EXERCISE 11.1 PAGE NO: 241

In each of the following Exercise 1 to 5, find the equation of the circle with

1. Centre (0, 2) and radius 2

Solution:

Given:

Centre (0, 2) and radius 2

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (0, 2) and radius (r) = 2

The equation of the circle is

$(x - 0)^2 + (y - 2)^2 = 2^2$

$x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

∴ The equation of the circle is $x^2 + y^2 - 4y = 0$

2. Centre (–2, 3) and radius 4

Solution:

Given:

Centre (-2, 3) and radius 4

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (-2, 3) and radius (r) = 4

The equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

$x^2 + y^2 + 4x - 6y - 3 = 0$

∴ The equation of the circle is $x^2 + y^2 + 4x - 6y - 3 = 0$

3. Centre (1/2, 1/4) and radius (1/12)

Solution:

Given:

Centre (1/2, 1/4) and radius 1/12

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (1/2, 1/4) and radius (r) = 1/12

The equation of the circle is

$(x – 1/2)^2 + (y – 1/4)^2 = (1/12)^2$

$x^2 – x + ¼ + y^2 – y/2 + 1/16 = 1/144$

$x^2 – x + ¼ + y^2 – y/2 + 1/16 = 1/144$
$144x^2 - 144x + 36 + 144y^2 - 72y + 9 – 1 = 0$
$144x^2 - 144x + 144y^2 – 72y + 44 = 0$
$36x^2 + 36x + 36y^2 – 18y + 11 = 0$
$36x^2 + 36y^2 - 36x – 18y + 11= 0$
∴ The equation of the circle is $36x^2 + 36y^2 - 36x – 18y + 11= 0$

4. Centre (1, 1) and radius √2
Solution:
Given:
Centre (1, 1) and radius √2
Let us consider the equation of a circle with centre (h, k) and
Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$
So, centre (h, k) = (1, 1) and radius (r) = √2
The equation of the circle is
$(x-1)^2 + (y-1)^2 = (\sqrt{2})^2$
$x^2 – 2x + 1 + y^2 -2y + 1 = 2$
$x^2 + y^2 - 2x -2y = 0$
∴ The equation of the circle is $x^2 + y^2 - 2x -2y = 0$

5. Centre (–a, –b) and radius $\sqrt{(a^2 – b^2)}$
Solution:
Given:
Centre (-a, -b) and radius $\sqrt{(a^2 – b^2)}$
Let us consider the equation of a circle with centre (h, k) and
Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$
So, centre (h, k) = (-a, -b) and radius (r) = $\sqrt{(a^2 – b^2)}$
The equation of the circle is
$(x + a)^2 + (y + b)^2 = (\sqrt{(a^2 – b^2)})^2$
$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 – b^2$
$x^2 + y^2 +2ax + 2by + 2b^2 = 0$
∴ The equation of the circle is $x^2 + y^2 +2ax + 2by + 2b^2 = 0$

In each of the following Exercise 6 to 9, find the centre and radius of the circles.

6. $(x + 5)^2 + (y – 3)^2 = 36$
Solution:
Given:
The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$

$(x - (-5))^2 + (y - 3)^2 = 6^2$ [which is of the form $(x - h)^2 + (y - k)^2 = r^2$]
Where, h = -5, k = 3 and r = 6
∴ The centre of the given circle is (-5, 3) and its radius is 6.

7. $x^2 + y^2 - 4x - 8y - 45 = 0$
Solution:
Given:
The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.
$x^2 + y^2 - 4x - 8y - 45 = 0$
$(x^2 - 4x) + (y^2 - 8y) = 45$
$(x^2 - 2(x)(2) + 2^2) + (y^2 - 2(y)(4) + 4^2) - 4 - 16 = 45$
$(x - 2)^2 + (y - 4)^2 = 65$
$(x - 2)^2 + (y - 4)^2 = (\sqrt{65})^2$ [which is form $(x-h)^2 + (y-k)^2 = r^2$]
Where h = 2, K = 4 and r = $\sqrt{65}$
∴ The centre of the given circle is (2, 4) and its radius is $\sqrt{65}$.

8. $x^2 + y^2 - 8x + 10y - 12 = 0$
Solution:
Given:
The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.
$x^2 + y^2 - 8x + 10y - 12 = 0$
$(x^2 - 8x) + (y^2 + 10y) = 12$
$(x^2 - 2(x)(4) + 4^2) + (y^2 - 2(y)(5) + 5^2) - 16 - 25 = 12$
$(x - 4)^2 + (y + 5)^2 = 53$
$(x - 4)^2 + (y - (-5))^2 = (\sqrt{53})^2$ [which is form $(x-h)^2 + (y-k)^2 = r^2$]
Where h = 4, K= -5 and r = $\sqrt{53}$
∴ The centre of the given circle is (4, -5) and its radius is $\sqrt{53}$.

