
According to the question
$A \subset B$ and $B \subset C$
Let us assume that,
$x \in A$
Then, we have,
$x \in B$
And,
$x \in C$
Therefore,
$A \subset C$

(iv) False
According to the question
$A \not\subset B$
Also,
$B \not\subset C$
Let us assume that,
A = {1, 2}
B = {0, 6, 8}
And,
C = {0, 1, 2, 6, 9}
$\therefore A \subset C$

(v) False
According to the question,
$x \in A$
Also,
$A \not\subset B$

Let us assume that,
$A = \{3, 5, 7\}$
Also,
$B = \{3, 4, 6\}$
We know that,
$A \not\subset B$
$\therefore 5 \notin B$

(vi) True
According to the question,
$A \subset B$
Also,
$x \notin B$
Let us assume that,
$x \in A$,
We have,
$x \in B$,
From the question,
We have, $x \notin B$
$\therefore x \notin A$

3. Let A, B and C be the sets such that A ∪ B = A ∪ C and A ∩ B = A ∩ C. show that B = C.
Solution:

According to the question,
$A \cup B = A \cup C$
And,
$A \cap B = A \cap C$
To show,
$B = C$
Let us assume,
$x \in B$
So,
$x \in A \cup B$
$x \in A \cup C$
Hence,
$x \in A$ or $x \in C$
When $x \in A$, then,
$x \in B$
$\therefore x \in A \cap B$

As, $A \cap B = A \cap C$
So, $x \in A \cap C$
$\therefore x \in A$ or $x \in C$
$x \in C$
$\therefore B \subset C$
Similarly, it can be shown that $C \subset B$
Hence, $B = C$

4. Show that the following four conditions are equivalent:
(i) $A \subset B$ (ii) $A - B = \Phi$
(iii) $A \cup B = B$ (iv) $A \cap B = A$
Solution:

According to the question,
To prove, (i) ↔ (ii)
Here, (i) = $A \subset B$ and (ii) = $A - B \neq \phi$
Let us assume that $A \subset B$
To prove, $A - B \neq \phi$
Let $A - B \neq \phi$
Hence, there exists $X \in A$, $X \neq B$, but since $A \subset B$, it is not possible
$\therefore A - B = \phi$
And $A \subset B \Rightarrow A - B \neq \phi$
Let us assume that $A - B \neq \phi$
To prove: $A \subset B$
Let $X \in A$
So, $X \in B$ (if $X \notin B$, then $A - B \neq \phi$)
Hence, $A - B = \phi \Rightarrow A \subset B$
$\therefore$(i) ↔ (ii)
Let us assume that $A \subset B$
To prove, $A \cup B = B$
$\Rightarrow B \subset A \cup B$
Let us assume that, $x \in A \cup B$
$\Rightarrow X \in A$ or $X \in B$
Taking Case I: $X \in B$
$A \cup B = B$
Taking Case II: $X \in A$
$\Rightarrow X \in B$ ($A \subset B$)
$\Rightarrow A \cup B \subset B$
Let $A \cup B = B$
Let us assume that $X \in A$

⇒ $X \in A \cup B$ ($A \subset A \cup B$)

⇒ $X \in B$ ($A \cup B = B$)

∴$A \subset B$

Hence, (i) ↔ (iii)

To prove (i) ↔ (iv)

Let us assume that $A \subset B$

$A \cap B \subset A$

Let $X \in A$

To prove, $X \in A \cap B$

Since, $A \subset B$ and $X \in B$

Hence, $X \in A \cap B$

⇒ $A \subset A \cap B$

⇒ $A = A \cap B$

Let us assume that $A \cap B = A$

Let $X \in A$

⇒ $X \in A \cap B$

⇒ $X \in B$ and $X \in A$

⇒ $A \subset B$

∴ (i) ↔ (iv)

∴ (i) ↔ (ii) ↔ (iii) ↔ (iv)

Hence, proved

5. Show that if $A \subset B$, then $C – B \subset C – A$.

Solution:

To show,

$C – B \subset C – A$

According to the question,

Let us assume that x is any element such that $X \in C – B$

∴ $x \in C$ and $x \notin B$

Since, $A \subset B$, we have,

∴ $x \in C$ and $x \notin A$

So, $x \in C – A$

∴ $C – B \subset C – A$

Hence, Proved.

6. Assume that P (A) = P (B). Show that A = B

Solution:

To show,

$A = B$

According to the question,

P (A) = P (B)

Let x be any element of set A,

$x \in A$

Since, P (A) is the power set of set A, it has all the subsets of set A.

$A \in P(A) = P(B)$

Let C be an element of set B

For any $C \in P(B)$,

We have, $x \in C$

$C \subset B$

$\therefore x \in B$

$\therefore A \subset B$

Similarly, we have:

$B \subset A$

SO, we get,

If $A \subset B$ and $B \subset A$

$\therefore A = B$

7. Is it true that for any sets A and B, P (A) ∪ P (B) = P (A ∪ B)? Justify your answer.

Solution:

It is not true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$

Justification:

Let us assume,

$A = \{0, 1\}$

And, $B = \{1, 2\}$

$\therefore A \cup B = \{0, 1, 2\}$

According to the question,

We have,

$P(A) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$

$P(B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$\therefore P(A \cup B) = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$

Also,

$P(A) \cup P(B) = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}\}$

$\therefore P(A) \cup P(B \neq P(A \cup B)$

Hence, the given statement is false

8. Show that for any sets A and B,
A = (A ∩ B) ∪ (A – B) and A ∪ (B – A) = (A ∪ B)

Solution:

To Prove,
$A = (A \cap B) \cup (A - B)$
Proof: Let $x \in A$
To show,
$X \in (A \cap B) \cup (A - B)$
In Case I,
$X \in (A \cap B)$
$\Rightarrow X \in (A \cap B) \subset (A \cup B) \cup (A - B)$
In Case II,
$X \notin A \cap B$
$\Rightarrow X \notin B$ or $X \notin A$
$\Rightarrow X \notin B$ $(X \notin A)$
$\Rightarrow X \notin A - B \subset (A \cup B) \cup (A - B)$
$\therefore A \subset (A \cap B) \cup (A - B)$ (i)
It can be concluded that, $A \cap B \subset A$ and $(A - B) \subset A$
Thus, $(A \cap B) \cup (A - B) \subset A$ (ii)
Equating (i) and (ii),
$A = (A \cap B) \cup (A - B)$
We also have to show,
$A \cup (B - A) \subset A \cup B$
Let us assume,
$X \in A \cup (B - A)$
$X \in A$ or $X \in (B - A)$
$\Rightarrow X \in A$ or ($X \in B$ and $X \notin A$)
$\Rightarrow$ ($X \in A$ or $X \in B$) and ($X \in A$ and $X \notin A$)
$\Rightarrow X \in (B \cup A)$
$\therefore A \cup (B - A) \subset (A \cup B)$ (iii)
According to the question,
To prove:
$(A \cup B) \subset A \cup (B - A)$
Let $y \in A \cup B$
$Y \in A$ or $y \in B$
($y \in A$ or $y \in B$) and ($X \in A$ and $X \notin A$)
$\Rightarrow y \in A$ or ($y \in B$ and $y \notin A$)
$\Rightarrow y \in A \cup (B - A)$
Thus, $A \cup B \subset A \cup (B - A)$ (iv)
$\therefore$From equations (iii) and (iv), we get:
$A \cup (B - A) = A \cup B$

9. Using properties of sets, show that:
(i) $A \cup (A \cap B) = A$
(ii) $A \cap (A \cup B) = A$.
Solution:

(i) To show: $A \cup (A \cap B) = A$
We know that,
$A \subset A$
$A \cap B \subset A$

$\therefore A \cup (A \cap B) \subset A$ (i)

Also, according to the question,
We have:
$A \subset A \cup (A \cap B)$ (ii)
Hence, from equation (i) and (ii)
We have:
$A \cup (A \cap B) = A$

(ii) To show,
$A \cap (A \cup B) = A$
$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$
$= A \cup (A \cap B)$
$= A$

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.
Solution:

Let us assume,
$A = \{0, 1\}$
$B = \{0, 2, 3\}$
And, $C = \{0, 4, 5\}$
According to the question,
$A \cap B = \{0\}$
And,
$A \cap C = \{0\}$
$\therefore A \cap B = A \cap C = \{0\}$
But,
$2 \in B$ and $2 \notin C$
Therefore, $B \neq C$

11. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that A = B. (Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law)
Solution:

According to the question,
Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X.
To show, A = B
Proof:
$A = A \cap (A \cup X) = A \cap (B \cup X)$ $[A \cup X = B \cup X]$
$= (A \cap B) \cup (A \cap X)$ [Distributive law]
$= (A \cap B) \cup \Phi$ $[A \cap X = \Phi]$
$= A \cap B$ (i)
Now, $B = B \cap (B \cup X)$
$= B \cap (A \cup X)$ $[A \cup X = B \cup X]$
$= (B \cap A) \cup (B \cap X)$ … [Distributive law]
$= (B \cap A) \cup \Phi$ $[B \cap X = \Phi]$
$= A \cap B$ (i)
Hence, from equations (i) and (ii), we obtain A = B.