9. $2x^2 + 2y^2 - x = 0$
Solution:
The equation of the given of the circle is $2x^2 + 2y^2 - x = 0$.
$2x^2 + 2y^2 - x = 0$
$(2x^2 + x) + 2y^2 = 0$
$(x^2 - 2(x)(1/4) + (1/4)^2) + y^2 - (1/4)^2 = 0$
$(x - 1/4)^2 + (y - 0)^2 = (1/4)^2$ [which is form $(x-h)^2 + (y-k)^2 = r^2$]
Where, h = ¼, K = 0, and r = ¼
∴ The center of the given circle is (1/4, 0) and its radius is 1/4.

10. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line 4x + y = 16.

Solution:

Let us consider the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$

We know that the circle passes through points (4,1) and (6,5)

So,

$(4 – h)^2 + (1 – k)^2 = r^2$ ....(1)

$(6 – h)^2 + (5 – k)^2 = r^2$ ....(2)

Since, the centre (h, k) of the circle lies on line 4x + y = 16,

$4h + k = 16$..................... (3)

From the equation (1) and (2), we obtain

$(4 – h)^2 + (1 – k)^2 = (6 – h)^2 + (5 – k)^2$

$16 – 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 15 – 10k + k^2$

$16 – 8h + 1 - 2k + 12h - 25 - 10k$

$4h + 8k = 44$

$h + 2k = 11$................ (4)

On solving equations (3) and (4), we obtain h=3 and k= 4.

On substituting the values of h and k in equation (1), we obtain

$(4 – 3)^2 + (1 – 4)^2 = r^2$

$(1)^2 + (-3)^2 = r^2$

$1+9 = r^2$

$r = \sqrt{10}$

so now, $(x – 3)^2 + (y – 4)^2 = (\sqrt{10})^2$

$x^2 - 6x + 9 + y^2 – 8y + 16 = 10$

$x^2 + y^2 - 6x – 8y + 15 = 0$

∴ The equation of the required circle is $x^2 + y^2 - 6x – 8y + 15 = 0$

11. Find the equation of the circle passing through the points (2, 3) and (–1, 1) and whose centre is on the line x – 3y – 11 = 0.

Solution:

Let us consider the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$

We know that the circle passes through points (2,3) and (-1,1).

$(2 – h)^2 + (3 – k)^2 = r^2$ ....(1)

$(-1 – h)^2 + (1 – k)^2 = r^2$ ....(2)

Since, the centre (h, k) of the circle lies on line x - 3y - 11= 0,

$h - 3k = 11$..................... (3)

From the equation (1) and (2), we obtain

$(2 – h)^2 + (3 – k)^2 = (-1 – h)^2 + (1 – k)^2$

$4 – 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 – 2k + k^2$

$4 – 4h + 9 - 6k = 1 + 2h + 1 - 2k$
$6h + 4k = 11$................ (4)
Now let us multiply equation (3) by 6 and subtract it from equation (4) to get,
$6h + 4k - 6(h-3k) = 11 – 66$
$6h + 4k – 6h + 18k = 11 – 66$
$22k = -55$
$K = -5/2$
Substitute this value of K in equation (4) to get,
$6h + 4(-5/2) = 11$
$6h – 10 = 11$
$6h = 21$
$h = 21/6$
$h = 7/2$
We obtain $h = 7/2$and $k = -5/2$
On substituting the values of h and k in equation (1), we get
$(2 – 7/2)^2 + (3 + 5/2)^2 = r^2$
$[(4-7)/2]^2 + [(6+5)/2]^2 = r^2$
$(-3/2)^2 + (11/2)^2 = r^2$
$9/4 + 121/4 = r^2$
$130/4 = r^2$
The equation of the required circle is
$(x – 7/2)^2 + (y + 5/2)^2 = 130/4$
$[(2x-7)/2]^2 + [(2y+5)/2]^2 = 130/4$
$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$
$4x^2 + 4y^2 - 28x + 20y - 56 = 0$
$4(x^2 + y^2 - 7x + 5y – 14) = 0$
$x^2 + y^2 - 7x + 5y – 14 = 0$
∴ The equation of the required circle is $x^2 + y^2 - 7x + 5y – 14 = 0$

12. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).
Solution:
Let us consider the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$
We know that the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.
So now, the equation of the circle is $(x – h)^2 + y^2 = 25$.
It is given that the circle passes through the point (2, 3) so the point will satisfy the equation of the circle.
$(2 – h)^2 + 3^2 = 2$
$(2 – h)^2 = 25-9$

$(2 - h)^2 = 16$
$2 - h = \pm \sqrt{16} = \pm 4$
If $2-h = 4$, then $h = -2$
If $2-h = -4$, then $h = 6$
Then, when $h = -2$, the equation of the circle becomes
$(x - 2)^2 + y^2 = 25$
$x^2 -12x + 36 + y^2 = 25$
$x^2 + y^2 - 4x + 21 = 0$

When $h = 6$, the equation of the circle becomes
$(x - 6)^2 + y^2 = 25$
$x^2 -12x + 36 + y^2 = 25$
$x^2 + y^2 -12x + 11 = 0$
$\therefore$ The equation of the required circle is $x^2 + y^2 - 4x + 21 = 0$ and $x^2 + y^2 -12x + 11 = 0$

13. Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.
Solution:
Let us consider the equation of the required circle be $(x - h)^2+ (y - k)^2 =r^2$
We know that the circle passes through (0, 0),
So, $(0 - h)^2+ (0 - k)^2 = r^2$
$h^2 + k^2 = r^2$
Now, The equation of the circle is $(x - h)^2 + (y - k)^2 = h^2 + k^2$.
It is given that the circle intercepts a and b on the coordinate axes.
i.e., the circle passes through points (a, 0) and (0, b).
So, $(a - h)^2+ (0 - k)^2 =h^2 +k^2$.................(1)
$(0 - h)^2+ (b- k)^2 =h^2 +k^2$..................(2)
From equation (1), we obtain
$a^2 - 2ah + h^2 +k^2 = h^2 +k^2$
$a^2 - 2ah = 0$
$a(a - 2h) =0$
$a = 0$ or $(a -2h) = 0$
However, $a \neq 0$; hence, $(a -2h) = 0$
$h = a/2$

From equation (2), we obtain
$h^2 - 2bk + k^2 + b^2= h^2 +k^2$
$b^2 - 2bk = 0$

$b(b - 2k) = 0$
$b = 0$ or $(b-2k) = 0$
However, $a \neq 0$; hence, $(b - 2k) = 0$
$k = b/2$
So, the equation is
$(x - a/2)^2 + (y - b/4)^2 = (a/2)^2 + (b/2)^2$
$[(2x-a)/2]^2 + [(2y+b)/2]^2 = (a^2 + b^2)/4$
$4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 = a^2 + b^2$
$4x^2 + 4y^2 - 4ax - 4by = 0$
$4(x^2 + y^2 - 7x + 5y - 14) = 0$
$x^2 + y^2 - ax - by = 0$
∴ The equation of the required circle is $x^2 + y^2 - ax - by = 0$

14. Find the equation of a circle with centre (2,2) and passes through the point (4,5).
Solution:
Given:
The centre of the circle is given as (h, k) = (2,2)
We know that the circle passes through point (4,5), the radius (r) of the circle is the distance between the points (2,2) and (4,5).
$r = \sqrt{[(2-4)^2 + (2-5)^2]}$
$= \sqrt{[(-2)^2 + (-3)^2]}$
$= \sqrt{[4+9]}$
$= \sqrt{13}$
The equation of the circle is given as
$(x - h)^2 + (y - k)^2 = r^2$
$(x - h)^2 + (y - k)^2 = (\sqrt{13})^2$
$(x - 2)^2 + (y - 2)^2 = (\sqrt{13})^2$
$x^2 - 4x + 4 + y^2 - 4y + 4 = 13$
$x^2 + y^2 - 4x - 4y = 5$
∴ The equation of the required circle is $x^2 + y^2 - 4x - 4y = 5$

15. Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?
Solution:
Given:
The equation of the given circle is $x^2 + y^2 = 25$.
$x^2 + y^2 = 25$
$(x - 0)^2 + (y - 0)^2 = 5^2$ [which is of the form $(x - h)^2 + (y - k)^2 = r^2$]
Where, h = 0, k = 0 and r = 5.
So the distance between point (-2.5, 3.5) and the centre (0,0) is

$\sqrt{[(-2.5 - 0)^2 + (-3.5 - 0)^2]}$
$\sqrt{(6.25 + 12.25)}$
$\sqrt{18.5}$
4.3 [which is < 5]
Since, the distance between point (-2.5, -3.5) and the centre (0, 0) of the circle is less than the radius of the circle, point (-2.5, -3.5) lies inside the circle.