12. Find sets A, B and C such that A ∩ B, B ∩ C and A ∩ C are non-empty sets and A ∩ B ∩ C = Φ.

Solution:

Let us assume, A {0, 1}
B = {1, 2}
And, C = {2, 0}

According to the question,
A ∩ B = {1}
B ∩ C = {2}
And,
A ∩ C = {0}
∴ A ∩ B, B ∩ C and A ∩ C are not empty sets
Hence, we get,
A ∩ B ∩ C = Φ

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Solution:

Let us assume that,
U = the set of all students who took part in the survey
T = the set of students taking tea
C = the set of the students taking coffee
Total number of students in a school, n (U) = 600
Number of students taking tea, n (T) = 150
Number of students taking coffee, n (C) = 225
Also, n (T ∩ C) = 100
Now, we have to find that number of students taking neither coffee nor tea i.e. n (T ∩ C')

∴ According to the question,
n (T ∩ C')= n(T ∩ C)'
= n (U) – n (T ∩ C)
= n (U) – [n (T) + n(C) – n (T ∩ C)]
= 600 – [150 + 225 – 100]
= 600 – 275
= 325
∴ Number of students taking neither coffee nor tea = 325 students

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Solution:

Let us assume that,
U = the set of all students in the group
E = the set of students who know English
H = the set of the students who know Hindi
∴ H ∪ E = U
Given that,
Number of students who know Hindi n (H) = 100
Number of students who knew English, n (E) = 50
Number of students who know both, n (H ∩ E) = 25
We have to find the total number of students in the group i.e. n (U)

∴ According to the question,
n (U) = n(H) + n(E) – n(H ∩ E)
= 100 + 50 – 25
= 125
∴ Total number of students in the group = 125 students

15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) The number of people who read at least one of the newspapers.
(ii) The number of people who read exactly one newspaper.
Solution:

(i) Let us assume that,
A = the set of people who read newspaper H
B = the set of people who read newspaper T
C = the set of people who read newspaper I

According to the question,
Number of people who read newspaper H, n (A) = 25
Number of people who read newspaper T, n (B) = 26
Number of people who read the newspaper I, n (C) = 26
Number of people who read both newspaper H and I, n (A ∩ C) = 9
Number of people who read both newspaper H and T, n (A ∩ B) = 11
Number of people who read both newspaper T and I, n (B ∩ C) = 8
And, Number of people who read all three newspaper H, T and I, n (A ∩ B ∩ C) = 3
Now, we have to find the number of people who read at least one of the newspaper
∴, we get.

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

= 25 + 26 + 26 – 11 – 8 – 9 + 3
= 80 – 28
= 52
∴ There are a total of 52 students who read at least one newspaper.

(ii) Let us assume that,
a = the number of people who read newspapers H and T only
b = the number of people who read newspapers I and H only
c = the number of people who read newspapers T and I only
d = the number of people who read all three newspapers

According to the question,
D = n(A ∩ B ∩ C) = 3
Now, we have:
n(A ∩ B) = a + d
n(B ∩ C) = c + d
And,
n(C ∩ A) = b + d
∴ a + d + c +d + b + d = 11 + 8 + 9

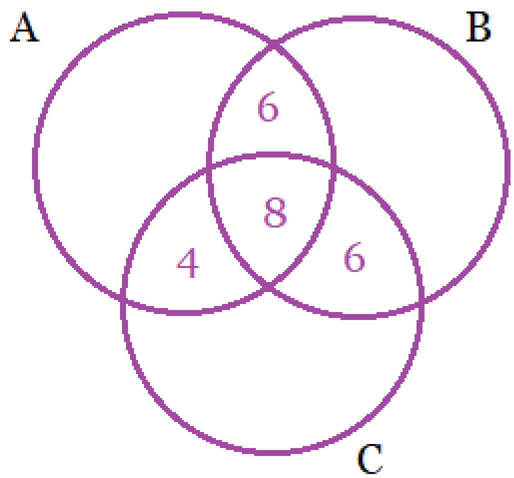
a + b + c + d = 28 – 2d
= 28 – 6
= 22
∴ Number of people read exactly one newspaper = 52 – 22
= 30 people

16. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Solution:

Let A, B and C = the set of people who like product A, product B and product C respectively.

Now, according to the question,
Number of students who like product A, n (A) = 21
Number of students who like product B, n (B) = 26
Number of students who like product C, n (C) = 29
Number of students who like both products A and B, n (A ∩ B) = 14
Number of students who like both products A and C, n(C ∩ A) = 12
Number of students who like both product C and B, n (B ∩ C) = 14
Number of students who like all three product, n (A ∩ B ∩ C) = 8



From the Venn diagram, we get,
Number of students who only like product C = {29 – (4 + 8 + 6)}
= {29 – 18}
= 11 students